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MASS RENORMALIZATION IN SUPERSYMMETRIC YANG-MILLS THEORIES

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One of the most attractive features of supersymmetric theories is the reduction of the number of parameters containing ultraviolet divergences. For example, in the massive Wess-Zumino model[1] supersymmetry invariance and a special form of the Lagrangian (determined by renormalizability requirements) lead to the equality of the renormalization constant for the chiral-multiplet propagator, the coupling-constant renormalization, and the mass renormalization. Instead of three expected constants there remains only one independent renormalization constant.

Recent three-loop calculations in the supersymmetric Yang-Mills (SSYM) theories[2,3] have shown that there is cancellation of ultraviolet divergences. For the $N=4$ SSYM theory divergences are absent at least up to three loops. In the $N=2$ SSYM theory there is only one-loop divergence in the coupling-constant renormalization, 2-and 3-loop divergences cancel[3]. Such a tendency is expected to hold in higher orders[4].

Supersymmetric theories claiming to be realistic ones must be gauge theories and undoubtedly they must contain massive fields. For this reason it is interesting to investigate not only the coupling-constant renormalization but also the mass renormalization.

In this paper we study the mass renormalization of matter fields for the theory describing interaction of a vector $N=1$ supermultiplet V with some chiral $N=1$ supermultiplets S_i. The action is :

$$S_h = S + S_m,$$

where

$$S = \frac{1}{8g^2c} \mathrm{Tr}\,\{\int dx\, d^2\theta\, W^\alpha W_\alpha\} + \frac{2}{c} \mathrm{Tr}\,\{\int dx\, d^2\theta\, d^2\bar\theta \times$$

$$\times \exp(-2gV)\, S_i \exp(2gV)\, S_i\} +$$

$$+ \frac{4i\gamma}{3!\,c}\epsilon_{ijk} \mathrm{Tr}\,\{\int dx\, d^2\theta\, S_i[S_j, S_k] + \int dx\, d^2\bar\theta\, \bar S_i[\bar S_j, \bar S_k]\} -$$

$$- \frac{1}{8\alpha c} \mathrm{Tr}\,\{\int dx\, d^2\theta\, d^2\bar\theta\, (D^2V)(\bar D^2 V)\} +$$

$$+ \frac{2}{c} \mathrm{Tr}\,\{\int dx\, d^2\theta\, d^2\bar\theta\, (\bar a' - a') L_{gV}[(a + \bar a) + \mathrm{Cth}\, L_{gV}(a - \bar a)]\},$$

$$S_m = -\sum_{i=1}^{n} \frac{m_i}{c} \mathrm{Tr}\,\{\int dx\, d^2\theta\, S_i S_i + \int dx\, d^2\bar\theta\, \bar S_i \bar S_i\}.$$

Here $W^\alpha = (-\frac{\bar D^2}{4})(\exp[-2gV]\, D^\alpha \exp[2gV])$, a and a' are chiral ghost fields, α is the gauge parameter, $i,j,k = 1,2,\ldots n$ and the Lie derivative $L_X Y = [X, Y]$. All fields are transformed under the adjoint representation of an arbitrary compact semisimple group:

$$V = V^a T^a, \qquad S_i = S_i^a T^a, \qquad a = a^a T^a,$$

$$[T^a, T^b] = i f^{abc} T^c, \qquad f^{acd} f^{bcd} = c\,\delta^{ab}.$$

When $m_i = 0$, $\gamma = 0$ and $n = 1$ we have the $N=2$ Yang-Mills theory in terms of $N=1$ superfields. For $m_i = 0$, $\gamma = g$, $n = 3$ we have the $N=4$ SSYM theory in terms of $N=1$ superfields[5]. When $m_i \neq 0$, we have a soft breaking of $N = 2,4$ supersymmetry.

Using the Wess-Zumino gauge we get the following on-shell action in terms of ordinary fields

$$S_h = \int dx\, \mathfrak{L},$$

$$\mathfrak{L} = \frac{1}{c} \mathrm{Tr}\{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(D_\mu A_i)^2 + \frac{1}{2}(D_\mu B_i)^2 - \frac{1}{2}\bar\phi_m i\hat D \phi_m +$$

$$+ \frac{ig}{2}\bar\phi_m[\alpha^i_{mn}A_i + \gamma^5 \beta^i_{mn} B_i, \phi_n] + \frac{g^2}{4}([A_i, A_j]^2 + [B_i, B_j]^2 +$$

$$+ 2[A_i, B_j]^2) + \sum_{i=1}^{n} \frac{i m_i \epsilon_{ijk}\,\gamma}{2}(A_i[A_j, A_k] - A_i[B_j, B_k] + 2B_i[A_j, B_k])\} -$$

$$- \frac{1}{2c} \mathrm{Tr} \sum_{i=1}^{n} (m_i^2 A_i^2 + m_i^2 B_i^2 - m_i \bar\psi_i \psi_i),$$

where $\phi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \\ \lambda \end{pmatrix}$ the α and β matrices obey the following relations:

$$[\alpha^i, \beta^j] = 0, \qquad \{\alpha^i, \alpha^j\} = \{\beta^i, \beta^j\} = -2\delta^{ij},$$

$$\mathrm{tr}(\alpha^r \alpha^t) = \mathrm{tr}(\beta^r \beta^t) = -4\delta^{rt}.$$

The main goal of our paper is to investigate first orders of the mass-renormalization:

$$m_{Ri} = Z_{S_i} m_i + \delta m_i$$

and the corresponding anomalous dimensions

$$\gamma_{m_i} = \frac{\partial \ln m_{Ri}}{\partial \ln \mu^2}.$$

According to the estimation of the degree of divergence of a general graph[7] in the theories under consideration there are no quadratic divergences, i.e., $\delta m_i = 0$, so, only the wave-function renormalization of S_i is required:

$$m_{Ri} = Z_{s_i} m_i = Z_m^{-1} m_i.$$

We have calculated Z_m both in the $N=1$ superfield techniques and in terms of ordinary fields up to two loops. Regularization by the dimensional reduction and minimal subtraction scheme[8] were adopted. In that scheme the introduction of masses does not change the coupling-constant renormalization, and consequently, the renormalization group β-function. Wave-function renormalizations are also the same as in the massless case.

For the $N=4$ SSYM theory the relation between β and γ can be obtained in all orders. According to the estimation of the degree of divergence[7] the $S_i S_j S_k$ vertex renormalization constant will be 1 (to all orders). That gives

$$g_R^2 = \frac{Z_{s_i}^3}{Z_{S_i^3}} g^2 = Z_{s_i}^3 g^2,$$

$$m_i = Z_{s_i}^{-1} m_{Ri}.$$

Differentiating the relation

$$\frac{g^2}{m_i^3} = \frac{g_R^2}{m_{Ri}^3}$$

with respect to $\ln \mu^2$ at the g^2 and m_i fixed, we get:

$$\beta(g_R^2) = 3 g_R^2 \gamma_{m_i}(g_R^2).$$

It is known that $\beta(g_R^2) = 0$ up to three loops, and therefore:

$$\gamma_{m_i}^{(3)}(g_R^2) = 0.$$

For $N=2$ the evaluation by the superfield techniques gives at the two-loop level:

$$\gamma_{m_i}^{(2)}(g_R^2) = -2c \frac{g_R^2}{(4\pi)^2}.$$

At the one-loop level, calculations in terms of ordinary fields allow us to get an analytic dependence of γ_{m_i} on N. (The latter is connected with the number of scalar fields n

$$N = n + 1.$$

The number of scalar fields, n, appears in the course of evaluation of Feynman diagrams containing scalar traces). At the one-loop level

$$\gamma_{m_i}^{(1)}(g_R^2) = (N-4)\, c \frac{g_R^2}{(4\pi)^2}.$$

In the two-loop approximation, because of the term

$$\sum_i m_i \epsilon_{ijk} \{ A_i [A_j, A_k] - A_i [B_j, B_k] + 2 B_i [A_j, B_k] \}$$

existing only for $N=4$ we cannot get the analytic dependence on N in an obvious way. For $N=4$ the contribution to γ_{m_i} from the diagrams containing that interaction is: $-12c^2 g_R^4/(4\pi)^4$. If we assume that in an arbitrary case these diagrams give the contribition

$$-6(N-2)\, c^2 \frac{g_R^4}{(4\pi)^4}.$$

and if we add contributions from other diagrams, where the analytic dependence on N follows trivially, then in the two-loop approximation

$$\gamma_{m_i}^{(2)}(g_R^2) = (N-4)\, c \frac{g_R^2}{(4\pi)^2} \left[1 - 2c(N-2) \frac{g_R^2}{(4\pi)^2} \right].$$

Comparing $\gamma_{m_i}^{(2)}$ with the expression for $\beta(g_R^2)$ derived earlier[3]:

$$\beta^{(2)}(g_R^2) = (N-4)\, c \frac{g_R^2}{(4\pi)^2} \left[1 - 2c(N-2) \frac{g_R^2}{(4\pi)^2} \right],$$

we have

$$\beta^{(2)}(g_R^2) = \frac{g_R^2}{(4\pi)^2}\gamma_{m_i}(g_R^2).$$

So, not only for $N=4$ but also for $N=2$ the charge and mass renormalization constants are related. They coincide at least up to two loops. That fact, as is shown above, takes place in all orders for $N=4$, and probably, also in higher orders for $N=2$. Our investigations allow the conclusion that the four-dimensional finite quantum field theory with massive fields may exist.

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Саздович Б.Т., Тарасов О.В. E2-83-213
Перенормировка масс в суперсимметричных теориях Янга-Миллса

Приводятся результаты исследований перенормировки масс в суперсимметричных теориях Янга-Миллса. Обнаружено, что по крайней мере в первых порядках теории возмущений константы ренормировки масс и заряда совпадают.

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Mass Renormalization in Supersymmetric Yang-Mills Theories

We study the mass renormalization in supersymmetric Yang-Mills theories. It is found that at least in first orders of perturbation theory the mass and charge renormalization constants coincide.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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