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исследований
дубна

E2-83-253

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STABILITY PROPERTIES OF SOLUTIONS TO NONLINEAR MODELS POSSESSING A SIGN-UNDEFINED METRIC

Submitted to "Acta Physica Austriaca"

1983

1. INTRODUCTION

In the last decade there was a rapidly growing interest to the nonlinear systems with noncompact global invariance groups. Noncompactness presents a number of attractive opportunities, for instance, extension of the set of permissible boundary conditions and consequently, spectrum of solutions /1/. Multicomponent magnetic systems /2/, nonlinear optics /3/, stationary axisymmetric gravitation /4/, supersymmetry /5/, and extended supergravity /6/ is the list of fields, though rather incomplete one, where noncompact models find their active application.

As in any nonlinear quantum theory, stability of the relevant classical solutions plays an important role for the class of models under consideration. In this note we analyze stability properties for systems of more general type, namely, for systems possessing sign-undefined metric of isotopic space (speaking otherwise, for those with sign-undefined kinetic term). The two models considered here are Lorentz-and Gallilean-invariant, respectively.

A traditional way of stability investigation consists in consideration of the appropriate energy functional in the vicinity of the given solution. As stated by Dirichlet's theorem, the stability is immediate if the solution appears to realize a local energy minimum. On the other hand, minimality is only a sufficient but by no means necessary condition for stability. The problem of inversion of Dirichlet's theorem is not solved completely even in the case of systems with finite degrees of freedom /7/. There are several well-known examples from classical mechanics which in spite of the absence of the energy minimum demonstrate some other stabilization mechanism, for instance, the gyroscopical one.

It turns out that similar situation exists in the theory of sign-undefined metric models. In the present note we show that an arbitrary solution of the corresponding evolution equations is not a local minimum of the energy. Moreover, it appears impossible to minimize the energy even conditionally, i.e., by imposing any number of physically reasonable integral constraints on trial perturbations. Dirichlet's theorem is, therefore, inapplicable. However, examining the equations of motion linearized with respect to the small fluctuation, we find that models from the said class can possess stable solutions. In the case of systems with the indefinite kinetic term, the stability criterion, ensuing from that examination, differs essentially from the minimality condition whereas for positive metrics /8/ they coincide (in the absence of velocity-dependent, gyroscopic forces).

In conclusion we remark that the above assertions on the nonexistence of the minima do not refer to systems with non-linearly realized internal symmetry group (σ-models, for instance). In the latter case the difficulty may be avoided by the construction of field-dependent positive metrics /6/.

2. CRITICAL POINTS OF ENERGY

We study two multicomponent field models in D spatial dimensions. The first one is Lorentz-invariant with lagrangian

$$\mathcal{L} = \int d^D x \{\phi_t^+ \gamma_0 \phi_t - \bar{\nabla}\phi^+ \gamma_0 \bar{\nabla}\phi - U(\phi^+, \phi)\} \qquad (1L)$$

and the second is Gallilean-invariant with lagrangian

$$\mathcal{L} = \int d^D x \{\frac{i}{2}[\phi^+ \gamma_0 \phi_t - \phi_t^+ \gamma_0 \phi] - \bar{\nabla}\phi^+ \gamma_0 \bar{\nabla}\phi - U(\phi^+, \phi)\}, \qquad (1G)$$

where ϕ is a column-vector formed of n complex functions ϕ_A, $A = 1,\dots,n$; ϕ^+ is a Hermitian-conjugate row. γ_0 is the metric tensor in isotopic space,

$$\gamma_0 = \mathrm{diag}\{+1,\dots,+1(\text{p times}); -1,\dots,-1(\text{q times})\},$$

$p + q = n$, and $\bar{\nabla} \equiv \{\partial/\partial x_1,\dots,\partial/\partial x_D\}$. Nonlinearity U must assure a non-vanishing interaction between the first set of components ϕ_A, where $A = 1,\dots,p$ and the second one, where $A = p+1,\dots,n$. The aim of this restriction will become clear later. The Euler-Lagrange equations for the ϕ-field take the form

$$\gamma_0(\partial^2/\partial t^2 - \bar{\nabla}^2)\phi + \partial U/\partial\phi^+ = 0, \qquad (2L)$$

$$\gamma\ (-i\partial/\partial t - \bar{\nabla}^2)\phi + \partial U/\partial \phi^+ = 0, \qquad (2G)$$

where the definition $\partial/\partial\phi^+$ stands for the column constructed from n operators $\partial/\partial\phi_A^*$. The related energy constants of motion are

$$E_L = \int d^D x \{\phi_t^+ \gamma_0 \phi_t + \bar{\nabla}\phi^+ \gamma_0 \bar{\nabla}\phi + U\},$$

$$E_G = \int d^D x \{\bar{\nabla}\phi^+ \gamma_0 \bar{\nabla}\phi + U\}.$$

In order to minimize the energy by some solution $\phi(x,t)$ of evo-

lution equations (2), the nearby $\tilde{\phi}(x,t)$ solutions should ensure the condition $\delta E = 0$ and positive definiteness of $\delta^2 E$. The latter functional looks like

$$\delta^2 E_L = 2\int d^D x \{\delta\phi_t^+ \gamma_0 \delta\phi_t + \bar{\nabla}\delta\phi^+ \gamma_0 \bar{\nabla}\delta\phi + \frac{1}{2}\delta^2 U\}, \qquad (3L)$$

$$\delta^2 E_G = 2\int d^D x \{\bar{\nabla}\delta\phi^+ \gamma_0 \bar{\nabla}\delta\phi + \frac{1}{2}\delta^2 U\}, \qquad (3G)$$

where we have put $\delta\phi(x,t) \equiv \tilde{\phi}(x,t) - \phi(x,t)$.

Since the difference $E[\tilde{\phi}(x,t)] - E[\phi(x,t)]$ is an integral of motion, it is not necessary to analyse the sign of it for arbitrary time-dependent perturbations $\delta\phi(x,t)$, and we can only restrict ourselves to the use of variable initial values

$$\delta\phi(x,0) \equiv \delta\psi(x); \quad \delta\phi_t(x,0) \equiv \delta\eta(x)$$

for the case of equation (2L), which is of the second order with respect to time, and

$$\delta\phi(x,0) \equiv \delta\psi(x) \quad \text{only}$$

for the case of the first order equation (2G). On the other hand, making the above restriction we are permitted not to care for the approximation

$$E[\tilde{\phi}(x,t)] - E[\phi(x,t)] \approx 1/2\,\delta^2 E$$

to be satisfied at any instant of time. It is quite sufficient if this approximation holds at the initial moment only.

Since we are going to prove the absence of extrema, we have the right to confine ourselves to certain choice of $\delta\psi$ and $\delta\eta$. So, having chosen $\delta\eta(x) \equiv 0$ in eq. (3L), one finds that $\delta^2 E_L = \delta^2 E_G \equiv \delta^2 E$. Keeping this equality in mind, hereafter we shall not make any distinctions between the two cases. Let us impose the following boundary conditions on the remained piecewise-smooth vector-functions $\delta\psi$:

$$\delta\psi(x) \to 0 \quad \text{as } |x| \to \infty,$$

and consider them small with respect to the metric of $(L_2)^{2n} \equiv \prod_{A,B=1}^{n} \otimes L_2(R^D)$ space of 2n-component vectors ξ:

$$\rho^2[\xi] = \int d^D x\, \xi^+(x)\xi(x) = \epsilon^2. \qquad (4)$$

In eq. (4) $\xi^+ = \{\delta\psi^+, \delta\psi\}$, and ξ stands for the Hermitian-conjugate column.

It is not difficult to check that the functional (3) is not bounded from below. Indeed, taking in the equation for $\delta^2 E$,

$$\delta^2 E = 2\int d^D x \{\bar{\nabla}\,\delta\psi^+ \gamma_0\, \bar{\nabla}\,\delta\psi + 1/2\,\delta^2 U\} \qquad (3)$$

the first p components of trial perturbation to be zero, we obtain the first term ("deformation energy contribution") as

$$-2\sum_{A=p+1}^{n} \int d^D x\; \bar{\nabla}\,\delta\psi^*_A\, \bar{\nabla}\,\delta\psi_A\,. \qquad (5)$$

The modulus of this negative expression can be made arbitrarily large even if eq. (4) is fulfilled, whereas the last term in (3), which depends quadratically on $\delta\psi$, is bounded thanks to constraint (4). Hence, the second variation can be lowered below zero, and the solution ϕ does not minimize the energy in $(L_2)^{2n}$ space. In this narrow sense one may call it "energetically unstable" with respect to the metric ρ.

What additional restrictions should the trial functions obey in order to make $\delta^2 E$ positive definite? We can try to bound the "deformation energy" introduced by $\delta\psi$, making the negative contribution (5) limited. Let us define the metric ρ_0 of Sobolev space $(W_2^1)^{2n} \equiv \prod_{A,B=1}^{n} \otimes W_2^1(R^D)$ and require the variations to be small now with respect to this metric:

$$\rho_0^2[\xi] \equiv \int d^D x\, \xi^+(x)\hat{R}\xi(x) = \epsilon^2, \qquad (6)$$

where $\hat{R}$ is

$$\hat{R} = -\bar{\nabla}^2 + \hat{a}\,, \qquad (7)$$

$\hat{a}$ being diagonal $2n\times 2n$ matrix with positive elements. Further we may impose some integral constraints on examined deviations rather than considering only unconditional "energetic stability", as we did before. For example, it is reasonable to demand the excited solution to have the same values of conserved quantities as the initial solution. The said integrals are charges in the case of Lorentz-invariant system /9/ and partial numbers of particles in the case of Gallilean-invariant model /10/.

3. INTEGRAL CONSTRAINTS

Suppose, the equations (2) are covariant under the action of s-parameter internal symmetry group, and, hence, possess s integrals of motion $F_i = F_i[\phi, \phi_t]$. In accordance with the Q-stability idea /9/, we require any r of them to be unperturbed, i.e.,

$$\begin{aligned} F_i[\phi(x,0);\ \phi_t(x,0)] &= F_i[\phi(x,t);\ \phi_t(x,t)] = \\ &= F_i[\phi(x,t)+\delta\phi(x,t);\ \phi_t(x,t)+\delta\phi_t(x,t)] = \\ &= F_i[\phi(x,0)+\delta\psi(x);\ \phi_t(x,0)]. \end{aligned} \qquad (8)$$

By considering the deviations infinitely small with respect to ρ or ρ_0 norms, we are led to a simpler form for eq. (8):

$$\mathcal{F}_i[\xi, \xi^+] = 0, \quad i = 1,\ldots, r \le s, \qquad (9)$$

where $\mathcal{F}_i$ are linear in ξ and ξ^+ functionals. Note, that these functionals also depend on $\phi(x,0)$ and $\phi_t(x,0)$.

The equation (3) may conveniently be rewritten as

$$\delta^2 E = \int d^D x\, \xi^+ H \xi, \qquad (10)$$

where Jacobi operator H is

$$H = -\sigma_0\gamma_0\bar{\nabla}^2 + V, \qquad (11)$$

and

$$V = V^+ = \begin{pmatrix} \dfrac{\partial^2 U}{\partial\phi^+\partial\phi} & \dfrac{\partial^2 U}{\partial\phi^+\partial\phi^+} \\[2ex] \dfrac{\partial^2 U}{\partial\phi\partial\phi} & \dfrac{\partial^2 U}{\partial\phi^+\partial\phi} \end{pmatrix}. \qquad (12)$$

Here $\sigma_0 = \mathrm{diag}\{1,1\}$ and $2n\times 2n$ matrix $\sigma_0\gamma_0$ is merely $\mathrm{diag}\{\gamma_0, \gamma_0\}$. ϕ^+- and ϕ-dependent matrix $\partial^2 U/\partial\chi\partial\beta$ is defined as the direct product of the colomn $\partial/\partial\chi$ and the row $\partial U/\partial\beta$.

Let us suppose, the perturbation ξ is described with the help of the metric ρ. The stationary points of $\delta^2 E$ under the condition (4) then may be found by solving Sturm-Liouville problem,

$$Hy(x) = \lambda y(x). \qquad (13)$$

Since the above operator contains the sign-undefined term $-\sigma_0\gamma_0\bar{\nabla}^2$, its continuous spectrum includes not only the positive semiaxis of λ, but the negative one as well /10/. As we shall see below, it is just this very fact that does not allow the energy to possess any minima. On the quantum level it makes impossible the construction of the Bethe ansatz /11/.

If the condition (4) is replaced by (6), the generalized Sturm-Liouville problem emerges as

$$H y(x) = \lambda \hat{R} y(x) . \tag{14}$$

Given the nonlinearity U, let us confine ourselves to the type of boundary conditions imposed on ϕ under which the V-matrix asymptotical value is independent of direction of the vector x:

$$V(x) \to V^{\circ} \quad \text{as} \quad |x| \to \infty, \quad x \in R^D.$$

Let, furthermore, V° be diagonal (this holds, for example, for the case of the vanishing conditions). Hence, as $|x| \to \infty$ we are led to the following $2n$ decoupled equations with common λ:

$$(-\bar{\nabla}^2 + V^{\circ}_{AA}) y_A(x) = \lambda(-\bar{\nabla}^2 + \alpha_{AA}) y_A(x), \quad 1 \le A \le 2p, \tag{15}$$

$$(+\bar{\nabla}^2 + V^{\circ}_{AA}) y_A(x) = \lambda(-\bar{\nabla}^2 + \alpha_{AA}) y_A(x), \quad 2p+1 \le A \le 2n . \tag{16}$$

The last $2q$ equations provide the existence of the negative branch of continuous spectrum, which, however, does not generally coincide with the whole semiaxis. Nevertheless, if the eq. (16) is represented in the form

$$\bar{\nabla}^2 y_A(x) = \alpha_{AA}(\lambda - V^{\circ}_{AA}\, \alpha^{-1}_{AA})(\lambda + 1)^{-1} y_A(x),$$

the existence of the non-vanishing vicinity of $\lambda = -1$, which is entirely filled by the continuous spectrum, becomes obvious. If $V_0^{p+1,p+1} = V_0^{p+2,p+2} \ldots = V_0^{nn} < 0$, we can shrink that vicinity to a single point through the special choice of $\alpha_{2p+1,2p+1} = \ldots = \alpha_{2n,2n} = -V^{\circ}_{2n,2n}$. In this case the infinite sequence of discrete negative eigenvalues converging to $\lambda = -1$ would emerge. For the nondiagonal V_0 the negative branch of the continuous spectrum still remains, but the high coupling of eqs. (15)-(16) complicates the proof.

When the infinite number of eigenvalues lying below zero is present, it is always possible to construct the trial function, which gives the functional (10) a negative value and satisfies eq. (9) at the same time.

To start with, let us form $(r+1)$ vectors in the following way

$$f_i(x) \equiv \int_{\Omega} d\lambda\, \chi_i(\lambda)\, y_{\lambda}(x), \quad i = 1, \ldots, r+1 .$$

Here Ω is the above-mentioned continuous spectrum domain, $\Omega \subset (-\infty, 0)$; $y_{\lambda}(x)$ is the n-component eigenfunction of the problem (11) (or (12)) and $\chi_i(\lambda) \in L_2(\Omega)$. Let f be defined as the linear combination

$$f(x) = \sum_{i=1}^{r+1} C_i f_i(x) \equiv \int_{\Omega} d\lambda\, \chi(\lambda)\, y_{\lambda}(x) . \tag{17}$$

Owing to the linearity of $\mathcal{F}_k$, the requirement

$$\mathcal{F}_k[f] = 0, \quad k = 1, \ldots, r,$$

is equivalent to the following set of equations

$$\sum_{i=1}^{r+1} q_{ki} C_i = 0, \quad k = 1, \ldots r,$$

where $q_{ki} \equiv \mathcal{F}_k[f_i]$. We can find $(r+1)$ unknown quantities C_i from these r equations and substitute them in eq. (15). The vector-column f obtained in this way satisfies all the r constraints (9) simultaneously. The value of $\delta^2 E$ is given by the formula

$$\delta^2 E = \tag{18}$$

$$= 2\int d^D x\, f^+ \gamma_0\, H f = 2 \int_{\Omega} d\lambda' \overset{*}{\chi}(\lambda) \chi(\lambda') \int d^D x\, y^+_{\lambda}(x) \gamma_0 H y_{\lambda'}(x).$$

After $y_{\lambda}(x)$ assumed to be orthonormalized, the last integral in (18) becomes equal to

$$\int d^D x\, y^+_{\lambda}(x) \lambda' \hat{R} y_{\lambda'}(x) = \lambda \delta(\lambda - \lambda'),$$

which finally yields $\delta^2 E = 2 \int d\lambda |\chi(\lambda)|^2 \lambda < 0$, since $\Omega \subset (-\infty, 0)$. Thus, the sign-undefined metric models have no any even conditional energy minima.

Note that our conclusion has no relation to the Lorentz vectors, though in nonlinear models of this type[12] the kinetic term is sign-undefined as well. The invariant local constraint $\partial_k \mathcal{A}^k = 0$ is usually imposed on the field $\mathcal{A}^k$ which can lead to the positive definiteness of $\delta^2 E$.

Suppose now that in lagrangian $\mathcal{L}$ the interaction between the set of components with $A \le p$ and that with $A \ge p+1$ is equivalent to zero and $\mathcal{L}$ can be expressed as the sum $\mathcal{L} = \mathcal{L}_1(\phi_1, \ldots \phi_p) + \mathcal{L}_2(\phi_{p+1}, \ldots \phi_n)$. This means that the same equations of motion might have been obtained from the lagrangian $\mathcal{L} = \mathcal{L}_1 - \mathcal{L}_2$. Therefore, the stability may be established through the investigation of the functional $\tilde{E} = E_1 - E_2$ which now possesses positive metric.

4. CRITERION FOR STABILITY

It is known that a solution to evolution equation is stable or not, greatly depends on the applied definition of stability. As a rule, the following definition is used. For any $\epsilon > 0$ it is possible to find such $\delta > 0$, that if initial data are given to be $\{\phi(x,0)+\delta\phi(x,0)\ ,\ \phi_t(x,0)+\delta\phi_t(x,0)\}$ ($\delta\phi$ and $\delta\phi_t$ being small in terms of some norm ρ_1, i.e.,

$$\rho_1[\delta\phi(x,0),\ \ \delta\phi_t(x,0)] < \delta),$$

then the subsequent time evolution keeps the solution near the unperturbed configuration, i.e.,

$$\rho_2[\delta\phi(x,t),\ \delta\phi_t(x,t)] < \epsilon,\quad t \in (0,\infty).$$

From the physical point of view, however, such a definition is unsuitable. Indeed, consider some localized (particle-like) solution. It is just this very type of solutions that is mostly applicable in physics. We can increase the velocity of its propagation by a small quantity,

$$v \to v' = v + \delta v$$

at the initial moment, and compare the two configurations after a certain time t. Then if the interval t is greater than the ratio

(the characteristic size of the "particle")/δv,

the difference will not be small anymore. In that way,in spite of that the shape of the solution is completely preserved, the above-mentioned definition classifies it as unstable. To remove such a discrepancy, we shall have to modify it.

By stability of particle-like solution we shall mean the stability of its shape. In other words, we demand that the equations of motion, when linearized about the analysed configuration, do not admit exponential growth in time. Note, that perturbations growing slower than exponentially with time (say, polynomially) do not indicate instability. They simply transform one localized solution into another but in no way destroy them[13].

Now let us derive the stability criterion for sign-undefined metric systems. As we shall see, it differs essentially from the minimal energy condition. We limit ourselves to static solutions $\phi(x)$ and to the solutions reducible to them via the reference frame transformations (through Lorentz and Gallilean boosts, respectively).

Let us begin with relativistically invariant system (2L). The associated linearized equation takes the form

$$-\gamma_0\xi_{tt} = H\xi, \tag{19}$$

where $\xi^+ = \{\delta\phi^+(x,t), \delta\phi(x,t)\}$ and H is given by (11) and (12). With a monochromatic ansatz

$$\xi(x,t) = y(x)\exp\{\omega t\}.$$

eq. (19) may be written as

$$\gamma_0 H y(x) = \lambda y(x), \tag{20}$$

where $\lambda = -\omega^2$.The existence of negative eigenvalues λ of the operator $\gamma_0 H$ is closely connected with instability. This operator differs from the one in (13) (minimality condition) by the factor γ_0. Therefore, the continuous spectrum of $\gamma_0 H$,

$$\gamma_0 H = -\sigma_0 \overset{-2}{\nabla} + \gamma_0 V$$

contains the positive semiaxis only. Let, for instance,

$$V(x) \longrightarrow V^\circ \quad \text{as} \quad |x| \to \infty, \quad V^\circ = \mathrm{diag}\{V^\circ_A\}.$$

Along with this, if we impose $(\sigma_0\gamma_0)_A V^\circ_A \geq 0$ for any $A = 1,\dots,2n$, then there won't be even a limited domain of continuous spectrum below zero; only a finite number of discrete eigenvalues may be located there. In such a situation the complete absence of the negative discrete eigenvalues would evidently mean stability. On the other hand, if they do exist, the possibility of solution $\phi(x)$ to be conditionally stable should be explored.

In the case of nonstatic ϕ the H operator in eq. (19) is again of the form (11), though the "potential" V is no longer given by formula (12). The latter observation cannot, however, prevent us from expanding our conclusions to arbitrary time-dependent fields.

Investigation of the nonrelativistic model (1G) may be carried out in the same way. Since the related equations of motion (2G) are of the first order with respect to time, the emerging eigenvalue problem is a symplectic one[8]. By squaring the involved operator, one easily reduces that problem to a familiar Sturm-Liouville equation for some new differential operator, which now is of the fourth spatial order. The possibility of stable solutions to exist ensues from the arguments completely analogous to the ones presented above.

5. CONCLUSION

Solving the small fluctuation equations one is led to the eigenvalue problem for certain differential operator. A similar problem arises in the energy functional minimization. Both operators possess at least the same spectrum structure (continuous spectrum occupies the positive semiaxis), provided the metric is positive definite. They merely coincide if, in addition to that, we restrict ourselves to the case of static fields, velocity-dependent forces being "switched off".

Introduction of the sign-undefined metric results in the multiplication of the operator, connected with the second variation of energy by the sign-mixing matrix γ_0^*. As a consequence of this, the spectrum structures become essentially different and Dirichlet's theorem turns out to be inapplicable. The energy in a model with indefinite kinetic term cannot have even a conditional minimum, but contrary to the untuitively appealing opinion on the equivalence of stability and minimality, classical solutions are not forbidden to be stable.

ACKNOWLEDGEMENTS

The author is grateful to B.Julia, V.G.Makhankov and O.K.Pashaev for helpful discussions. He would like to express his gratitude to A.Kundu and Yu.P.Rybakov for their valuable comments.

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Received by Publishing Department
on April 19, 1983.

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* For positive metrics γ_0 is simply a unit matrix.

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Барашенков И.В. E2-83-253

Свойства устойчивости решений для класса нелинейных моделей со знаконеопределенной метрикой

Рассматриваются многокомпонентные нелинейные модели со знаконеопределенной метрикой изотопического пространства, в частности с некомпактной группой внутренней симметрии. Показано, что энергия не может иметь даже условного локального минимума. Доказывается, что несмотря на это возможно существование стабильных частицеподобных решений соответствующих эволюционных уравнений.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1983

Barashenkov I.V. E2-83-253

Stability Properties of Solutions to Nonlinear Models Possessing a Sign-Undefined Metric

We investigate multicomponent field systems possessing a sign-undefined internal space metric, in particular models with a noncompact global invariance group. It is shown that the energy cannot have even a conditional relative minimum. We demonstrate, nevertheless, that the corresponding nonlinear equations of motion are permitted to possess stable particle-like solutions.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1983