объединенный
институт
ядерных
исследований
дубна

E2-83-104

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U(1)-SUPERSYMMETRIC EXTENSION OF THE LIOUVILLE EQUATION

Submitted to "Physics Letters B"

1983

1. During recent years, it has been found that many integrable two-dimensional equations admit a supersymmetrization /1-8/. From the group-theoretic standpoint, it means that one goes to the zero-curvature representation on some superalgebra containing the algebra of an analogous representation of the original bosonic equation as an even subalgebra /5,6/. In doing so, the systems with simple as well as extended supersymmetries can be obtained. For instance, the $N=1$ supersymmetric enlargements exist for the sine-Gordon /1,2/ and Liouville /3/ equations. The unique examples of integrable systems having $N=2$ supersymmetry were till now supersymmetric Kähler nonlinear σ-models /1,4,5,7/ and the supersymmetrically extended complex sine-Gordon equation /8/.

In this paper we describe one more integrable system with $N=2$ supersymmetry, the U(1)-supersymmetric Liouville equation. We construct for it the zero-curvature representation (on superalgebra osp (2|2)), the corresponding linear set and the general solution. The group structure of the obtained equation is analyzed and the relation to the $N=1$ supersymmetric Liouville equation is explained. We show that a close correspondence exists between the $N=0$, $N=1$, $N=2$ Liouville equations and the contact (super) algebras $K(1)$, $K(1|1)$, $K(1|2)$ from Kac's classification /9/.

2. We make use of a general approach which is based on nonlinear realizations of infinite dimensional (super) symmetries and provides a uniform description of the Liouville equation and its various supersymmetric extensions. To explain the basic ideas of our construction, let us begin with the simplest case of the ordinary $(N=0)$ Liouville equation:

$$u_{+-} = m^2 \exp(-2u) \qquad (1)$$

(Here $u_{+-} \equiv \partial_+\partial_- u$, $x^{\pm} = x^0 \pm x^1$ are light-cone coordinates of the two-dimensional Minkowski space, $[m^2] = L^{-2}$). It is known that eq. (1) can be viewed as the zero-curvature condition for some differential 1-form $\Omega \in s\ell(2,R)$ properly defined in terms of the field $u(x)$ and its derivatives u_+, u_- /10/. Our main observation is that the needed parametrization of Ω is attained automatically when $s\ell(2,R)$ is embedded into a more extensive infinite dimensional algebra $\mathcal{G}$. The latter is given by the direct sum of two contact algebras $K^{\pm}(1)$ (i.e., the algebras of formal vector fields on a line) with the generators $L^n_\pm$ *:

$$i[L^n_\pm, L^m_\pm] = (n-m)L^{n+m}_\pm, \ (n,m = -1,0,1,\ldots) \qquad (2)$$

($\mathcal{G}$ actually coincides with the algebra of the two-dimensional conformal group). The subalgebra $s\ell(2,R)$ we are interested in is generated by the following linear combinations of $L^n_\pm$:

$$R_+ = L^{-1}_+ + m^2 L^1_-, \quad R_- = L^{-1}_- + m^2 L^1_+, \quad U = L^0_+ - L^0_- . \qquad (3)$$

Let us consider a nonlinear realization of the group G with the algebra $\mathcal{G}$ in the coset space G/H , H being the O(1,1) subgroup generated by U defined in (3). We identify this O(1,1) with the Lorentz group of the two-dimensional Minkowski space ($L^{-1}_\pm$ are the corresponding translation generators) and parametrize the coset G/H as follows

$$g \equiv G/H = e^{ix^\pm L^{-1}_\pm} e^{iz^\pm_1(x) L^1_\pm} e^{iz^\pm_2(x) L^2_\pm} \ldots e^{iu(L^0_+ + L^0_-)}, \qquad (4)$$

where $u(x)$, $z^\pm_1(x)$, $z^\pm_2(x), \ldots$ constitute an infinite set of parameters-fields. The group G acts on g from the left.

The geometry of the coset space G/H is described by the Cartan forms introduced by the standard decomposition [11,12]

$$g^{-1}dg = i\Omega \equiv i(\Omega_0 + \Omega_1). \qquad (5)$$

Here, Ω_0 is defined to lie in the algebra $s\ell(2,R)$ (3) and Ω_1 in the orthogonal complement. The whole form Ω, by its definition, obeys the Maurer-Cartan equation on the algebra $\mathcal{G}$

$$d\Omega = i\Omega\wedge\Omega \qquad (6)$$

(Here, symbol d stands for the exterior differentiation). At this stage, eq.(6) has no any dynamical content, it is purely kinematic. The dynamics arises as a result of covariant reduction of G/H to the coset SL(2,R)/H. This reduction is effected by putting to zero the G/SL(2,R)-form Ω_1 **:

$$\Omega_1 = 0. \qquad (7)$$

---

* We basically follow the terminology of Kac[9]. The standard Virasoro algebra is the central extension of $K(1)$ continued to all negative dimensions.

** This condition is a particular case of constraints of the inverse Higgs phenomenon[3].

This matrix equation actually embodies an infinite set of differential equations one of which is just the Liouville equation (1) while the others express the parameters - fields $z^\pm_1(x)$, $z^\pm_2(x), \ldots$ in terms of the single dilaton $u(x)$ and its derivatives. Respectively, the form Ω reduces to the form $\Omega^{Red}_0 \subset s\ell(2,R)$. Explicitly:

$$\Omega^{Red} = \Omega^{Red}_0 = e^{-u}dx^+ R_+ + e^{-u}dx^- R_- + (u_- dx^- - u_+ dx^+)U. \qquad (8)$$

This form meets the zero-curvature condition on the algebra $s\ell(2,R)$ which arises in a natural way as a consequence of the original Maurer-Cartan equation (6) and the reduction constraint (7). One directly checks that requiring the curvature of the 1-form (8) to vanish yields the equation (1).

The described mechanism of implementing the zero-curvature representation is advantageous in that the needed structure of the basic 1-form is completely fixed within its framework by the choice of the extended algebra $\mathcal{G}_1$, the stability subgroup H, and the subalgebra $\mathcal{G}_0$, for which the above representation is constructed. In the present case we have taken H to be the two-dimensional Lorentz group in order to have an orthonormal coset with the minimal set of essential parameters-fields (consisting of the single dilaton $u(x)$). With any wider H, the number of essential parameters increases. Note a striking similarity with chiral fields: our approach suggests that the Liouville equation theory can be interpreted as a kind of the nonlinear σ model for the conformal group in two dimensions ($x^\pm$ and $u(x)$ are direct analogs of Goldstone fields of conventional nonlinear σ models).

3. Applying this scheme to direct sum of two contact superalgebras $K^\pm(1|1)$ [9]* and choosing $\mathcal{G}_0$ to be the superalgebra osp(1|2) (which is a minimal spinorial extension of $s\ell(2,R)$) we immediately recover the N = 1 supersymmetric Liouville equation of refs.[3]. The N = 2 supersymmetric Liouville equation emerges when $\mathcal{G}$ is taken to be the U(1)-supersymmetric extension of (2), that is the superalgebra $K^+(1|2) \oplus K^-(1|2)$ [15,9] with the following structure relations:

---

* The Neveu-Schwartz superalgebra[14] is a central extension of $K(1|1)$ continued to negative dimensions.

$$
\begin{cases}
i[L^n_\pm, L^m_\pm] = (n-m) L^{n+m}_\pm , \\
\{G^\alpha_\pm, G^\beta_\pm\} = \{\bar G^\alpha_\pm, \bar G^\beta_\pm\} = 0, \\
\{G^\alpha_\pm, \bar G^\beta_\pm\} = 2L^{\alpha+\beta}_\pm + 2(\alpha-\beta) T^{\alpha+\beta}_\pm , \\
i[L^n_\pm, T^s_\pm] = -s T^{n+s}_\pm , \\
i[T^s_\pm, T^p_\pm] = 0, \\
i[L^n_\pm, G^\alpha_\pm] = (\frac{n}{2}-\alpha) G^{\alpha+n}_\pm , \; i[L^n_\pm, \bar G^\alpha_\pm] = (\frac{n}{2}-\alpha) \bar G^{\alpha+n}_\pm , \\
i[T^s_\pm, G^\alpha_\pm] = \frac{1}{2} G^{\alpha+s}_\pm , \; i[T^s_\pm, \bar G^\alpha_\pm] = -\frac{1}{2} \bar G^{\alpha+s}_\pm \\
(n, m = -1,0,1,\ldots ; \; \alpha,\beta = -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \ldots ; \; s,p = 0,1,2,\ldots).
\end{cases} \tag{9}
$$

Note that the superalgebra (9) contains two local U(1)-algebras with the generators T^s_+, T^p_-.

In constructing the $N=2$ Liouville equation, we shall proceed in analogy with the purely bosonic $N=0$ case. The zero-curvature representation subalgebra $\mathcal{G}_0$ is now the superalgebra $osp(2|2)$ generated by the set $\{R_+, R_-, U, \bar Q_+, Q_+, \bar Q_-, Q_-, T\}$, where

$$
Q_+ \equiv G^{-1/2}_+ + m G^{1/2}_- , \quad Q_- \equiv G^{-1/2}_- - m G^{1/2}_+ , \quad T \equiv T^0_+ + T^0_- \tag{10}
$$

and R_+, R_-, U are defined by eqs. (3). The orthonormal coset space with the minimal number of essential parameters corresponds to the choice of H with the algebra $\mathcal{H} = O(1,1) \oplus O(2) = \{U,T\}$. It contains, besides the superspace coordinates $x^\pm$, $\theta^\pm$, $\bar\theta^\pm$ associated with the translation generators $L^{-1}_\pm$, $\bar G^{-1/2}_\pm$, $G^{-1/2}_\pm$, also two essential parameters-superfields $u(x,\theta,\bar\theta)$ and $\phi(x,\theta,\bar\theta)$ corresponding to the generators $L^0_+ + L^0_-$, $T^0_+ - T^0_-$. An element of the coset can be parametrized in the following way

$$
g \equiv G/H = e^{ix^\pm L^{-1}_\pm} \, e^{(\bar\theta^\pm G^{-1/2}_\pm + \theta^\pm \bar G^{-1/2}_\pm)} \, e^{i z^\pm_1 L^1_\pm} \ldots e^{iu(L^0_+ + L^0_-)} \, e^{\phi (T^0_+ - T^0_-)} . \tag{11}
$$

Isolating the $osp(2|2)$-component in the whole Cartan form $g^{-1}dg$ and nullyfying the remainder, we express, just as in the case of the $N=0$ Liouville equation, all higher parameters-superfields in (11) through a pair of essential superfields u and ϕ which satisfy now the following equations:

$$
\mathcal{D}^+ v = 0, \qquad \bar{\mathcal{D}}^- v = 0, \tag{12}
$$

$$
\bar{\mathcal{D}}^+ \mathcal{D}^- v = -2im\, e^{-v^+} \tag{13}
$$

with $v = u + i\phi$ and

$$
\mathcal{D}^\pm = -i\bar\theta^\pm \frac{\partial}{\partial x^\pm} + \frac{\partial}{\partial \theta^\pm} , \qquad \bar{\mathcal{D}}^\pm = -i\theta^\pm \frac{\partial}{\partial x^\pm} + \frac{\partial}{\partial \bar\theta^\pm} . \tag{14}
$$

The constraints (12) are the Grassmann analyticity conditions [16]. They reduce the complex $N=2$ superfield v to a complex $N=1$ superfield:

$$
v(x^+, x^-, \bar\theta^+, \theta^+, \bar\theta^-, \theta^-) = v(\xi^+, \xi^-, \bar\theta^+, \theta^-), \tag{15}
$$

$$
\xi^+ = x^+ - i\bar\theta^+\theta^+ , \quad \xi^- = x^- + i\bar\theta^-\theta^- . \tag{16}
$$

The equation (13) is the desirable U(1)-supersymmetric extension of the Liouville equation. To analyze it in more detail, let us pass to components:

$$
v = u(\xi^+, \xi^-) + i\phi(\xi^+, \xi^-) + \bar\theta^+ \psi_+ (\xi^+, \xi^-) + \theta^- \bar\psi_- (\xi^+, \xi^-) + \bar\theta^+ \theta^- \Phi(\xi^+, \xi^-). \tag{17}
$$

An auxiliary field $\Phi(x)$ is eliminated in terms of $u(x)$ and $\phi(x)$:

$$
\Phi(x) = 2im \exp(-u(x) + i\phi(x)) \tag{18}
$$

while the physical fields u, ϕ, ψ_+, ψ_- obey the following system of equations:

$$
\begin{cases}
\dfrac{\partial \psi_-}{\partial x^+} = m e^{-(u+i\phi)} \psi_+ , \\
\dfrac{\partial \bar\psi_+}{\partial x^-} = -m e^{-(u-i\phi)} \bar\psi_- , \\
\dfrac{\partial^2 u}{\partial x^+ \partial x^-} = m^2 e^{-2u} - \dfrac{im}{4} (\bar\psi_+ \psi_- + \psi_+ \bar\psi_-), \\
\dfrac{\partial^2 \phi}{\partial x^+ \partial x^-} = -\dfrac{m}{4} (\bar\psi_+ \psi_- - \psi_+ \bar\psi_-).
\end{cases} \tag{19}
$$

It is clear from (19) that eq. (13) represents in fact the U(1)-supersymmetric extension of the system

$$
u_{+-} = m^2 e^{-2u} , \quad \phi_{+-} = 0. \tag{20}
$$

The necessity of the field $\phi(x)$ and, accordingly, an additional equation in the bosonic sector follow yet from the standard condition that the number of bosonic and fermionic degrees of freedom be the same on-shell. The equation (13) can be obtained from the superfield action of the form (with the constraints (12) solved):

$$S = \int d^2x\, d\theta^+ d\bar{\theta}^- (v^+ \bar{\mathfrak{D}}^+ \mathfrak{D}^- v - 2im\, e^{-v^+}) + \text{h.c.} \tag{21}$$

The supergroup G associated with the superalgebra (9) acts on the coset (11) from the left and induces for superspace coordinates $x^\pm$, $\theta^\pm$, $\bar{\theta}^\pm$ and the coordinate-superfields v, v^+ the following transformations:

$$\begin{cases} \delta x^\pm = f^\pm(x^\pm, \bar{\theta}^\pm) + \bar{f}^\pm(x^\pm, \theta^\pm), \\ \delta\theta^\pm = i\bar{\mathfrak{D}}^\pm f^\pm(x^\pm, \bar{\theta}^\pm), \\ \delta v = i\mathfrak{D}^+\bar{\mathfrak{D}}^+ f^+(x^+, \bar{\theta}^+) + i\bar{\mathfrak{D}}^-\mathfrak{D}^-\bar{f}^-(x^-, \theta^-), \end{cases} \tag{22}$$

where

$$f^\pm(x^\pm, \bar{\theta}^\pm) = \frac{1}{2}[f_0^\pm(x^\pm) + ih_0^\pm(x^\pm)] - i\bar{\theta}^\pm \chi_0^\pm(x^\pm), \quad \bar{f}^\pm \equiv (f^\pm)^+ \tag{23}$$

and $f_0^\pm(x^\pm)$, $\chi_0^\pm(x^\pm)$, $h_0^{\pm\prime}(x^\pm)$ are, respectively, the parameters of the conformal, local supersymmetry and local U(1)-transformations ($h_0^\pm(x^\pm)$ enter into (22) only through their derivatives). The invariance of the action (21) under (22) is checked immediately.

Now, let us construct a linear set for equations (12), (13). Only the spinor components of the reduced Cartan form

$$\frac{1}{i} g^{-1} dg = \Omega^{Red} \subset osp(2|2)$$

are independent; the vector components are expressed in terms of spinor ones due to the anticommutation relations

$$\{\mathfrak{D}^\pm, \bar{\mathfrak{D}}^\pm\} = -2i\frac{\partial}{\partial x^\pm}. \tag{24}$$

So, the minimal linear set should be of the form:

$$\begin{cases} \mathfrak{D}^- V = \Omega^- V, \\ \bar{\mathfrak{D}}^+ V = \bar{\Omega}^+ V. \end{cases} \tag{25}$$

Here, V is a column of three complex N=2 superfields belonging to the fundamental representation (1|2) /9/ of OSp(2|2) and Ω^-, $\bar{\Omega}^+$ are the coefficients of differentials $d\theta^-$, $d\bar{\theta}^+$ in Ω^{Red}. Explicitly, Ω^- and $\bar{\Omega}^+$ are

$$\begin{cases} \Omega^- = \frac{i}{2}\mathfrak{D}^- v(U - T) + e^{-\frac{v^+}{2}}\bar{Q}_-, \\ \bar{\Omega}^+ = -\frac{i}{2}\bar{\mathfrak{D}}^+ v(U - T) + e^{-\frac{v^+}{2}} Q_+. \end{cases} \tag{26}$$

The generators $U-T$, Q_+, $\bar{Q}_-$ form the closed subalgebra in osp(2|2):

$$\{Q_+, \bar{Q}_-\} = -2m(U-T), \quad i[U-T, \begin{pmatrix} Q_+ \\ \bar{Q}_- \end{pmatrix}] = 0. \tag{27}$$

Choosing for them the simplest representation by matrices 3x3 with zero supertrace, we write the spectral problem as:

$$\mathfrak{D}^- V = \begin{pmatrix} \frac{i}{2}\mathfrak{D}^- v & 0 & 0 \\ \eta m^{1/2} e^{-\frac{v^+}{2}} & \frac{i}{2}\mathfrak{D}^- v & 0 \\ \eta m^{1/2} e^{-\frac{v^+}{2}} & \frac{i}{2}\mathfrak{D}^- v & 0 \end{pmatrix} V; \quad \bar{\mathfrak{D}}^+ V = \begin{pmatrix} \frac{i}{2}\bar{\mathfrak{D}}^+ v & -2m^{1/2}\eta^{-1} e^{-\frac{v^+}{2}} & 0 \\ 0 & -\frac{i}{2}\bar{\mathfrak{D}}^+ v & 0 \\ 0 & -\frac{i}{2}\bar{\mathfrak{D}}^+ v & 0 \end{pmatrix} V, \tag{28}$$

η being a complex spectral parameter introduced by the combined action of the Lorentz and U(1) transformations on the coset element (11) from the right. The equation (13) is easily recognized as one of the integrability conditions of the linear set (28). The analyticity conditions (12) are also contained in (28); they follow from requiring (28) to be consistent with the relations $\mathfrak{D}^-\mathfrak{D}^- = \bar{\mathfrak{D}}^+\bar{\mathfrak{D}}^+ = 0$. The linear problem in the component notation can be obtained from (28) with the use of eqs. (24).

To close this Section, we mention that the reduction to the N=1 supersymmetric Liouville equation is performed by setting $\phi(x) = 0$, $\bar{\psi}_\pm = \psi_\pm$ everywhere. Accordingly, one should regard θ^+, θ^- to be real and properly restrict the supergroup (22) (it will be reduced to $K^+(1|1) \oplus K^-(1|1)$ -transformations).

4. We construct here the general solution of eq. (13). The strategy we keep to is as follows. Since the reduced 1-form $i\Omega^{Red} = g^{-1}dg$ lies in the superalgebra osp(2|2) and its curvature vanishes, it is representable as

$$i\Omega^{Red} = g_0^{-1} dg_0, \tag{29}$$

g_0 being an element of the supergroup OSp(2|2). We parametrize g_0 in the following way:

$$g_0 = e^{i\alpha R_+} e^{i\beta R_-} e^{(\bar\zeta^+ Q_+ + \zeta^+ \bar Q_+)} e^{(\bar\zeta^- Q_- + \zeta^- \bar Q_-)} e^{i\gamma U} e^{\lambda T} , \quad (30)$$

where α, β, ζ^+, ζ^-, γ, λ are arbitrary functions of $\{x^\pm, \theta^\pm, \bar\theta^\pm\}$, and then express v through these functions by the condition (29). The structure of Ω^{Red} in terms of v and v^+ is strictly fixed that results in certain relations between the above functions; besides, their coordinate dependence is restricted in a definite way. The general solution, in its final form, reads

$$e^{-v} = \bar{\mathcal{D}}^+ \bar\zeta^+ \mathcal{D}^- (\beta\kappa^-)[1 + im^2 \beta \zeta^+ \bar\zeta^+ + 2im\beta\bar\kappa^- \zeta^+ - m^2\beta^2 \bar\kappa^- \kappa^- \bar\zeta^+ \zeta^+], \quad (31)$$

where

$$\begin{cases} \alpha = \alpha(x^+, \theta^+, \bar\theta^+), \\ \beta = (m^2\alpha - \omega)^{-1} \ ; \ \omega = \omega(x^-, \theta^-, \bar\theta^-), \\ \zeta^+ = \zeta^+(x^+ + i\bar\theta^+\theta^+, \theta^+), \\ \zeta^-(1 - im\bar\zeta^-\zeta^+)\beta^{-1} \equiv \kappa^- = \kappa^-(x^- + i\bar\theta^-\theta^-, \theta^-) \end{cases} \quad (32)$$

and the following constraints hold:

$$\begin{cases} \mathcal{D}^+\alpha + i\bar\zeta^+\mathcal{D}^+\zeta^+ = 0, \\ \bar{\mathcal{D}}^-\omega + i\bar\kappa^-\bar{\mathcal{D}}^-\kappa^- = 0 . \end{cases} \quad (33)$$

Unfortunately, we did not suceed in finding a superfield solution of eqs. (33). Nevertheless, they can easily be solved in components. Parametrizing the superfunctions α, ζ^+, ω, κ^- as

$$\begin{aligned} &\alpha(x^+, \theta^+, \bar\theta^+) = \alpha_0 + \theta^+\bar\eta^+ e^{-iA} - \bar\theta^+\eta^+ e^{iA} + \bar\theta^+\theta^+ c_0 , \\ &\omega(x^-, \theta^-, \bar\theta^-) = \omega_0 + \theta^-\bar\eta^- e^{-iB} - \bar\theta^-\eta^- e^{iB} + \bar\theta^-\theta^- b_0 , \\ &\zeta^+ = \zeta_0^+ + \theta^+\rho e^{-iA} , \ \kappa^- = \kappa_0^- + \theta^-\sigma e^{-iB} \end{aligned} \quad (34)$$

we get

$$\begin{aligned} &\zeta_0^+ = -i\eta^+\rho^{-1} , \\ &c_0 = -\frac{\partial}{\partial x^+}(\rho^{-2}\bar\eta^+\eta^+), \\ &\rho^{-1} = \left(\frac{\partial\alpha_0}{\partial x^+}\right)^{-1/2}\left[1 - \frac{i}{2}\left(\frac{\partial\alpha_0}{\partial x^+}\right)^{-2}\left(\frac{\partial\bar\eta^+}{\partial x^+}\eta^+ - \bar\eta^+\frac{\partial\eta^+}{\partial x^+}\right) + \frac{7}{4}\left(\frac{\partial\alpha_0}{\partial x^+}\right)^{-4}\frac{\partial\bar\eta^+}{\partial x^+}\eta^+\bar\eta^+\frac{\partial\eta^+}{\partial x^+}\right] \end{aligned} \quad (35)$$

and similar relations between the components of superfields ω and κ^-. Thus, the general component solution of eq. (13) is composed of four real scalar functions $\alpha_0(x^+)$, $A(x^+)$, $\omega_0(x^-)$, $B(x^-)$ and four spinor functions $\eta^+(x^+)$, $\bar\eta^+(x^+)$, $\eta^-(x^-)$, $\bar\eta^-(x^-)$.

Once the reduction to the $N = 1$ case is made, the constraints (33) can be solved in terms of superfields and the general solution (31) takes the compact form:

$$e^{-u} = \frac{\mathcal{D}^+\left(\frac{\mathcal{D}^+\alpha}{\sqrt{\partial_+\alpha}}\right)\mathcal{D}^-\left(\frac{\mathcal{D}^-\omega}{\sqrt{\partial_-\omega}}\right)}{\left(m^2\alpha - \omega + m\frac{\mathcal{D}^-\omega\,\mathcal{D}^+\alpha}{\sqrt{\partial_-\omega\,\partial_+\alpha}}\right)} \quad (36)$$

with $\alpha = \alpha(x^+, \theta^+)$, $\omega = \omega(x^-, \theta^-)$. We have verified that the components of the superfield (36) coincide, up to several redefinitions, with the general component solution of the $N = 1$ Liouville equation found in ref. /14/.

5. In conclusion, we would like to emphasize once more that searching for $N > 1$ supersymmetric extensions of nonlinear equations is rather a delicate task because a system of bosonic equations should be supersymmetrized, as it is illustrated by present work. For instance, the $O(n)$-supersymmetric extension of the Liouville equation is expected to contain, in its bosonic sector, equally with the dilaton $u(x)$, also $1/2\,n(n-1)$ scalar fields parametrizing the coset $O(n) \times O(n)/O(n)$. Thus, the Liouville equation will be combined with the equations of the nonlinear σ model for the principal chiral field on the group $O(n)$. An advantage of the scheme applied here should be seen in minimality of its initial principles. As a matter of fact, they are reduced to the choice of the (super) algebra $\mathcal{G}$ and the zero-curvature representation subalgebra $\mathcal{G}_0$. The results presented here show that this set of postulates is capable enough to construct new integrable equations and to find their general solutions. It is tempting to suggest that inspecting in this way all infinite-dimensional (super) algebras, one will obtain a kind of the group theoretic classification of integrable systems. This will be the subject of our further study.

A few words on possible physical applications of the equation (13) are in order. By analogy with the ordinary Liouville equation, one may think that its $N = 1$ and $N = 2$ counterparts correspond to the choice of a certain ansatz for fields of some supersymmetric four-dimensional Yang-Mills theories which generalizes the cylindrically-symmetric ansatz of ordinary Yang-Mills /15/. Besides, it is natural to expect that the equation (13) is relevant to the description of the U(1)-superstring in the geometrical approach of refs. /16/.

We are sincerely thankful to P.P.Kulish, D.A.Leites, Yu.I.Manin, and V.I.Ogievetsky for very useful discussions and remarks.

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Received by Publishing Department
on February 21, 1983.

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Иванов Е.А., Кривонос С.О. E2-83-104
U(1)-суперсимметричное расширение уравнения Лиувилля

Построено $N=2$ -суперсимметричное расширение уравнения Лиувилля. Для него найдены представление нулевой кривизны /на супералгебре $osp(2|2)$ / и соответствующая линейная задача, а также общее решение. Обсуждается редукция к $N=1$ случаю. Установлена внутренняя связь $N=0$, $N=1$ и $N=2$-уравнений Лиувилля с бесконечномерными контактными /супер/ алгебрами $K(1)$, $K(1|1)$ и $K(1|2)$ из списка Каца.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1983

Ivanov E.A., Krivonos S.O. E2-83-104
U(1)-Supersymmetric Extension of the Liouville Equation

The $N=2$ supersymmetric extension of the Liouville equation is presented. We construct for it the zero-curvature representation (on superalgebra $osp(2|2)$) together with an associated linear set, find its general solution and discuss the reduction to the $N=1$ case. An intrinsic connection of the $N=0$, $N=1$, and $N=2$ Liouville equations with the infinite dimensional contact (super) algebras $K(1)$, $K(1|1)$ and $K(1|2)$ from Kac's list is established.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1983