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NEUTRINO OSCILLATIONS WITH LARGE OSCILLATION LENGTH IN SPITE OF LARGE (MAJORANA) NEUTRINO MASSES?

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1. INTRODUCTION

The question[1] on finite neutrino masses and neutrino oscillations remains one of the utmost importance. At present a large number of search experiments related to this question are either being performed or planned at various facilities [2]. Recently[3,4] a few improved experiments on neutrino oscillations have been carried out. In these investigations oscillations were not found.

As to the claim to the discovery of oscillations[5], today not only there is no conformation of it, but also there is a definite disproof of it in the recent, more accurate reactor experiments[4].

The new experiments[3,4] reduce the region of possible values of the parameters Δm^2 and $\sin^2 2\theta$ ($\Delta m^2 = |m_1^2 - m_2^2|$, m_1 and m_2 being the neutrino masses and θ the mixing angle). From a cosmic-ray experiments of the Chudakov group [3] in which there were registered muons produced by "atmospheric" ν_μ generated at the opposite side of the Earth, it follows that $\Delta m^2 < 6\cdot 10^{-3}$ eV^2 if $\sin^2 2\theta \simeq 1$. From a reactor experiment of the Mossbauer group[4], in which the process $\tilde{\nu}_e + p \to e^+ + n$ was observed, it follows that $\Delta m^2 \lesssim 0.016$ eV^2 if $\sin^2 2\theta \simeq 1$ and $\sin^2 2\theta \le 0.17$ if $\Delta m^2 \gtrsim 5$ ev^2*.

At present from the comparison of direct (^{3}H beta spectrum...) experiments ("large neutrino mass"!)[6] with the neutrino oscillation investigations [2,4] (at best small mass differences!), the suggestion follows that the neurtino mixing schemes considered by us in ref.[7] might be quite possible.

The essence of the schemes considered in ref.[7] is:

1) the neutrinos with definite masses are two Majorana particles;

2) the oscillation amplitude $\sin^2 2\theta$ has the maximum value 1.

Such a picture was justified by some symmetry conditions. In this paper we continue the discussion initiated in ref.[7] and consider different conditions that also lead to 1) and 2).

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*Notice that according to the usual practice we use the same notations (Δm^2 and θ) for quantities which may be entirely different.

2. GENERAL DISCUSSION OF THE SCHEMES AT ISSUE

The general neutrino mass term in all the schemes at issue has the form

$$\mathcal{L} = -\frac{1}{2}[m_L(\bar{\nu}_L)^c \nu_L + m_R \bar{\nu}_R (\nu_R)^c + \\ + m_D(\bar{\nu}_R \nu_L + (\bar{\nu}_L)^c (\nu_R)^c)] + \text{h.c.} = -\frac{1}{2}(\bar{n}_L)^c M n_L + \text{h.c.} \tag{1}$$

Here $\nu^c = C\bar{\nu}^T$ is the charge-conjugated spinor,

$$n_L = \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix}, \quad M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}. \tag{2}$$

It is easy to see that the mass term (1) implies either two active and two sterile states or four active (eigen) states (of the weak interacton). The latter case corresponds to the renewed scheme [8] of Zeldovich, Konopinsky and Mahmoud [9] (Z-K-M) with only one violated lepton charge which is equal for e^- and μ^+ and for ν_e and $\bar{\nu}_\mu$ (we use the standard notation for the weak interaction neutrinos ν_e, ν_μ, ν_τ, $\bar{\nu}_e$, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$). We shall neglect CP violation effects, that is we assume that the parameters m_L, m_R and m_D are real. The first two members in expression (1) are Majorana mass terms, the third one is a Dirac mass term.

The diagonalization of the type (1) general mass term was first given in ref. [10] (see also review [11]). For convenience, below we give such a general solution in a form useful for our discussion. We have

$$M = O m' O^T, \tag{3}$$

where

$$m' = \begin{pmatrix} m'_1 & 0 \\ 0 & m'_2 \end{pmatrix}, \quad O = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}. \tag{4}$$

The quantities m'_1, m'_2 and $tg 2\theta$ are related to the parameters m_L, m_R and m_D through the relations

$$m'_{1,2} = \frac{1}{2}(m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4m_D^2}),$$
$$tg 2\theta = \frac{2m_D}{m_L - m_R}. \tag{5}$$

The eigenvalues m'_1 and m'_2 of the matrix M may be positive as well as negative. Let us write

$$m'_i = m_i \eta_i, \tag{6}$$

where m_i is the modulus of m'_i and the factor η_i takes the values ± 1. Making use of eqs. (1), (3) and (6) we get

$$\mathcal{L} = -\frac{1}{2} \sum_{i=1,2} m_i \bar{\chi}_i \chi_i. \tag{7}$$

Here

$$\chi = O^T n_L + \eta (O^T n_L)^c = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}. \tag{8}$$

We have

$$\chi_i^c = \eta_i \chi_i. \tag{9}$$

This is a well-known result: in the case of the coexistence of Majorana and Dirac mass terms the mass eigenstates are Majorana's*. The factor η_i can be obviously thought of as the C-parity of the Majorana field χ_i.**

3. THE CONDITION $m_D \gg m_L, m_R$ (TWO ACTIVE AND TWO STERILE STATES)

In the previous paper [7] the condition $m_L = m_R$ was imposed. Here we assume

$$m_D \gg m_L, m_R. \tag{10}$$

In this paragraph we will consider the case of two active and two sterile particles. From eqs. (5) and (10) it follows that

$$\theta \simeq \frac{\pi}{4},$$
$$m'_{1,2} = \frac{1}{2}(m_L + m_R) \pm m_D. \tag{11}$$

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* The general case of an arbitrary neutrino type number was discussed in ref. [12].

** Of course, any talk about C parities of the neutrinos ν_e, ν_μ, ... has no meaning.

Obviously $m_1' > 0$, $m_2' < 0$ (for definiteness we have assumed that $m_D > 0$). Thus, conditions (10) lead to opposite C-particles of the two Majorana fields χ_1 and χ_2. From (8) we get

$$\chi_1 = \frac{1}{\sqrt{2}}(\nu + \nu^c),$$
$$\chi_2 = \frac{1}{\sqrt{2}}(-\nu + \nu^c). \qquad (12)$$

The corresponding particles ν_1 and ν_2 have Majorana masses

$$m_{1,2} = m_D \pm \frac{1}{2}(m_L + m_R). \qquad (13)$$

Clearly,

$$|m_1 - m_2| \ll m_1, m_2. \qquad (14)$$

Oscillations do arise between ordinary active left-handed neutrinos and sterile left-handed antineutrinos (let us say, $\nu_e \rightleftarrows \bar{\nu}_{eL(ster)}$). For eqs. (12) and (14) it follows that the oscillations are completely analogous to the neutral kaon (and also $n \rightleftarrows \bar{n}$) oscillations, in the sense that

1) the mass difference of the Majorana neutrinos is much less than the neutrino masses,

2) the oscillation amplitude is maximum,

3) the PC-parities of the Majorana neutrino fields are opposite.

In connection with the scheme under discussion the limiting case $m_L \to 0$, $m_R \to 0$ is instructive. Here the particles with definite masses are two Majorana neutrinos with the same mass m_D and opposite C-parities. The neutrino mass term is reduced in this limit only to the Dirac mass term. This result is well-known: a Dirac field can always be represented as an equal mixture of two Majorana fields of equal mass and opposite C-parities; of course, oscillations are impossible. The scheme considered by us is close to such limiting case.

4. THE CONDITION $m_D \gg m_L, m_R$ (THE ALTERNATIVE CASE OF FOUR ACTIVE STATES)

The scheme discussed in §3 implied sterile particles $\nu_{R(ster)}$ and $\bar{\nu}_{L(ster)}$ to exist. Below we will consider an alternative scheme described by a mass term of the form (1) but not implying sterile particles: the renewed Z-K-M scheme [8].

The mass term $-m_D \bar{\nu}\nu$ in eq. (1) does conserve the lepton charge and therefore can be properly classified as Dirac mass term.*

It is assumed now that the Dirac mass term is much larger than both the Majorana mass terms. This again implies a maximum oscillation amplitude and a mass difference $|m_1 - m_2|$ much smaller than m_1 and m_2 (see eqs. (11)-(14)). Obviously, neutrino oscillations here are of the type $\nu_e \rightleftarrows \nu_\mu$ or $\nu_e \rightleftarrows \nu_\tau$, ...

5. CONCLUSIONS

As far as oscillations are concerned all the practical consequences of the schemes discussed in the preceding paper [7] hold for the schemes considered here: large oscillation amplitudes and, especially for the present paper schemes, large oscillation lengths. Let us stress again that from the point of view expressed in the present paper and in ref. [7] only cosmic-neutrino and especially solar-neutrino experiments are adequate.

The schemes discussed here naturally accomodate large Majorana masses m_1 and m_2 with a small mass difference $|m_1 - m_2|$. If $m_1 \approx m_2$ is equal or larger than a few eV, the classical methods of measuring the neutrino mass by investigating the ^3H beta-spectrum is applicable.

As for neutrinoless double β-decay ($(2\beta)_{0\nu}$-decay), these schemes are not very encouraging from the experimental point of view, despite possibly large values of Majorana masses m_1 and m_2. As was noticed by Wolfenstein [13], in the $(2\beta)_{0\nu}$-decay matrix element there is a factor $\sum_i O^2_{ei} m_i \eta_i$, which in our case is equal to $\frac{1}{2}(m_1 - m_2) = \frac{1}{2}(m_L + m_R)$. Thus, under conditions (10) the contributions of ν_1 and ν_2 to the $(2\beta)_{0\nu}$-amplitude are opposite in sign and practically cancel.**

Let us notice, incidentally, that this conclusion may not hold under the condition $m_L = m_R$ discussed in the preceding paper [7].

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*Notice, however, that in the literature there has been used a different terminology for neutrino mass terms leading to Majorana mass eigenstates. In that terminology a purely Majorana mass term implied the absense of sterile states, the coexistence of Majorana and Dirac mass terms implied the existence of sterile states (to terminology questions we shall come back at the end of the paper).

** That such a cancellation must take place is obvious, because in the Dirac limit $m_L, m_R \to 0$ the $(2\beta)_{0\nu}$-decay does not occur.

Now let us summarize the experimental situation on the finite-neutrino mass problem

1. Nobody has proven the existence of neutrino oscillations at reactor, accelerator and cosmic-neutrino facilities[2-4]. As for solar neutrino experiments, there may be a slight indication of oscillation effects[14] but there is certainly no clear proof.

2. Nobody has seen neutrinoless double β-decay[5].

3. There is one courageous experiment[6] in which a finite neutrino mass of about ten eV has been claimed.

All this is well reconcilable with the schemes considered in the present paper.

If m_1 and m_2 are less than one eV, effects of their finite values cannot be "felt" by direct methods ($^3H\,\beta$-spectrum ...) known up to now and in double β-decay investigations. Solar neutrino experiments may in principle be sensitive[10,16] to $|m_1^2-m_2^2| \gtrsim 10^{-12}$ eV^2, i.e., at mass values ~ 1 eV to $|m_1 - m_2| \gtrsim 10^{-12}$eV. This makes such experiments especially interesting in connection with the schemes considered in ref.[7] and in the present paper. Such schemes with maximum oscillation amplitude would lead unambiguously to an average intensity of detectable solar neutrinos equal to 1/2 of the expected intensity in the absence of oscillations. Notice, however, that a substantial oscillation amplitude is quite possible in realistic cases with a number of Majorana neutrino more than two. It is sufficient that there is no more than one relevent oscillation.*

Our discussion suggests that the condition $m_D \gg m_R, m_L$, we have considered in the present paper is just what one should expect if the interaction responsible for the violation of lepton charge is small as compared with the interaction which somehow generates the Dirac mass term.

Let us come back to terminology questions. Within the terminology we have been using, the question may arise: what is implied then by a purely Majorana mass term? With two Majorana eigenstates, a purely Majorana mass term is obviously obtained in the limiting case $m_D = 0$, but then there are neither oscillations of the type $\nu_e \rightleftarrows \bar{\nu}_{eL(ster)}$, nor of the type $\nu_e \rightleftarrows \nu_\mu$, as there is no mixing ($\theta = 0$). The case $0 < m_D \ll m_R, m_L\, (m_R \neq m_L)$ corresponds obviously to a situation whereby oscillations have a very small amplitude and the weak-interaction particles ν_e ... are almost identical to the particles with definite masses $\nu_1, \ldots$. More accurately $\nu_e(\bar{\nu}_e)$ is practically identical on the left (right) component of ν_1, etc. The lepton charge, as useful notion, here is practically lost.

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*As a matter of fact, relevant oscillation was defined in ref.[7] as an oscillation with an adequate oscillation length (i.e., smaller than or about equal to the distance "source-detector") and a substantial amplitude.

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Биленький С.М., Понтекорво Б. E2-83-126

Осцилляции нейтрино с большой длиной осцилляций несмотря на большие /майорановские/ массы?

Построена такая схема смешивания нейтрино с майорановскими массами m_1 и m_2, в которой осцилляции нейтрино полностью аналогичны $K^\circ \rightleftarrows K^\circ$ осцилляциям: амплитуда осцилляций максимальна, разность масс $|m_1 - m_2|$ много меньше масс m_1 и m_2, PC-четности нейтрино Майорана противоположны. Представляется правдоподобным, что реалистическая схема с числом нейтрино Майорана, большим двух, будет обладать теми же особенностями.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

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Neutrino Oscillations with Large Oscillation Length in Spite of Large (Majorana) Neutrino Masses?

A model is given in which neutrino oscillations with two Majorana mass eigenstates are entirely similar to $K^\circ \rightleftarrows \bar{K}^\circ$ oscillations: a maximum oscillation amplitude, a mass difference $|m_1 - m_2|$ much less than m_1, m_2, opposite PC-eigenvalues of the Majorana neutrinos. It is plausible that a realistic situation with more than two Majorana neutrino could well have similar features.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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