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ACTION PRINCIPLE IN SUPERSPACE

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After the important papers of Wess and Zumino [1,2] which revived* the interest on the supersymmetry between bosons and fermions a further progress was achieved by Salam and Strathdee [5]. They have shown that for construction of the representations of spinor generators, along with the usual coordinates, new spinor (anticommuting) coordinates θ are necessary and introduced the concept of superfield $\Phi(x,\theta)$. The superfield $\Phi(x,\theta)$ is equivalent to a certain set of fields in the Minkowski space. The highest order coefficient in the development of $\Phi(x,\theta)$ in powers of θ changes by a 4-divergence under a supersymmetry transformation and the invariant action is usually written as a four-dimensional integral of such coefficients of an appropriate choice of products of superfields.

In this letter we show that an equal footing treatment of the usual x and spinor θ coordinates is further lucrative and that the action can be represented as an integral over the eight-dimensional superspace x,θ. This approach puts the theory in manifestly supersymmetry invariant form and considerably simplifies all the computations. Manifestly invariant equations of motion are de-

* The Fermi-Bose symmetries were also studied in the pioneering papers of Golfand and Lichtman [3]. Volkov and Akulov have investigated their non-linear realizations [4].

duced directly in terms of superfields by the use of the variation principle. An invariant perturbation theory is outlined and the model of scalar superfield [2] is treated as an example.

The formal integral over anticommuting variables is known as the Grassmann integral. In quantum field theory it is used for the definition of the generating functional in the path integral formulation, in the presence of Fermi fields. The Grassmann integral is defined and well presented by Berezin [5] *. Here we give only the rules of integration

$$\int d\theta_i = 0, \quad \int d\theta_i \, \theta_k = \delta_{ik} , \tag{1}$$

where $d\theta_i$ are the "differentials" of the anticommuting variables θ_i

$$\{d\theta_i , d\theta_k\} = \{d\theta_i , \theta_k\} = \{\theta_i , \theta_k\} = 0 .$$

The multiple integral is written as

$$\int d^n\theta \, f(\theta) = \int d\theta_1 \dots d\theta_n \, f(\theta_1 ,\dots,\theta_n).$$

An intriguing feature is the form which we have found for the δ type function on the Grassmann algebra. Namely, let us introduce the anticommuting elements θ_i' . Then the polynomial:

$$\delta^{\Gamma}(\theta-\theta') = (\theta_n - \theta_n') \dots (\theta_1 - \theta_1') \tag{2}$$

has the following properties:

$$\begin{aligned} &\int d^n\theta \, \delta^{\Gamma}(\theta-\theta') \, f(\theta) = f(\theta') \\ &(-)^{n+1} \int d^n\theta \, f(\theta) \, \partial^{\Gamma}_{\theta_a} \, \delta(\theta-\theta') = \partial_{\theta_a'} f(\theta') \end{aligned} \tag{3}$$

The mathematical aspects of supersymmetry are investigated in recent preprint of Kotecky [6] where also the possibility of using the Grassmann integral in defining the action is remarked.

like those of the usual δ function, however,

$$\delta^{\Gamma}(0) \equiv 0, \quad \delta^{\Gamma}(\theta-\theta')\,\delta^{\Gamma}(\theta-\theta') \equiv 0 \; ! \tag{4}$$

Salam and Strathdee [7] have developed a special technique in which the invariant action is an appropriate construction of superfields such that the θ dependent part of the Lagrangian is a 4-divergence. By the use of the Grassmann integral it is possible to construct the action without worrying about the θ dependence of the Lagrangian. The invariant superaction is simply an integral over the eight-dimensional superspace x, θ. For example, for the scalar superfield we have (we use the notation and formalism of Ferrara, Wess and Zumino [8])

$$\begin{aligned} S = \int d^4x \, d^4\theta \; \times \\ \times \Big\{ -\frac{\delta(\bar\theta)}{8} \Phi(x_\mu, \theta) \frac{\partial}{\partial \bar\theta^{\dot\alpha}} \epsilon_{\dot\alpha\dot\beta} \frac{\partial}{\partial \bar\theta^{\dot\alpha}} \Phi^{+}(x_\mu + 2i\theta\sigma_\mu \bar\theta, \bar\theta) \\ - \frac{m}{4} [\delta(\bar\theta)\Phi^2(x, \theta) - \delta(\theta)\Phi^{2+}(x, \bar\theta)] \\ + \frac{g}{3} [\delta(\bar\theta)\Phi^3(x, \theta) - \delta(\theta)\Phi^{3+}(x, \bar\theta)] \Big\} , \end{aligned} \tag{5}$$

where

$$\begin{aligned} &d^4\theta = d\theta^1 d\theta^2 d\bar\theta^{\dot 1} d\bar\theta^{\dot 2} \qquad && \theta^{+}_{\alpha} = -\bar\theta_{\dot\alpha} \\ &\delta(\theta) = \tfrac{1}{2} \theta^{\alpha} \epsilon_{\alpha\beta} \theta^{\beta} && \delta^{+}(\theta) = -\delta(\bar\theta). \end{aligned} \tag{6}$$

Quite generally the eight-dimensional integral of any superfields is an invariant quantity under sypersymmetry transformations because the supersymmetry generators act always as translations in the superspace:

$$Q_\alpha = -i \frac{\partial}{\partial \theta^\alpha} , \quad \bar Q_{\dot\alpha} = i \frac{\partial}{\partial \bar\theta^{\dot\alpha}} + 2(\theta\sigma_\mu)_{\dot\alpha} \partial^\mu . \tag{7}$$

The manifestly invariant equations of motion [8] are obtained from Eq. (5) by making use of the variational principle (one varies directly the superfield $\Phi(x,\theta) \to \Phi(x,\theta)+\delta\Phi(x,\theta)$).

$$-\frac{1}{4}\frac{\partial}{\partial\bar\theta^{\dot\alpha}}\epsilon_{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta^{\dot\beta}}\Phi^+(x_\mu+2i\theta\sigma_\mu\bar\theta,\bar\theta)-m\Phi(x,\theta)+ \qquad (8)$$
$$+2g\Phi^2(x,\theta)=0.$$

The condition of invariance under supersymmetry transformation fixes the commutator of two superfields up to a function which is defined by a nontrivial commutator of one of its components.

To get the supersymmetry invariant perturbation theory we represent the S-matrix in the form:

$$S = T\exp i\int d^4x\, d^4\theta\, \mathcal{L}_{int}(x,\theta). \qquad (9)$$

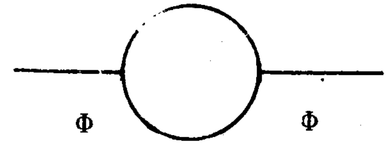
The superfield propagators which we use are similar in form with those deduced by Salam and Strathdee [7] and applied by Capper [9].

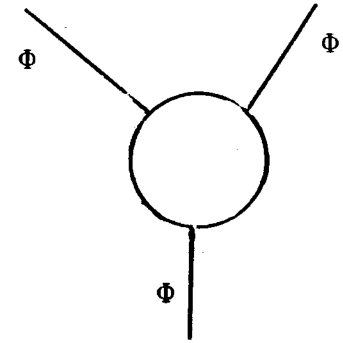
The Feynman rules are obtained by straightforward generalization of the usual ones. We will illustrate the adequacy of the approach by showing the almost trivial way in which the reduction of the number of infinities in the one loop diagrams takes place in the scalar superfield model of Ref. [2]. For this model the propagators are

$$\underbrace{\Phi^+(\ ,\bar\theta)\Phi(\ ,\theta')} \Rightarrow \frac{\exp-2\theta'\sigma_\mu\bar\theta k^\mu}{m^2-k^2-i\epsilon}$$

$$\underbrace{\Phi(\ ,\theta)\Phi(\ ,\theta')} \Rightarrow \frac{-2m\delta(\theta-\theta')}{m^2-k^2-i\epsilon}$$

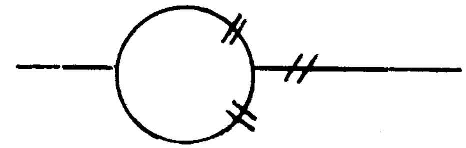
The suppression of infinities is obtained by making use of the properties of the Grassmann integral and of the above introduced $\delta^\Gamma(\theta-\theta')$. For example, the vertices which should give the mass and vertex renormalization





vanish because of $(\delta^\Gamma(\theta))^2 \equiv 0$.

It is easy to verify that the tadpole graphs vanish identically and that the possible divergent vertex $\Phi^2\Phi^+$ turns out convergent. There remains only one divergent diagram



in which also cancellations take place rendering it only logarithmically divergent. To obtain the supersymmetric counterterm we separate the divergent part of

$$-i^2\int d^4x_1\, d^4x_2\, d^4\theta_1\, d^4\theta_2\, \Phi^+(x_1,\bar\theta_1)\Phi(x_2,\theta_2)\times$$

$$\times\frac{1}{(2\pi)^4}\int d^4k\, e^{ik^\mu(x_1-x_2-2i\theta_2\sigma_\mu\bar\theta_1)}\times$$

$$\times -\frac{4g^2}{(2\pi)^4}\int d^4q\,\frac{\delta(\theta_1)\delta(\bar\theta_2)}{[m^2-(q+\frac{k}{2})^2][m^2-(q-\frac{k}{2})^2]}=$$

$$=Zi\int d^4x\, d^4\theta\,\frac{1}{4}\Phi^+(x_\mu+2i\theta\sigma_\mu\bar\theta,\bar\theta)\Phi(x,\theta)+\dots$$

where $$Z = \frac{4i\, g^2}{(2\pi)^4} \int \frac{d^4q}{(m^2 - q^2)^2}.$$

This term is identical with the first term in the Lagrangian (5) as could be seen when integrating by part over $\bar{\theta}$.

A detailed study of this and another supersymmetric models will be published elsewhere.

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