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$\pi\pi$-SCATTERING LENGTHS

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In papers [1-4] the model based on four-quark interactions has been presented which permits one to obtain the well-known phenomenological Lagrangians of the low energy meson interactions. All effective coupling constants are connected with one another and are expressed through the constants of two decays $\rho \to 2\pi (g_\rho)$ and $\pi \to \mu\bar{\nu}(F_\pi)$. It was assumed that all mesons are two-quark systems and may interact with one another through quark loops only. One may obtain simple relations between meson coupling constants if one considers only the divergent quark loops and neglects their finite parts.

In the present paper we take into account the finite parts of quark loops, called q^2-terms. These terms are necessary for describing electromagnetic radii of mesons, scattering lengths, slope parameters of different processes, etc. Efforts have been made to take account of these terms of the quark loops [5,6]. Here we follow these papers.

The relations between meson coupling constants obtained in [1-4] are approximate, because we neglect the masses of external mesons. Here we assume that these relations do not change essentially when we take into account the q^2-terms*. All the effective coupling constants in [1-4] have been described by logarithmically divergent quark loops. Therefore, the next step of the quark loop expansion needed for obtaining the q^2-terms leads to convergent integrals, and the coefficients of these terms are determined unambiguously. The problem is to redetermine the constant terms after the q^2-terms are picked out from divergent integrals. We shall proceed from the natural requirement that on the mass shell the form factor of the corresponding process should be equal to its physical constant. For example, the $\rho \to 2\pi$ decay form factor is

$$T_{\rho \to 2\pi} = -ig_\rho\left(1 + \frac{q^2 - m_\rho^2 + p_i^2 - m_\pi^2 + p_j^2 - m_\pi^2}{8\pi^2 F_\pi^2}\right) p_i^\nu \epsilon_{ijk} \pi^i \pi^j \rho_\nu^k , \quad (1)$$

where q, p_i, p_j are the moments of the ρ-meson and pions, respectively, m_ρ and m_π are their masses, $F_\pi = 95$ MeV is the pion decay constant and g_ρ is the ρ-decay constant $(\frac{g_\rho^2}{4\pi} \approx 3)$.

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* Calculations [3,6,7,8] have shown that the estimated values of the constants in [3] are in good agreement with the experimental data.

The $\epsilon \to 2\pi$ decay form factor is equal to

$$T_{\epsilon \to 2\pi} = 2m_q g \{ 1 - \frac{q^2 - m_\epsilon^2 - 3(p_1^2 - m_\pi^2 + p_2^2 - m_\pi^2)}{16\pi^2 F_\pi^2} \} \epsilon \vec{\pi}^2 , \qquad (2)$$

where q, p_1, p_2 are the moments of the ϵ-meson and pions, m_q is the quark mass and $g \approx \sqrt{2\pi} = m_q / F_\pi$ is the strong constant of pion-pion interactions ($g_\rho = \sqrt{6}\, g$).

The form factors (1) and (2) are easily obtained from the Lagrangian

$$\mathcal{L} = \bar{q} [i\hat{\partial} - m_q + g(\epsilon + i\gamma_5 \vec{\tau}\vec{\pi}) + \frac{g_\rho}{2} \vec{\tau}\hat{\vec{\rho}}] q , \qquad (3)$$

by using the method described in paper [3].

To describe the $\pi\pi$-scattering, it is necessary to take into account the quark box diagrams. Their contribution to the $\pi\pi$-scattering amplitude is equal to*

$$T_{4\pi} = - \frac{g^2}{2} [1 - \frac{s - c}{(2\pi F_\pi)^2}] (\vec{\pi}^2)^2 . \qquad (4)$$

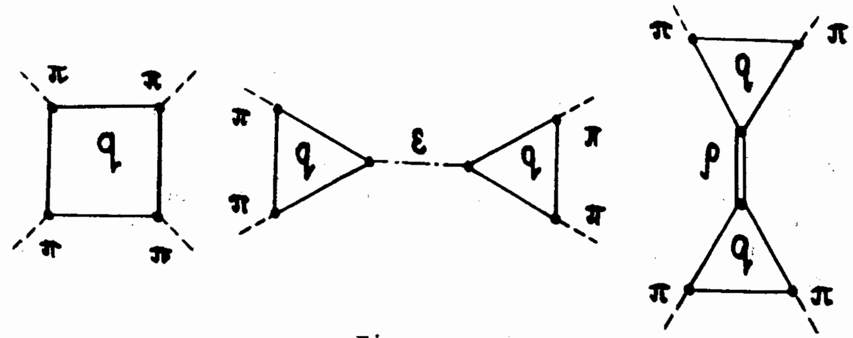
Here $s = (p_1 + p_2)^2$, where p_1 and p_2 are the moments of initial pions. c is the indefinite parameter. For the form factors (1) and (2) the same parameters were fixed by the requirement that on the mass shell the form factors equal the corresponding decay constants. Our definition of the c parameter for $T_{4\pi}$ is based on the correspondence of our amplitude of the $\pi\pi$-scattering and the low-energy current algebra theorems.

All diagrams which give their contribution to the $\pi\pi$ amplitude are plotted in the figure. They lead to the expression

$$\mathcal{A}(s,t,u) = -4g^2 \{ 1 - \frac{s - c}{(2\pi F_\pi)^2} + \frac{4m_q^2}{s - m_\epsilon^2} [1 - \frac{s - m_\epsilon^2}{(4\pi F_\pi)^2}]^2 \} + \qquad (5)$$

$$+ g_\rho^2 \{ \frac{s - u}{m_\rho^2 - t} (1 + \frac{t - m_\rho^2}{8\pi^2 F_\pi^2})^2 + \frac{s - t}{m_\rho^2 - u} (1 + \frac{u - m_\rho^2}{8\pi^2 F_\pi^2})^2 \} .$$

*The first term in (4) corresponds to the σ-model (see, for example, [3]).



Figure

If we now require that the constant part of the amplitude (5) should correspond to the "improved" Weinberg formula ($\mathcal{A}(s) = \frac{s - m_\pi^2}{F_\pi^2}$) [9], we obtain the following value of c*:

$$c \approx 2m_q^2 (1 + \frac{m_\epsilon^2}{32\pi^2 F_\pi^2}) \approx 6{,}4\, m_\pi^2 , \qquad (6)$$

$$(m_\epsilon^2 = m_\pi^2 + 4m_q^2).$$

The amplitude $\mathcal{A}(s,t,u)$ can now be written in the form

$$\mathcal{A}(s,t,u) = - \frac{m_\pi^2}{F_\pi^2} + 4g^2 [\frac{4m_q^2}{m_\epsilon^2 (m_\epsilon^2 - s)} + \frac{1 - (\frac{m_q}{4\pi F_\pi})^2}{(2\pi F_\pi)^2}] s + \qquad (7)$$

$$+ g_\rho^2 \{ \frac{s - u}{m_\rho^2 - t} (1 + \frac{t - m_\rho^2}{8\pi^2 F_\pi^2})^2 + \frac{s - t}{m_\rho^2 - u} (1 + \frac{u - m_\rho^2}{8\pi^2 F_\pi^2})^2 \} .$$

* The "improved" Weinberg formula is automatically obtained from (5), if we neglect the ρ-meson contributions and the q^2-terms in the quark loops (i.e. the combinations $\frac{q^2 - m_i^2}{aF_\pi^2}$ in (5))

$$\mathcal{A}(s,t,u) \approx -4g^2 (1 + \frac{4m_q^2}{s - m_\epsilon^2}) \approx \frac{s - m_\pi^2}{F_\pi^2} .$$

It is easy to see that the first term in the square bracket of (7) is very close to the Weinberg coefficient for $s \sim 1/F_\pi^2$. The second term is the result of the inclusion of the box diagram q^2-terms. It changes the coefficient of s, and as a result, the a_0^0-scattering length. On the other hand, the choice of the form factors in the form (1) and (2) leads to a very small influence of ρ-meson diagrams on the s-wave scattering lengths, which are in good agreement with the physical requirements.

Before the calculation of the scattering lengths, we should discuss the Weinberg relation

$$2a_0^0 - 5a_0^2 = \frac{3m_\pi}{4\pi F_\pi^2}. \tag{8}$$

In our case we obtain a similar relation

$$2a_0^0 - 5a_0^2 = \frac{12g^2 m_q^2 m_\pi}{\pi m_\epsilon^2 (m_\epsilon^2 - 4m_\pi^2)} + \frac{3g^2 m_\pi}{\pi (2\pi F_\pi)^2}\left(1 - \frac{m_q^2}{(4\pi F_\pi)^2}\right) + \frac{9g_\rho^2 m_\pi}{4\pi m_\rho^2}\left(1 - \frac{m_\rho^2}{8\pi^2 F_\pi^2}\right)^2. \tag{9}$$

It is easy to see that here the first term corresponds to the right-hand side of (8)*. The second term gives a 50% correction to the first one. It is connected with the q^2-terms of the box diagrams. The third term appears as a result of the ρ-meson diagrams and its value is small in comparison with other terms.

Let us now calculate the $\pi\pi$-scattering lengths. For the amlitude (7) in different channels we have

$$\mathcal{A}^0 = -5\frac{m_\pi^2}{F_\pi^2} + 8g^2\left[\frac{s + 2m_\pi^2}{(2\pi F_\pi)^2}\left(1 - \frac{m_q^2}{(4\pi F_\pi)^2}\right) + 2\frac{m_q^2}{m_\epsilon^2}\left(\frac{3s}{m_\epsilon^2 - s} + \frac{t}{m_\epsilon^2 - t} + \frac{u}{m_\epsilon^2 - u}\right)\right] +$$

$$+ 2g_\rho^2\left[\frac{s-u}{m_\rho^2 - t}\left(1 + \frac{t - m_\rho^2}{8\pi^2 F_\pi^2}\right)^2 + \frac{s-t}{m_\rho^2 - u}\left(1 + \frac{u - m_\rho^2}{8\pi^2 F_\pi^2}\right)^2\right], \tag{10a}$$

* Indeed, if we take into account the fact that $g = \frac{m_q}{F_\pi}$, $m_\epsilon^2 = m_\pi^2 + 4m_q^2$, we get $\frac{12g^2 m_q^2 m_\pi}{\pi m_\epsilon^2 (m_\epsilon^2 - 4m_\pi^2)} \approx \frac{3m_\pi}{4\pi F_\pi^2}$.

$$\mathcal{A}^1 = 4g^2\left[\frac{t-u}{(2\pi F_\pi)^2}\left(1 - \frac{m_q^2}{(4\pi F_\pi)^2}\right) + 4\frac{m_q^2}{m_\epsilon^2}\left(\frac{t}{m_\epsilon^2 - t} - \frac{u}{m_\epsilon^2 - u}\right)\right] +$$

$$+ g_\rho^2\left[2\frac{t-u}{m_\rho^2 - s}\left(1 + \frac{s - m_\rho^2}{8\pi^2 F_\pi^2}\right)^2 + \frac{t-s}{m_\rho^2 - u}\left(1 + \frac{u - m_\rho^2}{8\pi^2 F_\pi^2}\right)^2 + \frac{s-u}{m_\rho^2 - t}\left(1 + \frac{t - m_\rho^2}{8\pi^2 F_\pi^2}\right)^2\right], \tag{10b}$$

$$\mathcal{A}^2 = -2\frac{m_\pi^2}{F_\pi^2} + 4g^2\left[\frac{4m_\pi^2 - s}{(2\pi F_\pi)^2}\left(1 - \frac{m_q^2}{(4\pi F_\pi)^2}\right) + 4\frac{m_q^2}{m_\epsilon^2}\left(\frac{t}{m_\epsilon^2 - t} + \frac{u}{m_\epsilon^2 - u}\right)\right] +$$

$$+ g_\rho^2\left[\frac{s-t}{u - m_\rho^2}\left(1 + \frac{u - m_\rho^2}{8\pi^2 F_\pi^2}\right)^2 + \frac{s-u}{t - m_\rho^2}\left(1 + \frac{t - m_\rho^2}{8\pi^2 F_\pi^2}\right)^2\right]. \tag{10c}$$

These amplitudes lead to the values of $\pi\pi$-scattering lengths

$$a_0^0 = 0{,}37\, m_\pi^{-1}, \quad a_0^2 = -0{,}046\, m_\pi^{-1}, \quad a_1^1 = 0{,}046\, m_\pi^{-3},$$
$$a_2^0 = 19\cdot 10^{-4} m_\pi^{-5}, \quad a_2^2 = 13\cdot 10^{-4} m_\pi^{-5}, \quad a_3^1 = 1\cdot 10^{-4} m_\pi^{-7}. \tag{11}$$

Here we use the values for g, m_q, and m_ϵ which have been obtained in /3/*.

At present time the following experimental data for these scattering lengths are known /10/:

$$a_0^0 = (0{,}31 \pm 0{,}11) m_\pi^{-1}, \quad a_0^2 = (-0{,}028 \pm 0{,}012)\, m_\pi^{-1},$$
$$a_1^1 = (0{,}038 \pm 0{,}002)\, m_\pi^{-3},$$
$$a_2^0 = (17 \pm 3)\cdot 10^{-4} m_\pi^{-5}, \quad a_2^2 = (1{,}3 \pm 3)\cdot 10^{-4} m_\pi^{-5},$$
$$a_3^1 = (0{,}6 \pm 0{,}2)\cdot 10^{-4} m_\pi^{-7}. \tag{12}$$

The greatest disagreement with the experimental data has been observed for the a_2^2 scattering length, but this length has been measured with a large error, and we believe that more accurate experiments will change considerably this value.

Our estimations show that the values for a_2^0 and a_2^2 are close to each other, because the t- and u-dependent parts of $\mathcal{A}^0$ and $\mathcal{A}^2$, which give the main contributions to the amplitude d-

* If we take $m_\epsilon \approx 700$ MeV /10,11/, we get too low values for the scattering lengths $a_0^0 = 0{,}12\, m_\pi^{-1}$, $a_0^2 = -0{,}046\, m_\pi^{-1}$, $a_1^1 = 0{,}028\, m_\pi^{-3}$, $a_2^0 = 6\cdot 10^{-4} m_\pi^{-5}$, $a_2^2 = 5\cdot 10^{-6} m_\pi^{-5}$, $a_3^1 = 2\cdot 10^{-5} m_\pi^{-7}$.

wave, are almost equal. (They differ only by small ρ-meson pole diagrams).

The values of the a_0^0 and a_1^1 scattering lengths differ from previous theoretical estimations by Weinberg, since we have taken into account the q^2-terms of box quark diagrams. The a_0^2 length corresponds to Weinberg's result, because we fixed the parameter c according to the requirement of the coincidence between the constant part of our scattering amplitude and the "improved" Weinberg formula [9].

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Волков М.К., Осипов А.А. E2-83-177

Длины $\pi\pi$-рассеяния

В модели составных мезонов, основанной на рассмотрении четырехкварковых взаимодействий, получены оценки длин рассеяния a_0^0, a_0^2, a_1^1, a_2^0, a_2^2 и a_3^1. В расчетах использованы формфакторы распадов $\rho \to 2\pi$, $\epsilon \to 2\pi$ и учтена q^2 зависимость четырехугольных кварковых диаграмм.

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$\pi\pi$-Scattering Lengths

The a_0^0, a_0^2, a_1^1, a_2^0, a_2^2, and a_3^1 scattering lengths are calculated in the framework of the composite meson model which is based on four-quark interactions. The decay form factors of $\rho \to 2\pi$ and $\epsilon \to 2\pi$ are used. The q^2-terms of the quark box diagrams are taken into account.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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