

СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

2458/83

16/5-83

E2-83-101

**K.K.Gudima¹, G.Röpke², H.Schulz³,
V.D.Toneev**

**THE COALESCENCE MODEL
AND THE PAULI QUENCHING
IN HIGH-ENERGY HEAVY-ION COLLISIONS**

¹ Permanent address: Institute of Applied Physics,
Moldavian Academy of Sciences, Kishinev (USSR)

² Wilhelm Pieck University, Rostock (GDR)

³ Central Institute for Nuclear Research,
Rossendorf (GDR)

1983

In understanding the particle spectra of high-energy heavy-ion collisions the coalescence model plays an important role. The model bases on the assumption that nucleons, which have about the same momentum, coalesce to produce composites as deuterons, tritons, ^3He , α -particles, etc. In this note we do not intend to give a derivation of the model (cf. refs.^{1,2/} and references quoted therein) but try to point out some of the underlying assumptions. For simplicity we consider the deuteron formation. The reference to the deuteron is of a sufficient fundamental interest to justify the development of a general formalism which contains the formation of many-body clusters as well.

In spite of the fact that the deuteron formation is a rather complicated dynamical process, the coalescence model relies on the sudden approximation of quantum mechanics by calculating the overlap of the wave functions characterizing the initial and final state of the expanding fireball. The initial proton and neutron state is described by plane waves

$$\psi_i = e^{\frac{i}{\hbar}(\vec{p}_p \vec{r}_p + \vec{p}_n \vec{r}_n)}, \quad (1)$$

while the final state is assumed to be of the form

$$\psi_f = e^{\frac{i}{\hbar} \vec{P}_d \vec{R}_d} \phi_d^0(\vec{r}) \quad (2)$$

containing center-of-mass and internal motion of the deuteron. The coalescence formula is obtained by calculating the quantity $|\langle i | f \rangle|^2$ and folding it with the respective neutron and proton distributions in the momentum space (cf. ref.^{1/}). The representation of ψ_f in the form (2) is only justified if the deuterons move in the free space and are not affected by the surrounding medium. The important effect to be included is the Pauli quenching preventing the deuteron formation at comparatively low temperatures and/or high densities^{3/}. This can most easily be studied by solving a Bethe-Goldstone type equation for a deuteron embedded in a hot nuclear medium. The solution $\phi_d(\vec{r})$ of the Bethe-Goldstone equation becomes then a function of the density and temperature. Moreover, in contrast with the Schroedinger equation in free space the internal wave function $\phi_d^0(\vec{r})$ is affected by the total momentum \vec{P}_d of the deuteron. This momentum dependence leads to the effect that deuterons which move with a

high velocity relative to the surrounding matter are less degraded, because the quenching results essentially from the Pauli blocking. The density beyond which deuterons at given momentum P_d are destructed due to the change of the effective two-nucleon interaction is called the Mott density^{/3,4/}.

In the subsequent discussion we rely on the quantum mechanical sudden approximation in calculating the overlap $|\langle i|f\rangle|^2$ but improve the approach by considering in-medium corrections to the deuteron wave function. In view of a quantum statistical many-body description of hot nuclear matter the consideration of composites like deuterons is not sufficient, because the formation of larger clusters has to be accounted for as well. This is done by solving the corresponding equation of state in a self-consistent way. Describing the two-nucleon interaction by means of a Yamaguchi potential^{/5/} the resulting Bethe-Goldstone equation takes the form^{/3,4/}

$$\begin{aligned}
 (p^2 + a^2) \phi_d^{\vec{P}_d}(\vec{p}) &= \\
 &= \lambda_0 g(\vec{p}) \int \frac{d^3 p'}{(2\pi)^3} [1 - f(\vec{P}_{d/2} + \vec{p}') - f(\vec{P}_{d/2} - \vec{p}')] \phi_d^{\vec{P}_d}(\vec{p}'),
 \end{aligned}
 \quad (3)$$

where

$$f(p) = [\exp(p^2/2m - \mu)/T + 1]^{-1} \quad (4)$$

is the Fermi distribution function accounting for the phase space occupation due to the Pauli blocking. The quantity μ denotes the chemical potential containing the Hartree-Fock shift. It has to be determined self-consistently when solving the corresponding equation of state of hot nuclear matter^{/3,4,6/}. In the low density limit the chemical potential μ is connected with the density via $\rho = \frac{4}{\Lambda^3} e^{\mu/T}$, where $\Lambda = (2\pi\hbar^2/mT)^{1/2}$ is the thermal wave

length of a nucleon. From eq. (3) it can be seen that if either $\rho \rightarrow 0$ ($\mu \rightarrow -\infty$) or the velocity $P_d/2m$ of the deuterons relative to the medium is large, the solution for the free deuteron $\phi_d^0(\vec{p})$ is obtained. The Yamaguchi potential^{/5/} yields then the well-known Hulthén wave function of the deuteron. Since the deuteron-like clusters are extended in the coordinate space, their relative momentum distribution $\langle p^2 \rangle$ is rather narrow in momentum space. This can be seen from fig.1, where we compare p_0^2 calculated means of the Hulthén function of the isolated deuteron with those calculated by means of the biorthogonal solutions of eq. (3). In order to present the results in a transparent form the

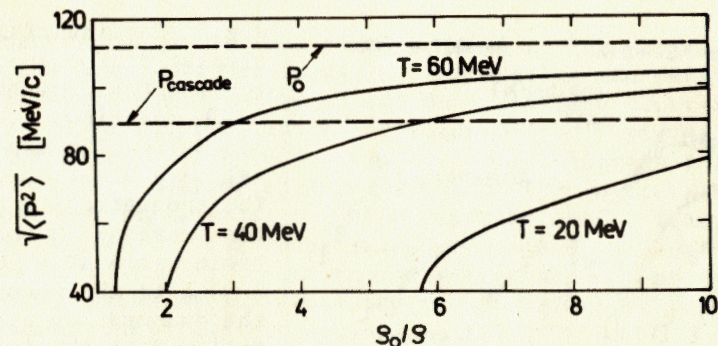


Fig.1. The expectation value $\langle p^2 \rangle$ of the deuteron-like cluster calculated by means of the solutions of eq. (3) as a function of the density at given temperature T and fixed total momentum $\frac{\langle P_d^2 \rangle}{4m} = \frac{3}{2} T$. $P_{\text{cascade}} = 90 \text{ MeV/c}$ represents the value of the momentum sphere adjusted in the cascade calculation, while $P_0 \approx 114 \text{ MeV/c}$ follows from the Hulthén deuteron wave function.

respective expectation values $\langle p^2 \rangle$ have been calculated for deuteron velocities according to the mean thermal energy $\langle P_d^2 \rangle/4m = \frac{3}{2} T$. It is interesting to compare these expectation values with those employed in the cascade calculation in order to reproduce the experimental spectra of protons and deuterons in high-energy heavy-ion collisions. In doing this we used a version of the Dubna cascade model^{/7/} extended to the case of composite particle formation. The coalescence mechanism for nucleons with a relative momentum \vec{p} is realized in the momentum space by a direct permutation of all nucleons produced as the intranuclear cascade has been completed^{/8/}. Such a procedure is performed for each nucleus-nucleus collision that allows us to take into account the energy-momentum conservation and the source geometry. In particular, it is assumed that a neutron and a proton whose momenta lie within a momentum sphere of the radius P_{cascade} coalesce and produce the deuteron. Rather good fits of such a cascade model calculations to both the proton and deuteron abundances at various angles for the reaction $\text{Ar} + \text{KCl}$ at 800 MeV/N ^{/9/} are shown in fig.2. The extracted value for the coalescence radius $P_{\text{cascade}} = 90 \text{ MeV/c}$ (cf. fig.1) turns out to be smaller than that given by the deuteron wave function (Hulthén function) of two isolated nucleons and falls just into the region determined by the solution of the equation of state which has explicitly allowed

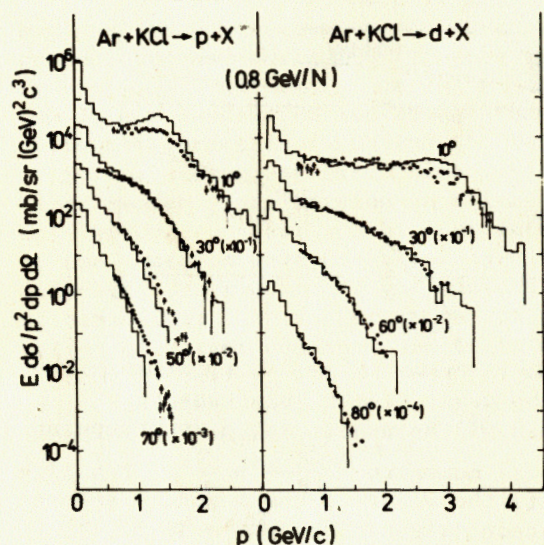


Fig.2. The invariant inclusive proton and deuteron spectra from the Ar + KCl reaction at 0.8 GeV/N. The experimental data are from ref.^{9/}. The theoretically predicted increase in the particle yield at $p < 0.5$ GeV/c is due to the account of the evaporation process followed by the intra-nuclear cascade. The account of the evaporation process has no influence on the proper choice of the coalescence radius $p_{\text{cascade}} = 90$ MeV/c.

for the formation of clusters. This fact can be interpreted as an effective account of the influence of the medium on the deuteron characteristics if the experimental spectra are fitted by means of the cascade model calculations. In this connection it is worthwhile to note that the same value of $p_{\text{cascade}} = 90$ MeV/c has been used in the cascade calculations for different projectile and target combinations and various incident energies with the result that the proton and deuteron spectra are equally well described^{8/}.

The relevance of the Pauli quenching for the deuteron production has been also discussed in ref.^{10/}, where the expanding fireballs have been described in a relativistic gas dynamical approach.

Using in the cascade calculations a coalescence radius determined by the free ground state deuteron wave function, the theoretically predicted deuteron abundances are overestimated and consequently the associated entropy per baryon is lowered. This might be the reason why in ref.^{11/} the cascade calculations gave a too small entropy value.

The account of the density and temperature dependence of the deuteron wave function in matter seems to be appropriate for a further improvement of the coalescence model and explains why the cascade model is able to describe experimental deuteron spectra with a momentum coalescence sphere, the radius of which is smaller than that deduced from the use of the wave function of the free deuteron. Concerning the cascade calculations represented in fig.2 the achieved good agreement of theory and experiment could further be improved by replacing the average

momentum sphere radius by that corresponding to the actual situation characterized mainly by the temperature and density of the medium and the velocity of the deuteron like clusters.

Finally let us mention that the cascade model^{8/} yields for the momentum spheres fitting spectra of larger clusters (^3H , ^3He , α -particles) smaller coalescence radii than expected from conventional considerations^{2/}. These findings are in line with the reasoning for the deuteron formation and can again be interpreted as an indication of the effective account of the Pauli blocking whose significance for the formation of larger clusters has already been established in refs.^{6,12/}.

Two of the authors (H.S and G.R.) are indebted to the Joint Institute for Nuclear Research for the hospitality extended to them.

REFERENCES

1. Kapusta J.I. Phys.Rev., 1980, C21, p. 1301.
2. Sato H., Yasaki K. Phys.Lett., 1981, 98B, p. 153.
3. Röpke G., Münchow L., Schulz H. NBI-preprint 81-21, Copenhagen, 1981; Nucl.Phys., 1982, A379, p. 536; Phys.Lett., 1982, B110, p. 21.
4. Röpke G. et al. Yad.Fiz., 1982, 25, p. 607.
5. Yamaguchi Y. Phys.Rev., 1954, 95, p. 1628.
6. Münchow L. et al. J.Phys.G. (Nucl.Phys.), 1982, 8, p. L135.
7. Gudima K.K., Toneev V.D. Yad.Fis., 1978, 27, p. 658; 1982, 31, p. 1455.
8. Gudima K.K., Toneev V.D. Invited Talk Int. Conf. on Nucleus-Nucleus Coll., East Lansing, 1982; Nucl.Phys. in press.
9. Nagamiya S. et al. Phys.Rev., 1981, C24, p. 917.
10. Barz H.W. et al. Z.Phys., 1982, A308, p. 187.
11. Bertsch G., Cugnon J. Phys.Rev., 1981, C24, p. 2514.
12. Schulz H. et al. NBI-preprint 82-10, Copenhagen, 1982; Phys.Lett., 1982, B119, p. 12.

Received by Publishing Department
on February 21, 1983.

Гудима К.К. и др.

E2-83-101

Модель коалесценции и эффект подавления Паули в высокоэнергетических столкновениях тяжелых ионов

Обсуждается механизм образования сложных частиц в реакциях столкновения тяжелых ионов. Показано, что эффективный учет влияния ядерной среды на волновую функцию связанного состояния дейтронного типа путем решения соответствующего уравнения Бете-Голдстоуна требует меньшего радиуса коалесценции в соответствующем импульсном пространстве по сравнению с величиной, полученной при использовании волновой функции двух изолированных нуклонов. Реализация механизма коалесценции в рамках каскадной модели для столкновений релятивистских тяжелых ионов дает значение радиуса коалесценции нуклонов, согласующееся с этими ожиданиями.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1983

Gudima K.K. et al.

E2-83-101

The Coalescence Model and the Pauli Quenching in High-Energy Heavy-Ion Collisions

A composite particle formation mechanism is discussed for high-energy heavy-ion collisions. It is argued that the effective account of the nuclear medium on the deuteron-like bound-state wave function by solving a Bethe-Goldstone type equation requires a smaller coalescence radius of the respective momentum sphere compared with that obtained by using a deuteron wave function of two isolated nucleons. Calculating the proton and deuteron spectra of relativistic heavy-ion reactions in the framework of the cascade model gives results in accordance with these expectations.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1983