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MESON LAGRANGIANS OF THE U(3) GROUP IN THE MODEL WITH FOUR-QUARK INTERACTIONS

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1. INTRODUCTION

In ref.[1] it is shown that one can construct a sigma-model for the pion interaction, a Yang-Mills Lagrangian for vector meson interaction, and a Lagrangian for mixed interactions of pions with vector mesons on the basis of a simple effective Lagrangian with four-quark interactions of scalar, pseudoscalar, and vector types. The relevant composite-hadron Lagrangians are derived in a straight-forward way by using the technique of path-integrals over collective fields. It is also shown that for electromagnetic interactions of quarks with photons this approach automatically leads to the well-known vector dominance model.

Our model is a further development of ideas expounded in refs.[2-4]. Let us present in brief main points of this approach. All mesons are considered to be composite two-quark systems. Interaction between mesons is mediated by quark loops. The same concerns the interaction of mesons with photons. Phenomenological Lagrangians are constructed only within one-loop approximation with divergent loops. The divergences in our model are removed by the renormalization of the meson fields. These renormalizations completely define the strength of meson interaction, phenomenological meson vertices. Renormalization of meson fields is determined by kinetic terms arising from loop diagrams with two meson legs. All meson vertices for strong interaction can be expressed via one vertex specific for the vector dominance model. This is the vertex g_ρ which describes the decay $\rho \to 2\pi$ ($g_\rho^2/4\pi \sim 3$). It turns out that the strong vertex describing the interaction of pions with each other and with quarks, g, is connected with g_ρ: $g_\rho = \sqrt{6}\,g$ and equals $g^2/4\pi = 1/2$. Thus, in the sector for pseudoscalar and scalar mesons there appear arguments for the use of the usual perturbation theory even for the description of strong interactions *.

In this paper the model proposed in[1] is generalized to the group U(3). Here we shall describe strong interactions of scalar, pseudoscalar, vector, and pseudovector meson nonets. The main point is the construction of phenomenological Lagrangians in which all interaction constants are again connected with each other uniquely and are expressed via the constant g (or g_ρ).

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* The same value for the constant g is obtained in ref.[5].

БИБЛИОТЕКА

At the same time we discuss mass formulae for the above particles. However, in deriving mass formulae it is not possible to get such unambiguous results as for the coupling constants of meson strong interactions because of a considerably greater number of arbitrary parameters. Indeed, if coupling constants of strong interactions of mesons are completely determined by renormalizations of meson fields, the definition of meson masses includes also parameters G_i of the strength of effective four-quark interactions and some other parameters (see /1/ and a further text of the present paper).

Thus, we pretend to quantitative results only for the phenomenological interaction constants of meson fields. As for the mass formulae of 36 sorts of mesons, only qualitative results can be expected.

We note that the Lagrangians obtained allow a satisfactory description of numerous decays of the mesons and of many intrinsic characteristics of mesons.

In the following section an effective Lagrangian will be derived for strong interactions of scalar, pseudoscalar, vector, and pseudovector nonets of mesons. In the third section a generalized sigma model is found. In the fourth section we discuss mass formulae for scalar and pseudoscalar mesons. In the fifth section we describe vector mesons and their interactions with scalar and pseudoscalar particles. Pseudovector particles and their interactions with other fields are discussed in section 6. The paper is concluded with a brief discussion of the results obtained.

2. EFFECTIVE LAGRANGIANS

Consider an effective quark Lagrangian corresponding to the group U(3) and describing four-quark interactions of scalar, pseudoscalar, vector, and pseudovector types*

$$\mathcal{L}(q,\bar{q}) = \bar{q}\{i\hat{\partial} - [m_0 + \Delta(\lambda_0 - \sqrt{2}\lambda_8)]\}q + \frac{G_1}{2}[(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2] - $$
$$- \frac{G_2}{2}[(q\gamma_\mu\lambda_a q)^2 + (q\gamma_5\gamma_\mu\lambda_a q)^2]\,. \qquad (1)$$

*We do not consider here tensor mesons though their inclusion in our model is not difficult in principle.

Here $\bar{q}=(\bar{u},\bar{d},\bar{s})$ are quark fields. It is assumed that they have colour indices over which the summation runs, λ_a are Gell-Mann matrices, $0 \le a \le 8$, $\lambda_0 = \sqrt{\frac{2}{3}}\,1$, $\mathrm{Tr}[\lambda_a, \lambda_\beta] = 2\delta_{a\beta}$, m_0 is the bare mass of u and d quarks ($m_0^u = m_0^d = m_0$, $m_0^s = m_0 + \sqrt{6}\Delta$).

Following ref. /1/ we consider a generating functional and introduce boson fields and then we can write down

$$W(\bar{\eta},\eta) = \frac{1}{N}\int d\bar{q}dq \exp\{i[\mathcal{L}(q,\bar{q}) + \eta\bar{q} + \bar{\eta}q]\} =$$
$$= \frac{1}{N}\int d\bar{q}dq \prod_{a=0}^{8} d\sigma_a^{in} d\phi_a dV_a dA_a \exp\{i[\mathcal{L}'(q,\bar{q},\sigma,\phi,V,A) + \eta\bar{q} + \bar{\eta}q]\}, \qquad (2)$$

where

$$\mathcal{L}'(q,\bar{q},\sigma,\phi,V,A) = \bar{q}\{i\hat{\partial} - [m_0 + \Delta(\lambda_0 - \sqrt{2}\lambda_8)] + \lambda_a\sigma_a^{in} + i\gamma_5\lambda_a\phi_a +$$
$$+ \lambda_a\hat{V}_a + \gamma_5\lambda_a\hat{A}_a\}q - \frac{1}{2G_1}(\sigma_a^{in\,2} + \phi_a^2) + \frac{1}{2G_2}(V_a^2 + A_a^2). \quad (\hat{V}_a = V_a^\nu\gamma_\nu). \qquad (3)$$

In formula (2) we may integrate over quark fields; as a result, we arrive at an expression in the exponential, which allows us to construct an effective potential for meson fields. Before doing this, we make some transformations for σ-fields in order to obtain a Lagrangian $\mathcal{L}'$ in the form more convenient for further calculations.

Mass terms in $\mathcal{L}'$ can be absorbed by fields σ_0^{in} and σ_8^{in}, as a result, the part of $\mathcal{L}'$ containing quark fields acquires the completely chiral-symmetric form (massless quarks). However, it turns out that in the effective meson Lagrangian the fields σ_0 and σ_8 will have nonzero vacuum expectation values. To make them zero requires additional shifts in those fields by parameters a and b (spontaneous breaking of chiral symmetry). As a result, we arrive at the following change of variables in $\mathcal{L}'$:

$$\tilde{\sigma}_0 - a = \sigma_0^{in} - \Delta - \sqrt{\frac{3}{2}}m_0, \quad \tilde{\sigma}_8 - b = \sigma_8^{in} + \sqrt{2}\Delta, \qquad (4)$$

where parameters a and b are defined by the condition

$$\langle\tilde{\sigma}_0\rangle_0 = \langle\tilde{\sigma}_8\rangle_0 = 0. \qquad (5)$$

In terms of new fields the Lagrangian $\mathcal{L}'$ assumes the form

$$\mathcal{L}' = \bar{q}\{i\hat{\partial} - M + \lambda_0\tilde{\sigma}_0 + \lambda_1\sigma_1 + \dots + \lambda_7\sigma_7 + \lambda_8\tilde{\sigma}_8 + \lambda_a(i\gamma_5\phi_a + \hat{V}_a + \gamma_5\hat{A}_a)\}q - \frac{1}{2G_1}(\sigma_a^{in\,2} + \phi_a^2) + \frac{1}{2G_2}(V_a^2 + A_a^2), \tag{6}$$

where

$$M = \begin{pmatrix} m_u & & \\ & m_u & \\ & & m_s \end{pmatrix}, \quad m_u = \frac{\sqrt{2}a + b}{\sqrt{3}}, \quad m_s = \frac{\sqrt{2}a - 2b}{\sqrt{3}}, \tag{7}$$

$$-a\lambda_0 - b\lambda_8 = -M = -m_u I + \frac{b}{\sqrt{2}}(\lambda_0 - \sqrt{2}\lambda_8).$$

It is seen that as a result of the spontaneous symmetry breaking in the effective meson Lagrangian (that will be obtained below) the bare masses of quarks $m_0^{(u,d)}$ and $m_0^{(s)}$ in (1) are changed to new masses m_u and m_s. Subsequent calculations show that the bare masses m_0 and $m_0^{(s)}$ coincide in magnitude with the quark current masses whereas the values of new masses m_u and m_s are close to the values of masses of quarks of composite particles (constituent quarks, $m_u \sim \frac{m_N}{3}$). For convenience of further calculations one more transformation can be made for σ-fields, after which all the propagators in quark loops will have equal masses:

$$\sigma_0 = \tilde{\sigma}_0 + \frac{b}{\sqrt{2}}, \quad \sigma_8 = \tilde{\sigma}_8 - b. \tag{8}$$

Now the Lagrangian $\mathcal{L}'$ assumes the form

$$\mathcal{L}' = \bar{q}\{i\hat{\partial} - m_u I + \lambda_a[\sigma_a + i\gamma_5\phi_a + \hat{V}_a + \gamma_5\hat{A}_a]\}q - \frac{1}{2G_1}(\sigma_a^{in\,2} + \phi_a^2) + \frac{1}{2G_2}(V_a^2 + A_a^2). \tag{9}$$

Integrating over quark fields in (2) we arrive at

$$W(\bar{\eta}, \eta) = \frac{1}{N}\int \prod_a d\sigma_a\, d\phi_a\, dV_a\, dA_a \exp\{i[\mathcal{L}''(\sigma, \phi, V, A) - \bar{\eta}\{i\hat{\partial} - m_u + \lambda_a[\sigma_a + i\gamma_5\phi_a + \hat{V}_a + \gamma_5\hat{A}_a]\}^{-1}\eta]\}, \tag{10}$$

where $\mathcal{L}''(\sigma, \phi, V, A)$ is the effective meson Lagrangian

$$\mathcal{L}''(\sigma, \phi, V, A) = -\frac{1}{2G_1}(\sigma_a^{in\,2} + \phi_a^2) + \frac{1}{2G_2}(V_a^2 + A_a^2) - i\,\mathrm{Tr}\ln\{1 + \frac{1}{i\hat{\partial} - m_u}\lambda_a[\sigma_a + i\gamma_5\phi_a + \hat{V}_a + \gamma_5\hat{A}_a]\}. \tag{11}$$

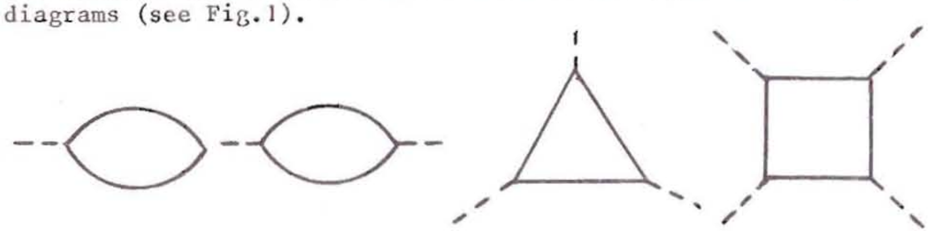
3. GENERALIZED SIGMA-MODEL

Let us take now the part of Lagrangian (11) containing only scalar and pseudoscalar fields. All meson fields in (11) are connected with each other via quark loops. We consider here only divergent quark loops. Divergent integrals may be of two types: quadratically divergent I_1 and logarithmically divergent I_2:

$$I_1 = -i\frac{3}{(2\pi)^4}\int\frac{d^4k}{m_u^2 - k^2}, \quad I_2 = -i\frac{3}{(2\pi)^4}\int\frac{d^4k}{(m_u^2 - k^2)^2}. \tag{12}$$

The integral I_1 enters into the definition of masses of mesons and of masses of bare (current) quarks; whereas the integral I_2, into expressions for coupling constants of mesons in the effective Lagrangian. Owing to this integral all meson vertices can be expressed in terms of one vertex describing the decay $\rho \to 2\pi$.

After these preliminary remarks we proceed to derive the effective Lagrangian for interactions of scalar and pseudoscalar mesons. On the whole there are met four types of divergent diagrams (see Fig.1).



a) b) c) d)

Fig.1

Their contributions to the Lagrangian $\mathcal{L}''$ are, respectively,

a) $-4\sqrt{6}\, I_1 m_u \sigma_0$,

b) $2I_2[(\partial_\mu\sigma_a)^2 + (\partial_\mu\phi_a)^2] + 4I_1(\sigma_a^2 + \phi_a^2) - 8I_2 m_u^2\sigma_a^2$,

c) $4I_2 m_u \mathrm{Tr}[\bar{\sigma}(\bar{\sigma}\bar{\sigma} + \bar{\phi}\bar{\phi})]$,

d) $-I_2 \mathrm{Tr}[(\bar{\sigma})^4 + (\bar{\phi})^4 + 4(\bar{\sigma})^2(\bar{\phi})^2 - 2\bar{\sigma}\bar{\phi}\bar{\sigma}\bar{\phi}]$, (13)

where $\bar{\sigma} = \lambda_a \sigma_a$, $\bar{\phi} = \lambda_a \phi_a$. After the change of variables

$$\sigma_0 = \sigma_0' + \sqrt{\frac{3}{2}}\, m_u \qquad (14)$$

the contribution of all this group of diagrams can be written down in a simple form

$$2I_2[(\partial_\mu \sigma_a')^2 + (\partial_\mu \phi_a)^2] + 4(I_1 + m_u^2 I_2)(\sigma_a'^2 + \phi_a^2) -$$
$$- I_2 \mathrm{Tr}[(\bar{\sigma}')^4 + (\bar{\phi})^4 + 4(\bar{\sigma}')^2(\bar{\phi})^2 - 2\bar{\sigma}'\bar{\phi}\bar{\sigma}'\bar{\phi}]. \qquad (15)$$

Returning to fields with zero vacuum expectation values ($\sigma_0' = \bar{\sigma}_0 - a$, $\sigma_8 = \bar{\sigma}_8 - b$) and renormalizing the fields so as to obtain a correct coefficient of the kinetic terms

$$\bar{\sigma}_a = g\sigma_a^R, \quad \phi_a = g\phi_a^R, \quad g = (4I_2)^{-1/2}, \qquad (16)$$

we arrive finally at the following Lagrangian for the interaction of scalar and pseudoscalar mesons*

$$\mathcal{L}(\sigma^R, \phi^R) = \frac{1}{2}[(\partial_\mu \sigma_a^R)^2 + (\partial_\mu \phi_a^R)^2] +$$
$$+ (\frac{I_1}{I_2} + m_u^2)[(\sigma_0^R - \frac{a}{g})^2 + (\sigma_8^R - \frac{b}{g})^2 + \sum_1^7 (\sigma_i^R)^2 + (\phi_a^R)^2] -$$
$$- \frac{1}{2G_1}[(g\sigma_0^R + \Delta + \sqrt{\frac{3}{2}}\, m_0 - a)^2 + (g\sigma_8^R - \sqrt{2}\,\Delta - b)^2 + g^2 \sum_1^7 (\sigma_i^R)^2 + g^2(\phi_a^R)^2] - \qquad (17)$$
$$- \frac{g^2}{4}\mathrm{Tr}\{[(\bar{\sigma}^R - \frac{M}{g})^2 + (\bar{\phi}^R)^2]^2 - [\bar{\phi}^R, (\bar{\sigma}_R - \frac{M}{g})]_-^2\} -$$
$$- i\,\mathrm{Tr}\ln\{1 + \frac{1}{i\hat{\partial} - m_u} g(\bar{\sigma} + i\gamma_5 \bar{\phi})\}'.$$

Now let us proceed to fix indefinite parameters of our model. At this stage we have six indefinite parameters: m_0, Δ, a,

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*Here and in what follows the prime of the last braces means that divergent loop diagrams are eliminated from the trace of logarithm.

b, g, and G_1 (or m_0, $m_0^{(s)}$, m_u, m_s, g, and G_1). These parameters may be fixed by the following six equations:

$$m_\pi^2 = g^2(\frac{1}{G_1} - 8I_1), \qquad (18)$$

$$m_K^2 = m_\pi^2 + 2m_s(m_s - m_u), \qquad (19)$$

$$\frac{\delta\mathcal{L}}{\delta\sigma_0^R}\Big|_{\sigma_a^R = 0,\ \phi_a^R = 0} = 0 \rightarrow \frac{m_0}{m_u} = 1 - 8I_1G_1 = G_1(\frac{m_\pi}{g})^2, \qquad (20)$$

$$\frac{\delta\mathcal{L}}{\delta\sigma_8^R}\Big|_{\sigma_a^R = 0,\ \phi_a^R = 0} = 0 \rightarrow m_\pi^2 + 2m_s(m_s + m_u) = \frac{g^2}{G_1}. \qquad (21)$$

These equations follow from the Lagrangian (17). If we construct an axial current on the basis of the same Lagrangian and describe with its help the decay $\pi^- \rightarrow \mu\bar{\nu}$ (ref. [1]), we obtain the fifth equation (the Goldberger-Treimann identity)

$$m_u = gF_\pi. \qquad (22)$$

The sixth equation is the equation following from the decay $K^- \rightarrow \mu\bar{\nu}$

$$m_u + m_s = 2gF_K. \qquad (23)$$

However, we choose, following [1], the last equation from the relation that holds in our model between coupling constants $g = g_{\pi\bar{q}q}$ (the coupling constant of pions with quark lines, see (6) and (16)) and $g_\rho = g_{\rho\bar{q}q}$ (the coupling constant of vector mesons with quark lines, see below). As will be shown in the fifth section (see also [1]), these coupling constants obey the simple relation

$$g_\rho = \sqrt{6}\, g. \qquad (24)$$

The coupling constant g_ρ can be fixed by the width of the decay $\rho \rightarrow 2\pi$:

$$\Gamma_{(\rho^0 \rightarrow \pi^+\pi^-)} = 158 \text{ MeV} \rightarrow \frac{g_\rho^2}{4\pi} \approx 3. \qquad (25)$$

Then relations (24), (22) and (19) produce

$$g \approx \sqrt{2\pi}, \quad g^2/4\pi \approx 1/2, \quad m_u \approx 240 \text{ MeV } (F_\pi = 95 \text{ MeV}), \quad m_s = 470 \text{ MeV}. \qquad (26)$$

If we consider now equation (23) as an equation for F_K, then we obtain for F_K a somewhat overestimated but quite reasonable value

$$F_K \approx 1.49\, F_\pi \,. \tag{27}$$

Using the cut-off regularization for the integrals I_1 and I_2 we obtain from (26) that the cut-off momentum $\Lambda = 1120$ MeV. Then $I_1 \approx 0.36\, m_u^2$ and for the remaining parameters from equations (18), (20), (21) we get

$$m_0^{(u,d)} = 4.5\ \text{MeV}, \quad m_0^{(s)} = 156\ \text{MeV}, \quad G_1 = 6 \cdot 10^{-6}\ \text{MeV}^{-2} \,. \tag{28}$$

These values are in good agreement with masses of current quarks [6-8] whereas the values (26) correspond to the quark masses in models of composite particles.

This section we conclude as follows. For the calculation of different physical processes like meson decays, form factors and so on we shall use only three parameters: g, m_u and m_s [1,9-11]. The other parameters are intrinsic parameters of the model.

4. MASS FORMULAE

From the Lagrangian (17) we get the following expression for the quadratic in pseudoscalar fields part

$$\mathcal{L}_m(\phi) = -\frac{1}{2}\{ m_\pi^2(\pi^{o2} + 2\pi^+\pi^-) + 2m_K^2(K^+K^- + \bar{K}^o K^o) + m_{\phi_8}^2 \phi_8^2 - \frac{4\sqrt{2}}{3}(m_s^2 - m_u^2)\phi_0\phi_8 + m_{\phi_0}^2 \phi_0^2 \} , \tag{29}$$

where masses m_π and m_K are fixed experimentally (see (18) and (19)), and $m_{\phi_8}^2$ and $m_{\phi_0}^2$ equal

$$m_{\phi_8}^2 = m_\pi^2 + \frac{4}{3}(m_s^2 - m_u^2), \quad m_{\phi_0}^2 = m_\pi^2 + \frac{2}{3}(m_s^2 - m_u^2) \,. \tag{30}$$

Introducing the angles of mixing

$$\phi_0 = \eta_0 \cos\phi - \eta_8 \sin\phi, \quad \phi_8 = \eta_0 \sin\phi + \eta_8 \cos\phi \tag{31}$$

and diagonalizing the last part of the Lagrangian (29) we get

$$m_{\eta_8}^2 = m_{\phi_8}^2 \cos^2\phi + m_{\phi_0}^2 \sin^2\phi + \frac{2\sqrt{2}}{3}(m_s^2 - m_u^2)\sin 2\phi = m_\pi^2 + 2(m_s^2 - m_u^2) ,$$

$$m_{\eta_0}^2 = m_{\phi_8}^2 \sin^2\phi + m_\phi^2 \cos^2\phi - \frac{2\sqrt{2}}{3}(m_s^2 - m_u^2)\sin 2\phi = m_\pi^2 , \tag{32}$$

$$\mathrm{tg}\, 2\phi = \frac{4\sqrt{2}(m_s^2 - m_u^2)}{3(m_{\phi_8}^2 - m_{\phi_0}^2)} = 2\sqrt{2} \,.$$

Thus, we arrive at an ideal angle of mixing at which η_8 will consist only of s-quarks, and η_0 will be the pure u-d quark state. The meson mass η_8 will approximately equal the experimental value ($m_{\eta_8} \approx 600$ MeV), whereas the η_0-meson mass will be too low as compared to that of η'-meson.

Problems of the description of masses of η and η' mesons in chiral theories are well known. They are usually solved as follows: either the annihilation channel is included [12], which violates the chiral symmetry and therefore is absent in the Lagrangian (1), or terms corresponding to gluon anomalies [13] are considered, or these effects are simultaneously taken into account [14]. Here we discuss in brief the situation occurring when gluon anomalies are included into the Lagrangian (17). A detailed consideration of such effects in chiral Lagrangian can be found in review [13] and in references therein. The inclusion of gluon anomalies produces in the Lagrangian (29) a term of the form $-\frac{d}{2}\phi_0^2$, where the parameter d is fixed, for instance, by the sum of squares of masses of η-mesons:

$$m_{\phi_0}^2 + m_{\phi_8}^2 + d = m_\eta^2 + m_{\eta'}^2 . \tag{33}$$

With this term the mixing angle is strongly changed, and we get

$$\theta \approx -11^\circ, \quad m_\eta \approx 500\ \text{MeV}, \quad m_{\eta'} \approx 1\ \text{GeV} \,. \tag{34}$$

These values are close to the experimental ones.

For scalar particles we have

$$\mathcal{L}_m(\sigma) = -\frac{1}{2}\{ m_\delta^2(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) + m_{\bar{K}}^2(\sigma_4^2 + \sigma_5^2 + \sigma_6^2 + \sigma_7^2) + m_{\sigma_0}^2 \sigma_0^2 - 4\sqrt{2}(m_s^2 - m_u^2)\sigma_0\sigma_8 + m_{\sigma_8}^2 \sigma_8^2 \} , \tag{35}$$

where masses are as follows

$$m_\delta^2 = m_\pi^2 + 4m_u^2\,, \qquad m_{\tilde K}^2 = m_\pi^2 + 2m_s(m_s + m_u)\,,$$
$$m_{\sigma_0}^2 = m_\pi^2 + 2(m_s^2 + m_u^2), \qquad m_{\sigma_8}^2 = m_\pi^2 + 4m_s^2, \tag{36}$$

that corresponds to the following values

$$m_\delta = 500\ \text{MeV}, \quad m_{\tilde K} = 840\ \text{MeV}, \quad m_{\sigma_0} = 760\ \text{MeV}, \quad m_{\sigma_8} = 950\ \text{MeV}.$$

The masses m_{σ_0} and m_{σ_8} are in good agreement with the values for scalar resonances ϵ and S^* whereas for δ and $\tilde K$-mesons one has too low values. This fact may be a consequence of large contributions to δ and $\tilde K$-mesons from four-quark states /15/.

The mass formula for σ_0 and σ_8-mesons contains a non-diagonal term that results in an ideal mixing of singlet and octet components of these mesons. However, the S^*-meson cannot consist only of s-quarks as in this case the decay $S^* \to 2\pi$ would be forbidden. For a better agreement with experiment one should take into account the annihilation channel in the scalar part of the Lagrangian (1), that changes the nondiagonal term in (35). To this end the quark Lagrangian (1) should be supplemented by terms of the type

$$\frac{G_1}{2}\kappa\,(\bar q\lambda_u q)(\bar q\lambda_s q), \quad \lambda_s = \frac{\lambda_0 - \sqrt2\lambda_8}{\sqrt3} = \sqrt2\begin{pmatrix}0&&\\&0&\\&&1\end{pmatrix}, \lambda_u = \frac{\sqrt2\lambda_0 + \lambda_8}{\sqrt3} = \begin{pmatrix}1&&\\&1&\\&&0\end{pmatrix}. \tag{37}$$

Then, the singlet-octet part of (35) is written in the form

$$\mathcal{L}_m(\sigma_0, \sigma_8) = -\frac12 (m_{\sigma_0}^2\sigma_0^2 - C\sigma_0\sigma_8 + m_{\sigma_8}^2\sigma_8^2), \tag{38}$$

where the coefficient C can be fixed by the width of the decay $\Gamma_{S^*\to 2\pi} = 40$ MeV. The part of Lagrangian (17) describing such

decays is of the form

$$\mathcal{L}(\epsilon, S^*, \pi) = \frac{2}{\sqrt3} g m_u (\sigma_8 + \sqrt2\sigma_0)\vec\pi^2 = 2gm_u\,[\cos(\theta_0-\theta)\epsilon + \sin(\theta_0-\theta)S^*]\,\vec\pi^2, \tag{39}$$

where θ_0 is an ideal angle of mixing, and θ is a real angle of mixing of singlet-octet components of σ-particles (see (31)). Upon introducing the mixing angle the quantities we are interested in are written as follows:

$$C = (m_{\sigma_8}^2 - m_{\sigma_0}^2)\,\mathrm{tg}2\theta = 2(m_s^2 - m_u^2)\mathrm{tg}2\theta\,,$$

$$m_\epsilon^2 = m_{\sigma_0}^2 + (m_s^2 - m_u^2)[2\sin^2\theta - \sin 2\theta\,\mathrm{tg}2\theta]\,,$$
$$m_{S^*}^2 = m_{\sigma_0}^2 + (m_s^2 - m_u^2)[2\cos^2\theta + \sin 2\theta\,\mathrm{tg}2\theta], \tag{40}$$
$$\Gamma_{S^*\to 2\pi} = \sin^2(\theta_0 - \theta)\frac{(gm_u)^2}{\pi m_{S^*}}\sqrt{1 - \left(\frac{2m_\pi}{m_{S^*}}\right)^2}\,.$$

Good agreement with experiment is achieved at $\theta = 6°$:

$$m_\epsilon = 760\ \text{MeV}, \quad m_{S^*} = 950\ \text{MeV}, \quad \Gamma_{S^*\to\pi^+\pi^-} = 27\ \text{MeV},$$
$$\Gamma_{S^*\to 2\pi} = 40\ \text{MeV}. \tag{41}$$

5. VECTOR MESONS

We proceed now to describe interactions of vector and axial-vector mesons. In the preceding section we have observed that the final results of the description of singlet-octet components of scalar and pseudoscalar mesons contain always the ideal mixing of these components. However, the inclusion of gluon anomalies and annihilation channels results in a considerable deviation from the ideal mixing. Since in the description of vector and axial-vector mesons such reasons for the deviation from the ideal angle of mixing should not arise, it will be convenient to work in Lagrangians (1) and (11) directly with the matrices λ_u and λ_s instead of λ_0 and λ_8 and with components $V_u(A_u)$ and $V_s(A_s)$ instead of $V_0(A_0)$ and $V_8(A_8)$. Then, as we shall see in what follows, no nondiaginal terms will arise in the quadratic part of the effective meson Lagrangian.

Summing up divergent loop diagrams with two, three, and four vector legs we arrive at the expression

$$-\frac13 I_2 \mathrm{Tr}\{\bar V_{\mu\nu} - i[\bar V_\mu, \bar V_\nu]\}^2\,, \tag{42}$$

where $\bar V_{\mu\nu} = \lambda_a[\partial_\mu V_\nu^a - \partial_\nu V_\mu^a]$, and $[\bar V_\mu, \bar V_\nu]$ is the commutator of operators $\bar V_\mu = \lambda_a V_\mu^a$. Upon the renormalization

$$V_\nu^a = \sqrt{\frac32}\, g V_\nu^{aR} \tag{43}$$

the Lagrangian of interaction of vector fields acquires the form *

$$\mathcal{L}(V) = \frac{m_V^2}{2}(V_\mu^a)^2 - \frac{1}{8}\mathrm{Tr}\{\bar V_{\mu\nu} - i\frac{g_\rho}{2}[\bar V_\mu, \bar V_\nu]_-\}^2 - \qquad (44)$$

$$- i\,\mathrm{Tr}\ln\{1 + \frac{1}{i\hat\partial - m_u}\frac{g_\rho}{2}\hat{\bar V}\}',$$

where

$$g_\rho = \sqrt{6}\,g, \quad m_V^2 = \frac{3}{2}\frac{g^2}{G_2}.$$

Considering divergent triangular and box diagrams of a mixed type, where besides the vector mesons in interaction there take part also scalar and pseudoscalar mesons (see Fig.2), we arrive at the following interaction Lagrangian

$$\mathcal{L}(\sigma, \phi, V) = \frac{1}{4}\mathrm{Tr}\{D_\mu\bar\sigma'' D^\mu\bar\sigma'' + D_\mu\bar\phi D^\mu\bar\phi\}, \qquad (45)$$

where

$$\bar\sigma'' = \bar\sigma - \frac{m_s - m_u}{\sqrt{2}\,g}\lambda_s, \quad D_\mu\bar a = \partial_\mu\bar a + i\frac{g_\rho}{2}[\bar V_\mu, \bar a]$$

is a covariant derivative of scalar and pseudoscalar fields.

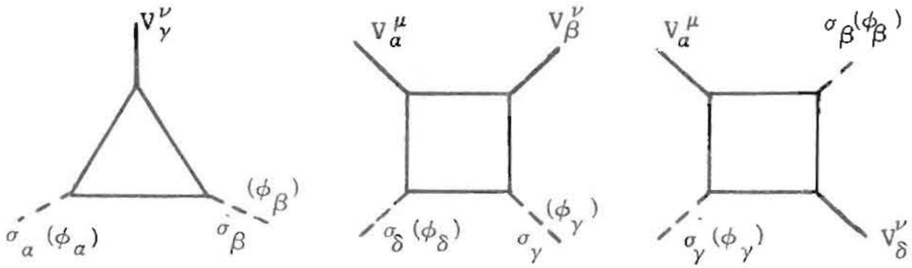

Fig.2

* To write the formulae in a more simple form, the index R of fields V will be omitted hereafter. For the gradient-noninvariant regularization we obtain $m_V^2 = \frac{3}{2}g^2[\frac{1}{G_2} - 4(I_1 + m_u^2 I_2)]$ (see /4/).

The Lagrangian obtained contains terms describing the decay $\rho \to 2\pi$ the width of which fixes the constant g_ρ (see formula (25) and ref. /1/). If the constants G_2 are assumed to be the same for vector-type interactions of all sorts of quarks, the Lagrangians (44) and (45) produce the following mass formulae for vector mesons

$$m_\rho^2 = m_\omega^2 = m_\phi^2 = \frac{3g^2}{2G_2^V}, \quad m_{K^*}^2 = m_\rho^2 + \frac{3}{2}(m_s - m_u)^2. \qquad (46)$$

The ω-meson consists only of u- and d-quarks; while ϕ-meson, of s-quarks. Only the mass of ϕ-meson is in poor agreement with experiment. This causes us to assume the coupling constant of the four s-quarks interaction G_2^s in (1) to differ from other constants of the vector interaction $G_2^{(V)}$*.

6. PSEUDOVECTOR MESONS

Interaction between pseudovector mesons is described by divergent loop diagrams of two types: with two and four meson legs (Fig.3). Upon the renormalization like for vector fields $A_\nu^a = \frac{g_\rho}{2}A_\nu^{aR}$ we get for the pseudovector Lagrangian the expression

$$\mathcal{L}(A) = \frac{(m_V^2 + 6m_u^2)}{2}(A_\mu^a)^2 - \frac{1}{4}(A_{\mu\nu}^a)^2 + \frac{g_\rho^2}{32}\mathrm{Tr}\{[\bar A_\nu, \bar A_\mu]_-\}^2 - \qquad (47)$$

$$- i\,\mathrm{Tr}\ln\{1 + \frac{1}{i\hat\partial - m_a}\frac{g_\rho}{2}\hat{\bar A}\gamma_5\}'.$$

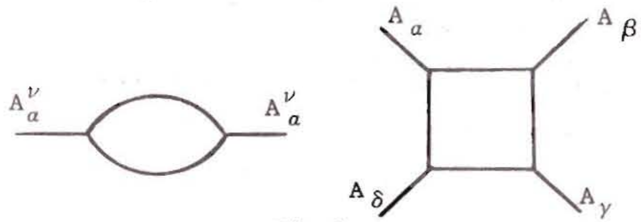

Fig.3

* Inequality $m_\phi(m_s) > m_\rho(m_u) = m_\omega(m_u)$ is fulfilled at $G_2^s = G_2^{(V)}$, but if we take into account the dependence of divergent integrals on quark masses $(I_1(m_s) < I_1(m_u),\ I_2(m_s) < I_2(m_u)$, see (12) and the cut-off regularization, §3).

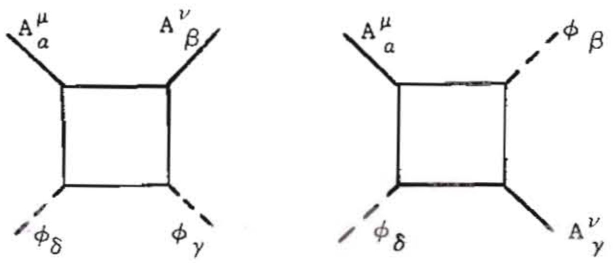

Fig.4

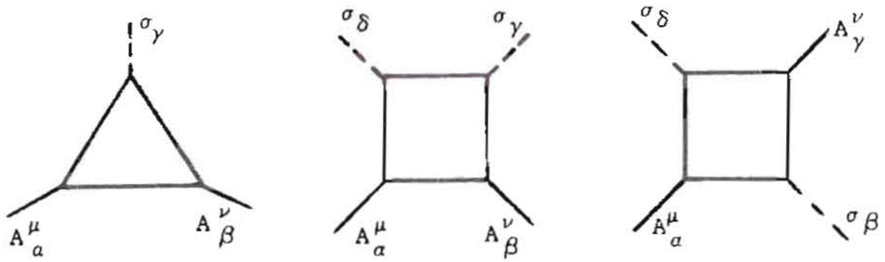

Fig.5

Interaction of pseudovector mesons with pseudoscalar mesons is described by divergent diagrams of the box type only (Fig.4). They lead to the following Lagrangian

$$\mathcal{L}(\phi, A) = \frac{g_\rho^2}{16}\,\mathrm{Tr}\{\bar\phi, \bar A_\mu\}_+^2 , \qquad (48)$$

where under the Tr there is the anticommutator squared.

The contribution to the interaction Lagrangian of scalar and pseudovector mesons comes from divergent diagrams of both triangular and box types (Fig.5). In terms of the σ-fields from the Lagrangian (11) this contribution is written as follows:

$$\mathcal{L}_\Delta(\sigma, A) = -3m_u g\,\mathrm{Tr}(\bar\sigma \bar A_\mu \bar A_\mu),$$
$$\mathcal{L}_\Box(\sigma, A) = \frac{g_\rho^2}{16}\,\mathrm{Tr}\{\bar\sigma, A_\mu\}_+^2 . \qquad (49)$$

Taking the σ-fields with zero vacuum expectation values we get for the sum contribution (49)

$$\mathcal{L}(\sigma, A) = -3m_u^2 (A^a_\mu)^2 + \frac{g_\rho^2}{16}\,\mathrm{Tr}\{\bar\sigma''', \bar A_\mu\}_+^2 , \qquad (50)$$

where $\bar\sigma''' = \bar\sigma - \frac{1}{g}(m_u\lambda_u + \frac{m_s}{\sqrt{2}}\lambda_s)$. Here $\bar\sigma'''$ differs from $\bar\sigma''$ in (45), as in going from (49) to (50) we have made the change of variables (14).

From the Lagrangians (47) and (50) we obtain the mass formulae for pseudovector mesons

$$m^2_{A_1} = m^2_\rho + 6m^2_u , \qquad m^2_{A_{1/2}} = m^2_\rho + \frac{3}{2}(m_s + m_u)^2 ,$$
$$m^2_{A_u} = m^2_\rho + 6m^2_u , \qquad m^2_{A_s} = m^2_\rho + 6m^2_s , \qquad (51)$$

that gives the numerical values

$$m_{A_1} \approx 1\ \mathrm{GeV},\quad m_{A_{1/2}} \approx 1200\ \mathrm{MeV},\quad m_{A_u} = 1\ \mathrm{GeV},\quad m_{A_s} = 1400\ \mathrm{MeV}.$$

Experimental masses of pseudovector mesons are [16]

$$m_{A_1} = 1100\text{–}1300\ \mathrm{MeV},\ m_{A_{1/2}=Q_1} = 1280\ \mathrm{MeV},\ m_D = 1284\ \mathrm{MeV},\ m_E = 1418\ \mathrm{MeV}. \qquad (52)$$

Somewhat low theoretical values for masses of A_1- and D-mesons can again be explained by that the corresponding constants G_2 are not equal to the constant $G_2^V = \frac{3}{2}\frac{g^2}{m_\rho^2}$.

To conclude this section, we will write the Lagrangians of mixed interactions of mesons σ–ϕ–A and ϕ–A–V

$$\mathcal{L}(\sigma, \phi, A) = \frac{g_\rho}{4}\,\mathrm{Tr}[\bar A_\nu(\{\partial_\nu \bar\sigma'', \bar\phi\}_+ - \{\bar\sigma'', \partial_\nu \bar\phi\}_+)], \qquad (53)$$

$$\mathcal{L}(\phi, A, V) = i\frac{3}{4}g\,\mathrm{Tr}(\{\bar A_\nu, \bar\phi\}_+ [M, \bar V_\nu]_- + \{\bar A_\nu, M\}_+ [\bar V_\nu, \bar\phi]_-). \qquad (54)$$

7. CONCLUSION

The main result of this work is the construction of effective Lagrangians which describe interactions of 36 different mesons (scalar, pseudoscalar, vector, and pseudovector nonets) with three parameters g, m_u and m_s only. The strong constant of pion-quark interactions g appears to be small enough so that it is possible to use the standard perturbation theory in this coupling constant. Introducing into the consideration electromagnetic interactions, like in ref. [1], and weak interactions [17], we may describe by our model practically all the decays of the above-mentioned mesons and their important characteristics (scattering lengths, electromagnetic and weak form factors, polarizabilities, etc.). Calculations are in satisfactory agreement with experiment (see refs. [1,9,10,11,18]).

Less exact results follow in deriving mass formulae. It is to be noted that in this case we have much more arbitrary parameters. Besides parameters g, m_u, and m_s there are parameters G_1 and G_2. As to the latter, it nowise follows that there should exist only two such parameters. From the requirement of chiral symmetry we may conclude that the constants G_1^a for scalar and pseudoscalar mesons with the same index a should be equal to each other, whereas all the other constants G_1^α, $G_{2(V)}^\alpha$, $G_{2(A)}^\alpha$, in general, may differ from each other, however, we have taken them equal from the considerations of maximal simplicity. The mass formulae found under these simplest assumptions are in general in a qualitative agreement with experiment except for three cases. The masses predicted for scalar mesons $\vec{\delta}$ and $\tilde{K}$ are approximately twice as small as experimental ones that may be a result either of the strong violation of chiral symmetry $(G_1(\vec{\pi}) \neq G_1(\vec{\delta}))$, or of the presence of four-quark states in these mesons [15]. The mass of ϕ-meson appears to equal the masses of ρ and ω-mesons if $G_2(\phi) = G_2(\rho) = G_2(\omega)$ and $g(m_u) = g(m_s)$ is assumed. If we take into account the dependence of divergent integrals on quark masses $(I_1(m_u) > I_1(m_s),\ I_2(m_u) > I_2(m_s))$ then we obtain $m_\phi > m_\rho = m_\omega$.

Interesting results are obtained in estimating the quark masses arising in this model. It turns out that the bare quark masses in the initial pure quark Lagrangian coincide in magnitude with the current quark masses, while the quark masses appearing after introducing the mesons and the spontaneous breaking of chiral symmetry are equal to the constituent quark masses. This fact testifies to the consistency of the model.

APPENDIX I

Let us write in more detail the part of the Lagrangian (17) for the interaction of scalar and pseudoscalar mesons of the third and fourth power in the fields

$$\mathcal{L}_{int}(\sigma,\phi) = -\frac{4g}{3}\{\frac{g}{4}(\sigma_0^2+3\sigma_a^2+\phi_\alpha^2)^2 + g[\phi_0^2(\phi_a^2-\sigma_a^2)-(\sigma_a\phi_a)^2] +$$

$$+ 2g[\sigma_0\phi_0\sigma_a\phi_a - (\sigma_a^2)^2] - (a\sigma_0 + 3b\sigma_8)(\sigma_0^2 + 3\sigma_a^2 + \phi_\alpha^2) +$$

$$+ 2b[\phi_8(\sigma_a\phi_a) + \sigma_8\phi_0^2] - 2\phi_0(a\sigma_a\phi_a + b\sigma_0\phi_8) + 8b\sigma_8\sigma_a^2\} -$$

$$-2\sqrt{\frac{2}{3}}g^2\{\sigma_0 d_{abc}\sigma_a(\sigma_b\sigma_c + \phi_b\phi_c) + \phi_0 d_{abc}\phi_a(\sigma_b\sigma_c + \phi_b\phi_c)\} -$$

$$-3g^2 d_{ab\ell}\sigma_a\sigma_b d_{cd\ell}\phi_c\phi_d + 2g^2(d_{abc}\sigma_a\phi_b)^2 -$$

$$-\frac{g^2}{2}[(d_{abc}\sigma_a\sigma_b)^2 + (d_{abc}\phi_a\phi_b)^2] + 2\sqrt{\frac{2}{3}}g\{b\sigma_0 d_{8aa}(\phi_a^2 + 3\sigma_a^2) +$$

$$+ ad_{abc}\sigma_a(\sigma_b\sigma_c + \phi_b\phi_c) + 2b\phi_0 d_{8aa}\sigma_a\phi_a\} +$$

$$+ g\{6bd_{8bb}d_{bac}\sigma_b\phi_a\phi_c - 4bd_{8bb}d_{bac}\phi_b\sigma_a\phi_c +$$

$$+ \frac{2}{3}b\sigma_8^3 + 2bd_{8cc}d_{cab}\sigma_c\sigma_a\sigma_b\,|_{a,b,c\neq 8} +$$

$$+ 2b\sigma_8 d_{8aa}(2d_{8aa} - \frac{1}{\sqrt{3}})\sigma_a^2\,|_{a\neq 8}\}.$$

Summation runs over dummy indices. Latin indices vary from 1 to 8; Greek ones, from 0 to 8; and d_{abc} are structure constants of SU(3) group:

$$a = \frac{2m_u + m_s}{\sqrt{6}},\quad b = \frac{m_u - m_s}{\sqrt{3}},\quad g = \sqrt{2\pi}.$$

APPENDIX II

The part of the effective Lagrangian for the interactions of the vector and axial-vector mesons with each other and with the rest of mesons can be written in a very compact form following the work [4]

$$\mathcal{L}(\sigma,\phi,A,V) = -\frac{1}{8}\mathrm{Tr}(G_V^{\mu\nu}G_{V\mu\nu}) - \frac{1}{8}\mathrm{Tr}(G_A^{\mu\nu}G_{A\mu\nu}) +$$

$$+\frac{1}{4}\mathrm{Tr}(D_\mu\bar{\sigma}'' - \frac{g_\rho}{2}\{\bar{A}_\mu,\bar{\phi}\}_+)^2 + \frac{1}{4}\mathrm{Tr}(D_\mu\bar{\phi} + \frac{g_\rho}{2}\{\bar{A}_\mu,\bar{\sigma}''\}_+)^2,$$

where

$$G_V^{\mu\nu} = \partial^\mu\bar{V}^\nu - \partial^\nu\bar{V}^\mu - i\frac{g_\rho}{2}\{[\bar{V}^\mu,\bar{V}^\nu]_- + [\bar{A}^\mu,\bar{A}^\nu]_-\},$$

$$G_A^{\mu\nu} = \partial^\mu\bar{A}^\nu - \partial^\nu\bar{A}^\mu - i\frac{g_\rho}{2}\{[\bar{A}^\mu,\bar{V}^\nu]_- + [\bar{V}^\mu,\bar{A}^\nu]_-\}.$$

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Мезонные лагранжианы группы U(3) в модели с четырехкварковыми взаимодействиями

На основе эффективного лагранжиана с четырехкварковыми взаимодействиями построены феноменологические лагранжианы, описывающие взаимодействия скалярных, псевдоскалярных, векторных и псевдовекторных нонетов. Получены формулы для масс этих мезонов. Даны оценки для токовых и составных кварковых масс.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1983

Kreopalov D.V., Volkov M.K. E2-83-19

Meson Lagrangians of the U(3) Group in the Model with Four-Quark Interactions

On the basis of an effective Lagrangian with four-quark interactions we have constructed phenomenological Lagrangians for interactions of scalar, pseudoscalar, vector, and pseudovector meson nonets. The formulae are obtained for meson masses. Estimations are found for current and constituent quark masses.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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