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Z.Omboo, V.V.Uzhinskii

LIST OF DIAGRAMS FOR ^{4}He-^{4}He INTERACTIONS

1983

Investigations of nucleus-nucleus scattering at high energies are very popular now. They stimulate the theoretical efforts to describe these interactions. As a result we have many models of inelastic reactions. But the theory of elastic scattering - the eikonal theory has not changed since 1969. Only new effective methods were proposed in the theory of heavy ion collisions/1/. So we begin our introduction with basic formulae of eikonal approximation for the elastic scattering.

The expression for the scattering amplitude of two nuclei A and B with mass numbers A and B, respectively, was given by some authors /2/, practically simultaneously, in the following form

$$\mathcal{F}_{AB}(\vec{q}) = \frac{iP_B}{2\pi} \int d^2b e^{i\vec{q}\vec{b}} \Gamma(\vec{b}), \qquad (1)$$

$$\Gamma(\vec{b}) = \langle \psi_A^f ; \psi_B^f | 1 - \prod_{i=1}^{A} \prod_{j=1}^{B} (1 - \gamma(\vec{b} - \vec{s}_i + \vec{r}_j)) | \psi_B^i ; \psi_A^i \rangle , \qquad (2)$$

where P_B is the momentum at the projectile nucleus B; $\vec{q}$ is the transverse momentum transferred to it; ψ_A^i, ψ_B^i and ψ_A^f, ψ_B^f are the wave functions of nuclei A and B in the initial and final states, respectively. The averaging with respect to these latter is denoted by the broken brackets.

$\gamma(\vec{b})$ is the amplitude of elastic NN-scattering in the impact parameter representation. $\{\vec{s}_A\}$, $\{\vec{r}_B\}$ are the coordinates of the nucleons of nuclei A, B within the plane of the impact parameter $\vec{b}$ (in the plane perpendicular to the momentum P_B).

As is seen from (2) the scattering amplitude is determined by the sum of different terms representing different rescattering processes. To take into account all of them, one can put into correspondence to each of them the picture like fig.1, where circles correspond to the interacting nuclei; black and light points, to nucleons; solid lines, to interactions between nucleons. Now one can say how many the diagrams of whatever type are. Since it is a combinatorical problem, it is quite natural to use the graph theory.

In the graph theory the graphs like that in fig.1 are called two-coloured labelled graphs. Graphs can be represented with the help of an adjacency matrix. The adjacency matrix $S = [S_{ij}]$ of the labelled graph G of order p (with p points) is the $p \times p$ matrix in which $S_{ij} = 1$ if points i and j are adjacent (are connected with a line) and $S_{ij} = 0$ in other cases. In our case

$$S = \begin{pmatrix} Z_1 & Q \\ Q^T & Z_2 \end{pmatrix},$$

where all elements of the $B \times B$ matrix Z_1 and the $A \times A$ matrix Z_2 are equal to zero. The element q_{ij} of the $B \times A$ matrix Q is equal to one if point j of the set B and point j of the set A are adjacent. The matrix Q can be represented by the set of crossing points of B-horizontal and A-vertical lines with dark circles in the places corresponding to the elements $q_{ij} \neq 0$. The last representation gives the so-called line graphs. The examples of graphs, matrices Q corresponding to them and line graphs, are shown in fig.2. We shall refer to line graphs as to scattering diagrams.

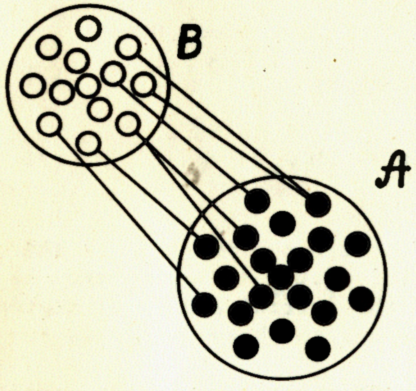


Fig. 1

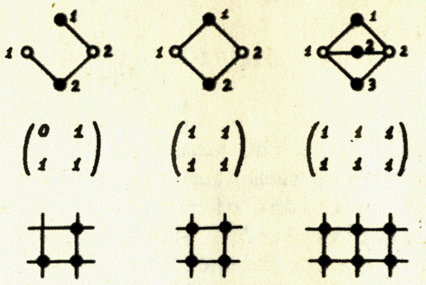


Fig. 2

Each term in expansion (2) has the form

$$-\langle \psi_A^f ; \psi_B^f \mid \prod_{\substack{i,j \\ (i,j) \in M \subset \{I_A\} \otimes \{I_B\}}} [-\gamma(\vec{b} - \vec{s}_i + r_j)] \mid \psi_B^i ; \psi_A^i \rangle \tag{3}$$

$$\{I_A\} = (1,2,3,\ldots,A), \quad \{I_B\} = (1,2,3,\ldots,B).$$

Because it can be represented by the graphs G, we shall consider the expressions like (3) as the graph functions ($g(G)$). Due to antisymmetrization of the wave functions the functions of isomorphic graphs are equal. So

$$\Gamma(\vec{b}) = \sum_{\mu} C_\mu \cdot g(G_\mu), \tag{4}$$

where summation runs on the set of all nonisomorphic graphs. From the graph theory we have that the combinatorical coefficient at the function of graph G_μ with ℓ-components, k_1 belonging to one class of isomorphism; k_2, to another class, etc., ($\ell = k_1 + k_2 + \ldots + k_j$) is equal to

$$C_\mu = \frac{A!}{(m_1!)^{k_1}(m_2!)^{k_2}\ldots(m_j!)^{k_j}(A - \sum_{i=1}^{j} m_i k_i)!} \times \tag{5}$$

$$\times \frac{B!}{(n_1!)^{k_1}(n_2!)^{k_2}\ldots(n_j!)^{k_j}(B - \sum_{i=1}^{j} n_i k_i)} \prod_{i=1}^{j} \left(\frac{m_i! n_i!}{s(G_i)}\right)^{k_i},$$

where m_i and n_i are the numbers of the points of the sets A and B, respectively, in the component belonging to the i-th class of isomorphism, and $s(G_i)$ is the number of symmetries of this component.

Now using coefficients C_μ and knowing the function $\gamma(\vec{b})$ and wave functions one can calculate the scattering amplitude $\mathcal{F}_{AB}(\vec{q})$. For example, in the case of elastic scattering if we determine $\gamma(\vec{b})$ as

$$\gamma(\vec{b}) = \frac{a}{\pi} \sigma e^{-a\vec{b}^2} \tag{6}$$

and

$$|\psi_A^i|^2 = C_A \cdot \delta\left(\sum_{j=1}^{A} \vec{r}_j / A\right) \prod_{j=1}^{A} \left(\frac{1}{\pi R_A^2}\right)^{3/2} e^{-\vec{r}_j^2/R_A^2}, \tag{7}$$

$$|\psi_B^i|^2 = C_B \cdot \delta\left(\sum_{j=1}^{B} \vec{r}_j / B\right) \prod_{j=1}^{B} \left(\frac{1}{\pi R_B^2}\right)^{3/2} e^{-\vec{r}_j^2/R_B^2}$$

so the graph function after elimination of the centre-of-mass correlations has the form

$$g(G) = -\left(-\frac{a\sigma}{\pi}\right)^{\nu} e \left(\frac{1}{\pi R_A^2}\right)^{A} \left(\frac{1}{\pi R_B^2}\right)^{B} \frac{\mathrm{Det}|D|}{\mathrm{Det}|C|} \times \tag{8}$$

$$\times \exp\left[-\vec{b}^2 \frac{\mathrm{Det}|C|}{\mathrm{Det}|D|}\right],$$

$$D = \begin{pmatrix} P & -aQ \\ -aQ^T & T \end{pmatrix},$$

$$P_{ij} = \left(\frac{1}{R_B^2} + a\beta_i\right)\delta_{ij}, \quad T_{ij} = \left(\frac{1}{R_A^2} + a\alpha_i\right)\delta_{ij},$$

$$C = \begin{pmatrix} D & V \\ V^T & a\nu_e \end{pmatrix},$$

$$\nu_e = \sum_{i=1}^{A} \alpha_i = \sum_{i=1}^{B} \beta_i,$$

$$V^T = (-a\beta_1, -a\beta_2, \ldots, -a\beta_B, a\alpha_1, a\alpha_2, \ldots, a\alpha_A),$$

where Q, as before, is the essential part of the adjacency matrix; $\alpha_i(\beta_i)$ are the degrees of the points belonging to the set A(B).

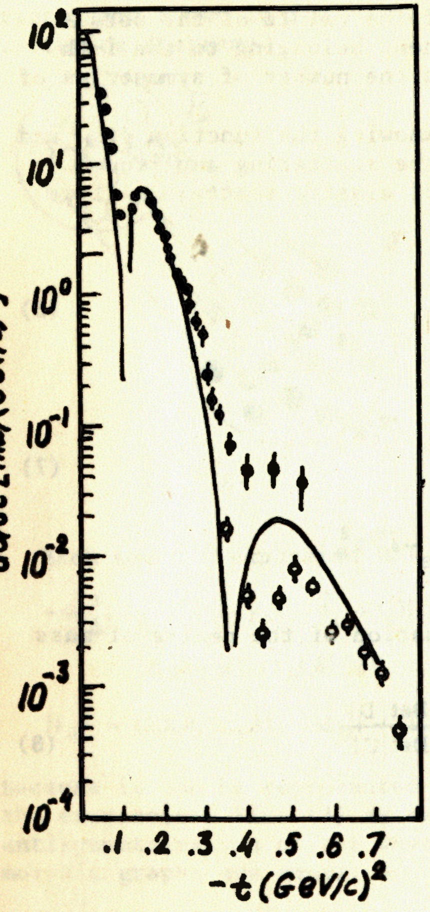


Fig. 3

The amplitude $\mathfrak{F}_{AB}(\vec{q})$ is given by the expression

$$\mathfrak{F}_{AB}(\vec{q}) = \frac{iP_B}{2\pi} \exp\left[\frac{R_A^2 \vec{q}^2}{4A} + \frac{R_B^2 \vec{q}^2}{4B}\right] \times \qquad (9)$$

$$\times \int d^2 b\, e^{i\vec{q}\vec{b}} \sum_{\mu} C_\mu\, g(G_\mu).$$

Using relations (8), (9) one can obtain the amplitude and the elastic cross section. A calculation of such type at A = B = 4, σ = 20 mb, R_A= 1.36 fm, a=1 fm^{-1} together with the experimental data /3/ is shown in fig.3. On performing this calculation we took into account all possible diagrams given in the appendix with corresponding coefficients C_μ.

As is seen, the calculation of the scattering amplitude of the heavy nuclei by this method may be sufficiently complicated, because the number of the diagrams may be very large. But here one can take into account only the graphs called trees and forest. Their yield produces a considerable contribution to the amplitude /4/ and their enumeration is quite a simple task.

It is more difficult to determine inelastic cross sections. The point is that one has to select from the total inelastic cross section

$$\sigma_{AB}^{in} = \int d^2 b \{1 - \langle\psi_A^i; \psi_B^i | \prod_{i=1}^{A}\prod_{j=1}^{B} (1 - \sigma h(\vec{b} - \vec{s}_i + \vec{r}_j)) | \psi_B^i; \psi_A^i\rangle\}, \qquad (10)$$

$$\sigma h(\vec{b}) = \gamma(\vec{b}) + \gamma^*(\vec{b}) - \gamma(\vec{b})\gamma^*(\vec{b})$$

the cross sections of different processes. As it was shown earlier /5/ the cross section of all processes with a fixed number ν of inelastic nucleon-nucleon interactions is given by

$$\sigma_\nu = (-1)^{\nu+1} \frac{\sigma^\nu}{\nu!} \frac{d\sigma_{AB}^{in}}{d\sigma}, \qquad (11)$$

$$\sigma_{AB}^{in} = \sum_\nu \sigma_\nu .$$

Introducing the graph function as

$$g(G) = -\int d^2 b \langle\psi^i; \psi^i | \prod_{\substack{i,j \\ (i,j) \in M \subset \{I_A\} \otimes \{I_B\}}} [-\sigma p(\vec{b} - \vec{s}_i + \vec{r}_j)] | \psi_B^i; \psi_A^i\rangle \qquad (12)$$

we rewrite equations (10), (11) in the form

$$\sigma_{AB}^{in} = \sum_\mu C_\mu\, g(G_\mu), \qquad (10')$$

$$\sigma_\nu = (-1)^{\nu+1} \sum_\mu \binom{\nu_e^\mu}{\nu} C_\mu\, g(G_\mu). \qquad (11')$$

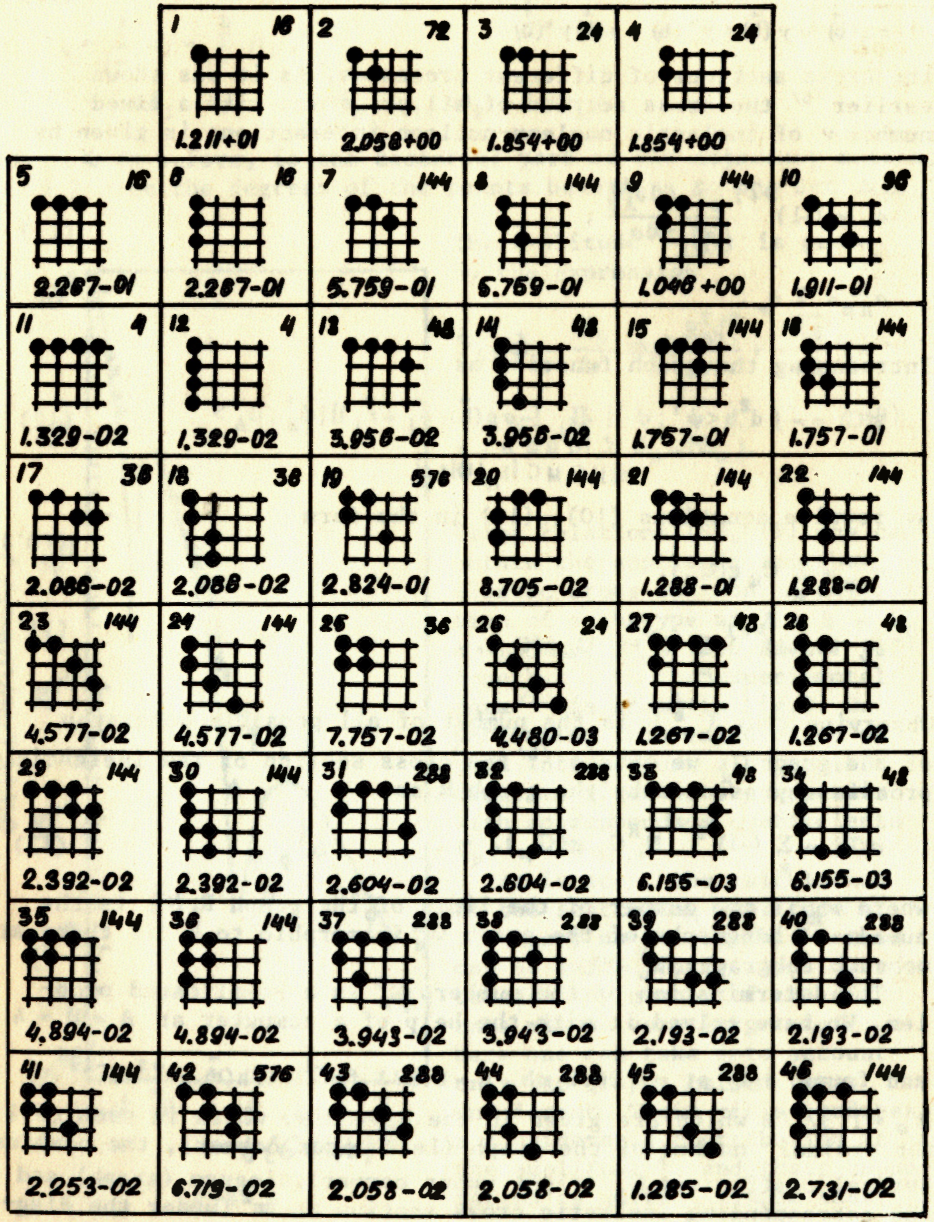
Observing that $\binom{\nu_e^\mu}{\nu}$ is the number of all possible subgraphs of the graph G_μ we obtain /5/ the cross section of the inelastic process represented by the graph R as

$$\sigma(R) = \sum_\mu (-1)^{\nu_R+1} N_\mu^R C_\mu\, g(G_\mu). \qquad (13)$$

where ν_R is the number of the lines of the graph R, N_μ^R is the number of subgraphs of the graph G_μ^R isomorphic to R (if there are no such subgraphs, N_μ^R=0).

The determination of the numbers N_μ^R is a complicated problem. We have solved it with the help of a computer at A = B = 4 and found $\sigma(R)$ at σ = 26.2 mb, a = 0.811 fm^{-1} $\left(\sigma h(\vec{b}) = \frac{\sigma a}{\pi} e^{-a\vec{b}^2}\right)$, R_a = 1.37 fm which are given in the appendix, where in each cell the ordinal number of the graph (left upper corner), the combinatorical coefficient C_μ (right upper corner), diagram (graph) and the corresponding inelastic cross section in fm^2 (under the diagram are presented. Using these values one can find, for example, the multiplicity distribution of the "wounded" nucleons and other usefull quantities.

Appendix



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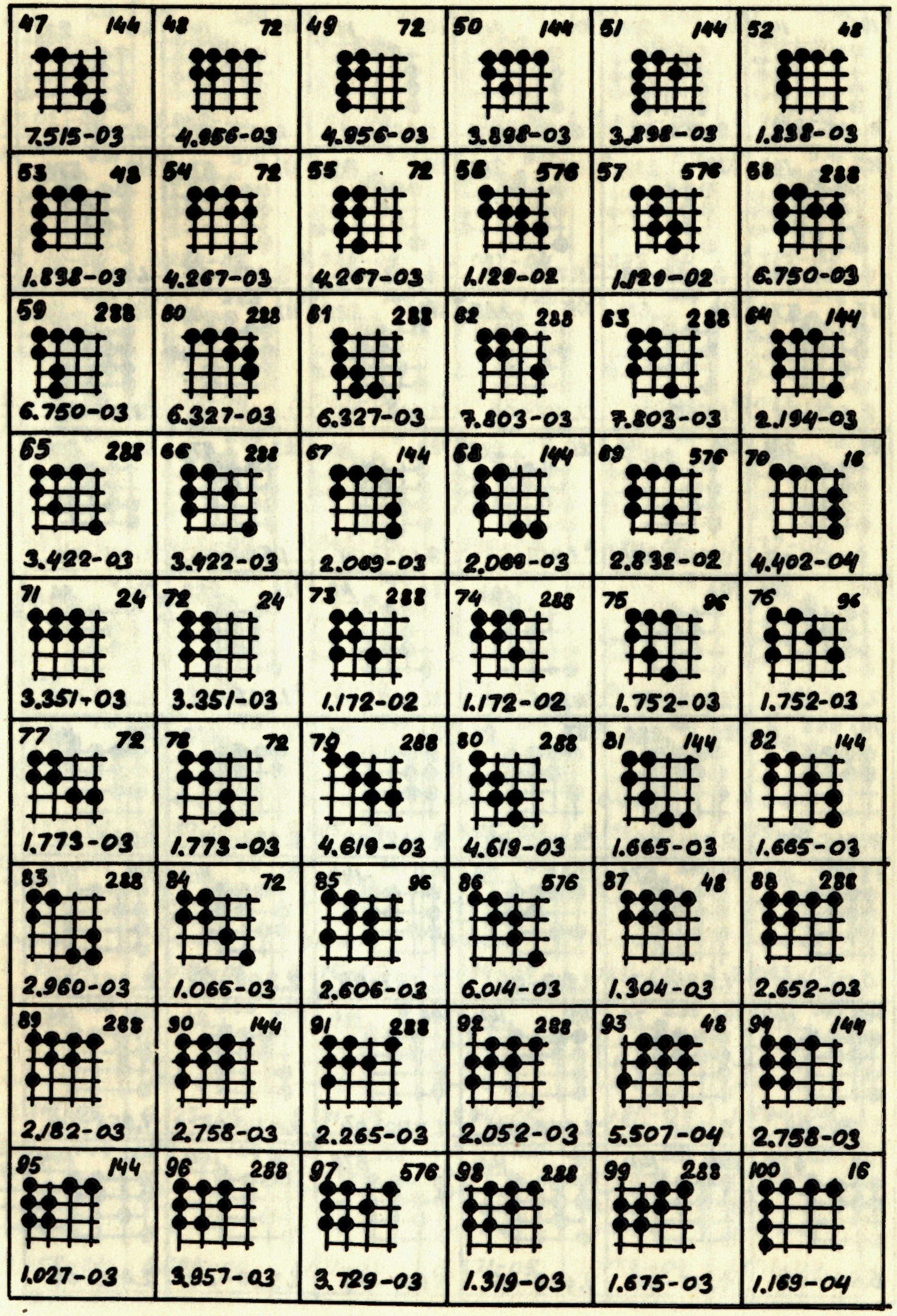
11 (4): 1.329-02	12 (4): 1.329-02	13 (48): 3.956-02	14 (48): 3.956-02	15 (144): 1.757-01	16 (144): 1.757-01
17 (36): 2.086-02	18 (36): 2.088-02	19 (576): 2.824-01	20 (144): 8.705-02	21 (144): 1.288-01	22 (144): 1.288-01
23 (144): 4.577-02	24 (144): 4.577-02	25 (36): 7.757-02	26 (24): 4.480-03	27 (48): 1.267-02	28 (48): 1.267-02
29 (144): 2.392-02	30 (144): 2.392-02	31 (288): 2.604-02	32 (288): 2.604-02	33 (48): 6.155-03	34 (48): 6.155-03
35 (144): 4.894-02	36 (144): 4.894-02	37 (288): 3.943-02	38 (288): 3.943-02	39 (288): 2.193-02	40 (288): 2.193-02
41 (144): 2.253-02	42 (576): 6.719-02	43 (288): 2.058-02	44 (288): 2.058-02	45 (288): 1.285-02	46 (144): 2.731-02



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59 (288): 6.750-03	60 (288): 6.327-03	61 (288): 6.327-03	62 (288): 7.803-03	63 (288): 7.803-03	64 (144): 2.194-03
65 (288): 3.422-03	66 (288): 3.422-03	67 (144): 2.069-03	68 (144): 2.069-03	69 (576): 2.832-02	70 (16): 4.402-04
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179 144 2.309-04	180 144 1.389-04	181 144 1.303-04	182 288 2.362-04	183 18 2.015-05	184 288 1.743-04
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311 96 1.010-06	312 24 2.140-07	313 24 2.140-07	314 72 5.252-07	315 16 8.188-08	316 1 3.647-09

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Омбоо З., Ужинский В.В. E2-83-250

Список диаграмм для взаимодействий ^4He - ^4He

Найдена связь между различными диаграммными представлениями, используемыми в теории ядро-ядерного рассеяния. Представлено полное множество графов для рассеяния ^4He-^4He. Приведены основные формулы эйкональной теории упругого и неупругого рассеяния вместе с рассчетами неупругих сечений различных реакций. При их использовании можно определить, например, распределение по числу "раненых" нуклонов и т.д..

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Omboo Z., Uzhinskii V.V. E2-83-250

List of Diagrams for ^4He-^4He Interactions

The linkage between different diagram representations used in the nucleus-nucleus scattering theory is found. The full set of graphs for ^4He-^4He scattering is presented. The basic formulae of eikonal theory for elastic and inelastic scattering and calculations of inelastic cross sections of different reactions are given. One can determine, e.g., using the last ones the multiplicity distribution of "wounded" nucleons and so on.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1983