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ANALYTICITY AND DETERMINATION OF SPIN-ZERO LIGHT NUCLEUS CHARGE RADIUS

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1. Introduction

The elastic electron scattering is the main experimental source of information on the electromagnetic size of nucleus. These experiments consist of scattering of high-energy electrons from target nuclei and studying the energy and angular distributions of the scattered electrons.

Theoretically the electron-nucleus interaction is well understood (it is the electromagnetic interaction with the nuclear charge and current densities) and for light nuclei it is possible to analyse the scattering within the conventional framework of the first Born approximation. The differential cross section can be expressed through the kinematical variables (energy and scattering angle) and electromagnetic form factor $F(t)$ which is a function of the momentum transfer $q(t=-q^2)$ and is the Fourier transform of the charge distribution $\rho(r)$.

The electromagnetic size of the spin-zero nucleus is characterized by the root-mean-square (r.m.s.) charge radius $\langle r^2\rangle^{1/2}$. Many attempts have been undertaken to determine this parameter for different nuclei (see refs.[1–5] and reference cited therein). The procedure commonly followed in the determination of $\langle r^2\rangle^{1/2}$ from analysis of the elastic electron scattering experiments is to test various model-dependent forms of charge distributions containing a number of free parameters. By χ^2 minimization technique these are chosen to be such as to optimize the agreement between the calculated and experimentally measured values of cross sections. The best values of these parameters are finally used in $\rho(r)$ to determine the r.m.s. charge radius defined by $\langle r^2\rangle^{1/2} = \{ \int_0^{r_{max}} r^2 \rho(r) d^3 r \}^{1/2}$.

From some previous papers one can immediately see a significant model-dependent effect in the determined values of $\langle r^2 \rangle$. For example, for C^{12} authors of paper [4], using the Fermi distribution function for $\rho(r)$ with two free parameters, have obtained the value $\langle r^2\rangle^{1/2} = 2.53 \pm 0.02$ fm. On the other hand, using the modified harmonic oscillator model for $\rho(r)$ they have found the value $\langle r^2\rangle^{1/2} = 2.44 \pm 0.02$ fm from the same experimental data. As a consequence the determined magnitudes of $\langle r^2\rangle^{1/2}$ are scattered in a rather broad interval: 1.63 fm $\le \langle r^2\rangle^{1/2}_{He^4} \le$ 1.71 fm (ref. [1]), 2.35 fm $\le \langle r^2_{C^{12}}\rangle^{1/2} \le$ 2.53 fm (refs. [2-5]) and 2.65 fm $\le \langle r^2_{O^{16}}\rangle^{1/2} \le$ 2.73 fm (refs. [2,3]).

In this paper we propose a new model-independent method of determination of $\langle r^2\rangle^{1/2}$ based on the hypothesis of analyticity of form factor in the complex momentum-transfer-squared t-plane. The method is described in Sect. 2. In Sect. 3 we present some concrete numerical results and discuss some indications that the diffraction minima of elastic electron scattering on light nuclei might be interpreted as zeros of corresponding electromagnetic form factors. We complete with Sect. 4 where inconsistencies of experimental data and underestimation of their errors are noted and conclusions are drawn.

2. Analyticity Hypothesis Applied to Electromagnetic Form Factors of Spin-Zero Light Nuclei

It is generally believed that the electromagnetic form factors of elementary particles are analytic functions in the cut complex t-plane. We extend this hypothesis to electromagnetic nuclear form factors. It allows us to apply a model-independent method of determination of r.m.s. charge radius to nuclei. Moreover, taking into

account the fact that there are no near singular points on the first Riemann sheet of the t-plane (aside from the cut from $t = 4m_\pi^2$ to $+\infty$), it is difficult to understand the nature of the sharp change of the behaviour of $F(t)$ in the vicinity of the diffraction minimum of elastic electron-nucleus scattering. It seems for us to be natural to explain this phenomenon as an occurance of zero of the form factor in this t range. At least three points do not contradict this assumption. First, the existence of zeros of form factor for $t < 0$ is not forbidden by any of the fundamental principles. Second, it is practically impossible to measure experimentally whether the differential cross section is equal to zero at the diffraction minimum. Third, the differential cross section contains $|F(t)|^2$ being thus insensitive to the sign of the form factor after the diffraction minimum.

It is interesting to mention that in some papers the analyticity property of nuclear form factor has already been exploited, however, tacitly. We have in mind the use of the well-known expression

$$F(t) = 1 + \frac{1}{6} \langle r^2 \rangle t \qquad (1)$$

in fits of low-energy data on $F(t)$ at very small values of t (see, e.g., ref. [5]). Equation (1) is nothing but the two first terms of the Taylor series, provided that $F(t)$ is an analytic function inside the circle around the point $t = 0$. Its radius of convergence equals $R = 4m_\pi^2 \approx 0.0784$ GeV^2 ≈ 2.0 fm^{-2}. Although it is not so simple to specify the region of validity of the approximation (1), it is clear that one can use it with confidence only for $|t| \ll R$. Therefore, as long as one wants to use eq. (1) for determining $\langle r^2\rangle^{1/2}$ one must have data at very low energy measurements, so as to remain in the t region where $|t| \ll R$.

We propose to exploit the analyticity of $F(t)$ in the whole complex t-plane. We are able to write an expressions for the form factor which is convergent in the whole region: $-\infty < t < 4m_\pi^2$. This allows us to use all data abailable on $F(t)$, also at large values of $t = -q^2$, for determination of $\langle r^2\rangle^{1/2}$.

The goal is achieved by the use of the conformal mapping technique *. We map the entire cut t -plane onto an unifocal ellipse in the z -plane so that the last experimental point (with the largest $|t|$ value) is mapped in $z=-1$ and the point $t=0$ in $z=1$. The cut is situated on the ellipse.

To determine the charge radius we use a search of the form

$$F[z(t)] = 1 + \sum_{n=1}^{M} A_n B_n [T_n(z)-1], \qquad (2)$$

where the normalization $F(0)=1$ is taken into account automatically due to the property of the Tschebyscheff polynomials $T_n(1)=1$. Here $B_n = (R^{2(n-1)} + R^{-2(n-1)} + 2\delta_{n-1,0})^{-1/2}$, R is the sum of the semiaxes of the ellipse and A_n are coefficients to be found from a fit. After the fit one can take the limit

$$\lim_{t\to 0} 6 \frac{dF[z(t)]}{dt} = \langle r^2 \rangle \qquad (3)$$

whence it is straightforward to calculate $\langle r^2\rangle^{1/2}$ and its error.

3. Numerical Examples

To demonstrate our method practically we have chosen the measurements of differential cross sections at energies which cover also the region of diffraction minimum: eHe^4 scattering at 800 MeV from ref. /1/ , eC^{12} scattering and eO^{16} scattering at 374 MeV from ref. /3/.

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* For more detail about it see, e.g., ref. /6/ where it has been used for another problem of nuclear physics: determination of the nuclear spectroscopic factors.

Usually some approximations are made in well-known formulae which connect the differential cross section with the form factor. To be consistent in all cases, we have evaluated this relation once more in one-photon exchange relativistic approximation. It has the following form:

$$\frac{d\sigma}{d\Omega} = \frac{e^2 Z^2}{8\pi^2 s} \frac{|F(t)|^2}{4q^4(1-\cos\theta)^2} \{2(E_e E_A + q^2)(E_e E_A + q^2\cos\theta) -$$

$$- m_A^2 (E_e^2 - q^2\cos\theta) + m_e^2 m_A^2\},$$

Here $\frac{d\sigma}{d\Omega}$ is a differential cross section, θ is a scattering angle, q is a momentum, E_e and E_A are total energies of an electron and nucleus respectively. All these quantities are in c.m. system, m_e and m_A are masses of the electron and nucleus respectively, $s=(E_e+E_A)^2$ and Z is a charge number of corresponding nucleus.

By means of eq. (4) we have calculated $F(t)$ and its errors using the aforementioned data *. We show the evaluated values of form factors in Figs. 1-3, assuming that they alter the sign after diffraction minima.

At the beginning, we carried out the fits without introducing zeros of form factor at the diffraction minimum. Comparing these fits with those with zeros we came to the conclusion that the values of χ^2 were reduced several times in the latter case (see Tables 1-3).. We consider this as a practical support for our hypothesis about existence of form factor zeros in the region of diffraction minimum.

Nevertheless, as can be seen from Tables 2,3, the values of χ^2 (even with zeros) are too large in the cases of form factors of C^{12} and O^{16}. The analysis of the partial values of χ^2 revealed that just the points

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* Note that data on $\frac{d\sigma}{d\Omega}$ in original papers are given in lab. frame.

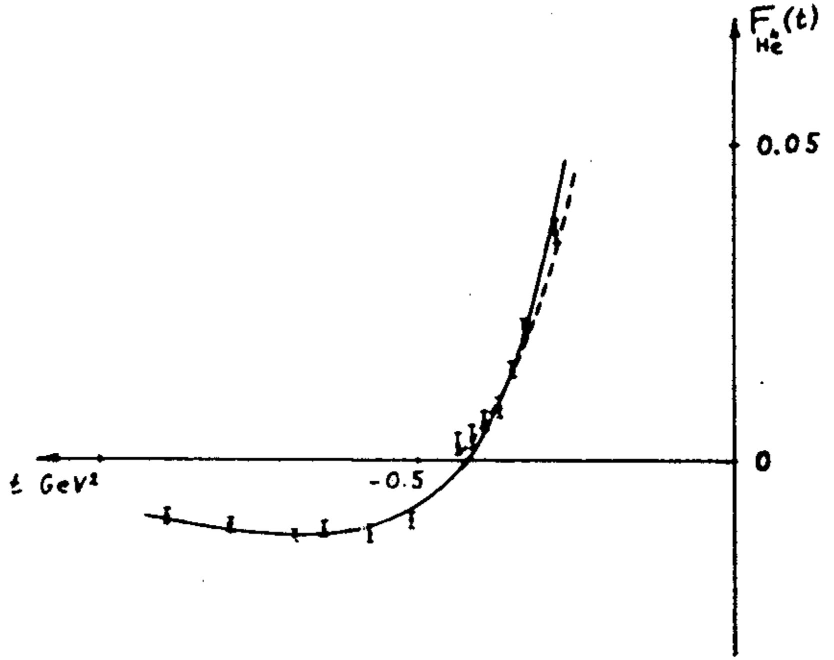

Fig. 1. The data on the eHe^4 elastic scattering at 800 MeV from ref./1/ and the fits to them with $M=3$. The solid line corresponds to the fit to all data and to the case when the form factor is assumed to have a zero. The dashed line corresponds to the fit of the data before the diffraction minimum only.

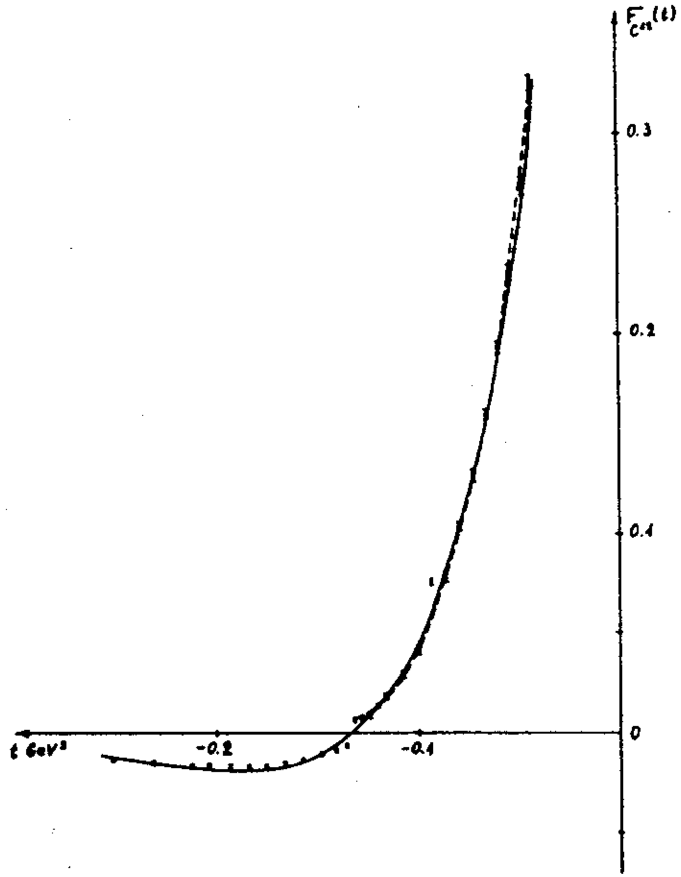

Fig. 2. The data on the eC^{12} elastic scattering at 374 MeV from ref./3/ and the fits to them with $M=3$. Conventions are the same as in Fig. 1.

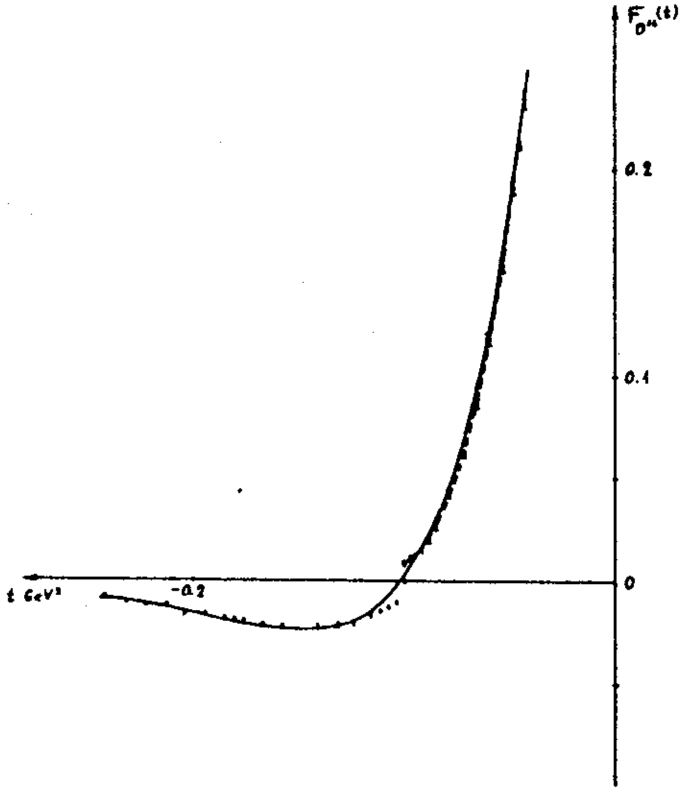

Fig. 3. The data on the eO^{16} elastic scattering at 374 MeV from ref./3/ and the fits to them with $M=3$. Conventions are the same as in Fig.1.

around diffraction minimum were responsible for such large values of the total value of χ^2. Recalling the experimental difficulties in measuring the differential cross section in the vicinity of diffraction minimum we conjecture that the errors of these particular values of $\frac{d\sigma}{d\Omega}$ have been underestimated.

To perform the conistency check of the determined values of radii we have also carried out the fits (see dashed lines in Figs. 1-3) only with the values of $F(t)$ before the diffraction minimum. Also in this case reasonable results have been obtained (see Tables 1-3).

4. Conclusions

We have proposed a new model-independent method of determination of $\langle r^2\rangle^{1/2}$ for spin-zero light nuclei. Some practical examples also have been considered.

In principle one could reanalyse all existing data on eHe^4, eC^{12} and eO^{16} elastic scattering using this method in order to find the most unbiased estimates of radii of corresponding nuclei. However, it does not seem to be so straightforward to carry out it practically due to inconsistencies of available data obtained in different experiments (compare, for instance, the data on eHe^4 scattering from ref. /1/ and ref. /7/). On the whole, we feel that the errors of the data have been underestimated, especially, as has already been mentioned, in the regions of diffraction minima. In any case, one should begin this work with thorough analysis of errors and mutual consistency of experimental data. We consider these questions as exceeding the scope of this paper.

At the end we would like to note that the model-independent fit of the form factor which can be obtained by our method can be used not only for determination of the radius but also, through the Fourier transform, for the model-independent determination of the charge distribution $\rho(r)$. The latter, subsequently, can be compared with predictions of different nonrelativistic nuclear models.

Table 1

Results of the fits to the data on eHe^4 elastic scattering at 800 MeV from ref. /1/. The truncation point of the series (2) should be determined by minimizing the quantity $X = \chi^2 + \Phi$ where Φ is the Cutkosky convergence test function (see, e.g., ref. /6/). The numbers in parenthesis at $M=3$ show the values of χ^2 and $\langle r^2\rangle^{1/2}$ when the form factor was assumed not to have a zero.

M	all data (13 points)			data before diffraction minimum only (7 points)		
	χ^2	X	$\langle r^2\rangle^{1/2} \pm \Delta\langle r^2\rangle^{1/2}$ (fm)	χ^2	X	$\langle r^2\rangle^{1/2} \pm \Delta\langle r^2\rangle^{1/2}$ (fm)
2	684	689	1.661±0.001	4.5	9.3	1.767±0.005
3	18.6	25	1.883±0.007	3.1	8.5	1.86±0.06
	(86)		(1.993±0.007)			
4	14.1	23	2.01±0.04	2.0	24	2.97±0.58
5	12.8	35	2.46±0.22	0.7	43	13.55±3.56

Table 2

Results of the fits to the data on eC^{12} elastic scattering at 374 MeV from ref. /3/. Conventions are the same as in Table 1.

M	all data (28 points)			data before the diffraction minimum only (15 points)		
	χ^2	X	$\langle r^2\rangle^{1/2} \pm \Delta\langle r^2\rangle^{1/2}$ (fm)	χ^2	X	$\langle r^2\rangle^{1/2} \pm \Delta\langle r^2\rangle^{1/2}$ (fm)
2	14996	15003	2.272±0.001	56.2	64	2.423±0.002
3	2375	2386	2.537±0.002	55.4	63	2.42±0.01
	(4633)		(2.750±0.002)			
4	2372	2383	2.518±0.007	45.6	72	2.81±0.03
5	2332	2364	2.72±0.02	30.5	77	1.44±0.25

Table 3

Results of the fits to the data on eO^{16} elastic scattering at 374 MeV from ref. /3/. Conventions are the same as in Table 1.

M	All data (33 points)			Data before the first diffraction minimum only (13 points)		
	χ^2	X	$\langle r^2\rangle^{1/2} \pm \Delta\langle r^2\rangle^{1/2}$ (fm)	χ^2	X	$\langle r^2\rangle^{1/2} \pm \Delta\langle r^2\rangle^{1/2}$ (fm)
2	60865	60872	2.402±0.001	23.3	32	2.643±0.002
3	3000	3013	2.751±0.001	4.0	13	2.70±0.01
	(4257)		(2.946±0.001)			
4	2971	2984	2.716±0.005	2.4	22	2.79±0.04
5	2907	2938	2.88±0.01	2.4	33	2.75±0.30

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