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Yu.A.Kubyshin*

CORRECTIONS TO THE ASYMPTOTIC FORMULA FOR HIGH ORDERS OF PERTURBATION THEORY

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*Institute of Nuclear Physics, Moscow State University

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It has been established for a variety of quantum-field models that the coefficients of the perturbation theory (PT) for the Green functions, anomalous dimensions, β -function (or Gell-Mann-Low function), etc., show the asymptotic behaviour of the type (see, for example, refs.[1,2]):

$$\beta(g)=\sum_n \beta_n (-g)^n, \tag{1}$$

$$\beta_n = \frac{1}{A^n}\, n!\, n^{\alpha-1} C\,(1+ b_1/n + O(1/n^2)). \tag{2}$$

The essence of the method of obtaining formula (2) is to evaluate the functional integral determining the Green function in the n-th order of PT by the steepest descent method by analogy with an ordinary integral. The classical solution of the equation of motion corresponds to the saddle point. A partial proof of this method for the lattice theory is given in [3]. If we suppose that β -function is analytic in coupling constant g in the complex plane cut along the negative real axis (as it takes place in the case of anharmonic oscillator[4]), we may write the dispersion relation

$$\beta(g)=\frac{1}{2\pi i}\int_{-\infty}^{0} \frac{\mathrm{disc}\,\beta(g')}{g'-g}\, dg', \tag{3}$$

where $\mathrm{disc}\,\beta(-g)=\beta(-g+i0)-\beta(-g-i0)$ and $\beta(-g\pm i0)$ is the analytic continuation of the functional integral, determining the Green function, from the positive semiaxis to the negative one through the upper or lower half-plane. The $\mathrm{disc}\,\beta(-g)$ can be also evaluated by the calculation of the functional integral by the steepest descent method [5,6]. If the asymptotic formula for $\mathrm{disc}\,\beta(-g)$ at $g\to +0$ is of the form

$$\mathrm{disc}\,\beta(-g)=-iB\,e^{-A/g}\, g^{-\alpha}\left(1+g\,\frac{a_1}{A}+O(g^2)\right), \tag{4}$$

we can substitute it in (3) and obtain (2) with

$$C=\frac{B}{2\pi A^{\alpha}}\,, \qquad b_1=a_1+\frac{\alpha(\alpha-1)}{2}\,. \tag{5}$$

Up to now only the leading asymptotic term (i.e., the numbers C, A, α) has been calculated in quantum field theory. But for example, in the case of the $\varphi^4_{(4)}$ model the comparison of the asymptotic coefficients (i.e., the coefficients calculated by eq. (2) in the leading approximation) for n=2,3,4,5 with the exact ones shows that the asymptotics does not set on in the fifth order in g [7]. The analysis of anharmonic oscillator enables one to make a conclusion that the situation improves considerably if the corrections b_1/n and b_2/n^2 in (2), which have been calculated in [8] and [9], are taken into account. That is why the calculation of the corrections in the realistic field models is of some interest. The values of $b_1, b_2,\ldots$ may be used to approximately restore the function $\beta(g)$ from several exact coefficients and the asymptotic formula (see ref.[10]).

The main purpose of this work is to give an account of the technique of calculation of the coefficients a_i in formula (4) and to calculate a_1 for massless scalar model $\varphi^4_{(4)}$. The feature differing this model from the anharmonic oscillator is the presence of ultraviolet divergencies and the necessity of renormalizations.

1. General formulas and method of regularization

Following[1,2] we shall examine the scalar model with action

$$S[\varphi]=\int dx\left[\frac{1}{2}\sum_{i=1}^{N}(\partial_\nu\varphi_i)^2+\frac{g}{4!}\Big(\sum_{i=1}^{N}\varphi_i^2\Big)^2\right]+S'[\varphi;g] \tag{6}$$

in the four-dimensional Euclidean space. Here $S'[\varphi;g]$ contains the counterterms which provide for subtraction of the ultraviolet divergencies, g is the renormalized coupling constant, μ is the parameter of renormalization

$$S'[\varphi;g]=\int dx\Big[\frac{1}{4!}(\varphi_i^2)^2\left(g^2\alpha_2(\mu)+g^3\alpha_3(\mu)+\ldots\right)+$$
$$\frac{1}{2}(\partial_\nu\varphi_i)^2\left(g^2c_2(\mu)+\ldots\right)+\frac{1}{2}\varphi_i^2\left(g\,d_1(\mu)+\ldots\right)\Big]\,. \tag{7}$$

By substituting $\varphi(x)\to\psi(x)/\sqrt{g}$ it can be shown that for small g the saddle point solution is determined by the part of the action without counterterms. For the β -function defined in a standard way [11] the discontinuity on the cut $\mathrm{disc}\,\beta(-g)$ $(g>0)$ is determined only by the $\mathrm{disc}\, G^{(4)}(p^2/\mu^2; -g)$ if the terms $O(g^2)$ in (4) are neglected.

$G^{(4)}$ is the four-point Green function including weak-connected and disconnected diagrams. It is defined by the functional integral as

$$G^{(4)}\left(\frac{p_1^2}{\mu_1^2},\dots,\frac{p_4^2}{\mu_4^2};g\right)(2\pi)^4\,\delta\Big(\sum_{n=1}^{4}p_n\Big)\,T_{i_1\dots i_4}=$$

$$=\prod_{n=1}^{4}p_n^2\int\exp\Big(i\sum_{m=1}^{4}p_m y_m\Big)\prod_{K=1}^{4}dy_K\;G^{(4)}_{i_1\dots i_4}(y_1,\dots,y_4;g); \qquad (8)$$

$$G^{(4)}_{i_1\dots i_4}(y_1,\dots,y_4;g)\equiv G^{(4)}_{i_K}(y_K;g)=\frac{1}{J_0}\int\prod_x\mathcal{D}\varphi_i(x)\prod_{K=1}^{4}\varphi_{i_K}(y_K)\,e^{-S[\varphi]};$$

$$J_0=\int\prod_x\mathcal{D}\varphi_i(x)\,\exp\Big[-\frac{1}{2}\int dx\,(\partial_\nu\varphi_i)^2\Big].$$

Here $T_{i_1\dots i_4}$ stands for the $O(N)$ rotation-group tensor:

$$T_{i_1\dots i_4}=\frac{2}{3}\left(\delta_{i_1i_2}\delta_{i_3i_4}+\delta_{i_1i_3}\delta_{i_2i_4}+\delta_{i_1i_4}\delta_{i_2i_3}\right).$$

To regularize the theory we shall operate in the space of dimension $d=4-2\varepsilon$ instead of the four-dimensional space and replace the constant g in (6) by the function $g_\varepsilon(x;g)$ which is sufficiently smooth in x, vanishes at $|x|\to\infty$, is regular in ε and g near the point $g=0$, $\varepsilon=0$ and satisfies the conditions

$$g_\varepsilon(x;g)\big|_{\varepsilon=0}=g,\qquad g_\varepsilon(x;g)\big|_{g=0}=0. \qquad (9)$$

In virtue of the local character of counterterms, the results of the calculations in PT with such a regularization after the transition to the limit $\varepsilon\to 0$ will not be different from the results obtained by any other method (for example, by the dimensional regularization). Let us consider for example the 4-point Green function in the one-loop approximation for N=1

$$G^{(4)}(y_K;g)=\int d^dx\,g_\varepsilon(x;g)\prod_{n=1}^{4}\Delta_0(y_n-x)+\frac{1}{2}\int d^dx_1\,d^dx_2\,\cdot$$

$$\cdot\,g_\varepsilon(x_1;g)\,g_\varepsilon(x_2;g)\Big\{\prod_{n=1}^{2}\Delta_0(y_n-x_1)\prod_{m=3}^{4}\Delta_0(y_m-x_2)\,\cdot$$

$$\cdot\Big[\Delta_0^2(x_1-x_2)+\delta(x_1-x_2)\frac{2}{3}\alpha_2(\mu)\Big]+\text{permutations of } y_n\Big\}+\dots$$

where $\alpha_2(\mu)$ is the factor from (7), $\Delta_0(x)$ is the free propagator.

After the subtraction of divergencies, the integrand in brackets is finite, and we may put $\varepsilon=0$ and our result will not differ from the one obtained with $g_\varepsilon(x;g)=g$=const. Let us note that with such a regularization the term with the derivative of the field in (7) must be rewritten as follows:

$$\int d^dx\,[\partial_\nu(g_\varepsilon(x;g)\,\varphi_i(x))]^2\,C_2(\mu).$$

We shall use the arbitrariness in the definition of the function $g_\varepsilon(x;g)$ to choose it as

$$g_\varepsilon(x;g)=g\left(\frac{2}{1+x^2}\right)^{4-d}. \qquad (10)$$

Then, it can easily be verified that the function

$$(\varphi_c)_i(x)=u_i\sqrt{\frac{3d(d/2-1)}{g}}\left(\frac{2}{1+x^2}\right)^{d/2-1}, \qquad (11)$$

where u_i stands for the arbitrary isotopic vector with $u_i^2=1$, is the solution for the saddle-point equation obtained by varying the regularized action with negative coupling constant $(-g)(g>0)$. The action calculated on the solution (11) is finite and at $d\to 4$ (11) turns out to be the solution used by Lipatov [1].

The conventional version of dimensional regularization with constant $g_\varepsilon(x;g)\equiv g$ was used to calculate the leading term in the asymptotic formula in [12]; in this case, however, the exact solutions for the equation of motion are not known. Moreover, the initial theory is invariant with respect to the O(5) group which may be realized as a linear transformation group by making the stereographic projection onto the sphere S^4 in five-dimensional Euclidean space. The transition to the space of dimension d and the co-ordinated introduction of the function (10) make the theory $O(d+1)$ invariant thereby considerably simplifying the calculations. With other regularization methods irrelevant to the dimensional regularization the equation of motion has the exact solution (11) with d =4, but the O(5) invariance disappears.

When expanding the action around $\varphi_c(x)$ the spectrum of the operator of quadratic fluctuations has the zero eigenvalues associated with rotational invariance in the theory and the eigenvalues proportional to ε and associated with translational and scale invariance since the latter is violated by the regularization. To correctly calculate the functional integral we shall use the Faddeev-

Popov method. First of all we shall pass to the space of dimension $d=4-2\varepsilon$, substitute in (8) the representation of the unity(see Appendix) and then replace the coupling constant g by the function

$$g_\varepsilon(x;g;x_0,\lambda)=\frac{1}{\lambda^{4-d}}\,g_\varepsilon\left(\frac{x-x_0}{\lambda};g\right),$$

where $g_\varepsilon(x;g)$ is determined by eq. (10).

We shall carry out the calculations in the scheme of momentum subtractions at the symmetrical point μ^2 ; the Green functions will be considered in the regime of symmetrical asymptotics with respect to momentum p_n : $(p_n+p_m)^2=4p_k^2/3$. At the subtraction point $\mu^2=(p_n+p_m)^2$ $(n,m,k=1,2,3,4)$. The constants in counterterms (7) are given in Appendix.

2. Expansion around saddle point solution

To facilitate the calculations, we shall operate on the sphere S^d in the $d+1$ -dimensional space with coordinates /1/

$$z_\mu=\frac{2x_\mu}{1+x^2}\;,\; z_{d+1}=\frac{x^2-1}{x^2+1}\;,\;\sum_{\alpha=1}^{d+1} z_\alpha^2=1\;,\;\mu=1,2,\dots,d\;,$$

$$\varphi_i(x)=\left(\frac{2}{1+x^2}\right)^{d/2-1}\psi(z)\;,$$

where $\psi(z)$ is the function defined on the sphere.

Fulfilling the expansion around the classical solution in a standard way /1,2/, we obtain

$$\operatorname{disc} G^{(4)}_{in}(y_n;-g)=\mathcal{D}\,g^{-\frac{d+4+N}{2}}e^{-A(\varepsilon)/g}\int_0^\infty d\lambda\,\lambda^{3-3d}\int d^dx_0\,\cdot \tag{12}$$
$$\cdot\int d^{(N-1)}v\,(1+u_N)^{N-1}\,\Phi_{i_1\dots i_4}\left[\frac{\delta}{\delta\theta}\right]\mathcal{Z}[\theta]\Big/_{\theta=0}\;;$$

$$\Phi_{i_1\dots i_4}[\psi]=\prod_{n=1}^{4}\left\{\frac{2}{1+\left(\frac{y_n-x_0}{\lambda}\right)^2}\right\}^{d/2-1}\left(u_{i_n}+\frac{1}{\psi_c}\psi_{i_n}(\varkappa_n)\right)\cdot \tag{13}$$
$$\cdot\det\left(\delta_{\rho\sigma}+\sqrt{g}\,a_{\rho\sigma}[\psi]\right)\exp\left(-\Delta S+\sqrt{g}\,S'_{1/2}+gS'_1+\dots\right);$$

$$\Delta S=-\int d\Omega_d\left[\sqrt{\frac{g}{6}\frac{d}{2}\left(\frac{d}{2}-1\right)}\,u_i\psi_i(z)\psi_j^2(z)+\frac{g}{4!}\left(\psi_i^2(z)\right)^2\right].$$

The terms $\mathcal{D},A(\varepsilon),v$ are presented in Appendix, u_i is the isotopic vector in (11), the argument $\varkappa_n$ of the function ψ corresponds to the point $(y_n-x_0)/\lambda$ in the d -dimensional Euclidean space, $d\Omega_d$ is the volume of integration over the sphere S^d . The terms $S'_{1/2}$, S'_1 are the coefficients before $\sqrt{g}$ and g in the expansion of counterterms, $a_{\rho\sigma}[\psi]$ arises from the expansion of $B_{\rho\sigma}$ (see Appendix) around the classical solution $(\psi_c)_i=u_i\psi_c$ defined on the sphere $(\psi_c=\sqrt{3d(d/2-1)g^{-1}})$. The generating functional $\mathcal{Z}[\theta]$ is equal to

$$\mathcal{Z}[\theta]=\frac{1}{J_0}\int\prod_z\mathcal{D}\psi_i\,\delta^{(d+N)}\left(\int d\Omega_d\,\psi(z)\eta(z)\right)\cdot \tag{14}$$
$$\cdot\exp\left[-S_2+\int d\Omega_d\,\theta_i(z)\,\psi_i(z)+S'_0\right]\;.$$

Here S'_0 is the part of counterterms independent of g,

$$S_2=\frac{1}{2}\int d\Omega_d\,\psi_i(z)\left[(\delta_{ij}-u_iu_j)(-\hat{L}^2)+u_iu_j\left(-\hat{L}^2-d(d/2-1)\right)\right]\psi_j(z) \tag{15}$$

is the part of the initial action square in quantum fluctuations ψ . The symbol $\eta(z)$ stands for the eigenfunctions corresponding to the zero (η^T) and proportional to $\varepsilon(\eta^L_\alpha,\alpha=1,\dots,d+1)$ eigenvalues of the operator of quadratic fluctuations (see ref. /1,2/), $\hat{L}^2$ is the angular part of the $d+1$ -dimensional Laplace operator.

To calculate the leading asymptotics (4) by the steepest descent method we have to limit ourselves to the term quadratic in fluctuations in the expansion of the action around the saddle solution(this term is independent of g)(see ref. /1/).To obtain the corrections to the leading asymptotics, we have to expand $\Phi[\psi]$ in g , i.e., carry out perturbation calculations around the classical solution. Used as a propagator is $G_{ij}(p(z,z'))$, the function of propagation of the field ψ in the external field ψ_c , which depends on $p(z,z')=\sum_\alpha z_\alpha z'_\alpha$, the distance between the points z and z' on the sphere, which is invariant with respect to $O(d+1)$ transformations. The propagator has the longitudinal and transverse, with respect to the isotopic vector u_i (see eq. (11)), components G^L and G^T satisfying the equations determined by S_2 :

$$\left[-\hat{L}^2_{z'}-d(d/2-1)\right]G^L(p(z',z''))=\delta(z',z'')-\sum_\alpha\eta^L_\alpha(z')\eta^L_\alpha(z'')\;; \tag{16}$$
$$-\hat{L}^2_{z'}\,G^T(p(z',z''))=\delta(z',z'')-\eta^T(z')\eta^T(z'')\;.$$

These equations may be solved either by the Fock method of fifth parameter (see ref./13/) or by presenting G^L and G^T to be a series in spherical functions of the $d+1$ -dimensional space (see ref./1/) and using the addition theorem for the Gegenbauer polynomials and the generalized Dugoll equality (see in Appendix). Eventually we get

$$G^L(p)=\frac{\Gamma(\frac{d-1}{2})}{4\pi^{(d+1)/2}}\left[-\frac{\pi}{\sin\pi\varkappa}C_\varkappa^{\frac{d-1}{2}}(-p)-\frac{d+1}{d(2-d/2)}C_1^{\frac{d-1}{2}}(p)\right],$$

$$G^T(p)=-\frac{\Gamma(\frac{d-1}{2})}{4\pi^{(d+1)/2}}\left[\frac{d}{ds}C_s^{\frac{d-1}{2}}(-p)\Big|_{s=0}+\frac{1}{d-1}\right],$$

where $C_\varkappa^\nu$ is the Gegenbauer function (see ref./14/), and $\varkappa=-(d-1)/2+\sqrt{(d-1)^2/4+d(d/2-1)}=1-4\varepsilon/5+O(\varepsilon^2)$. We shall present also the expressions of the propagators G^L and G^T in the external classical field at $\varepsilon=0$:

$$G^L(p)=\frac{1}{8\pi^2}\left[\frac{3p^2-1}{2}\,\frac{1}{1-p}-3p\ln\frac{1-p}{2}-\frac{3+7p}{2}-\frac{3}{5}p\right];$$

$$G^T(p)=\frac{1}{8\pi^2}\left[\frac{1+p}{2}\,\frac{1}{1-p}-\ln\frac{1-p}{2}-11/6\right].$$

3. Calculation of the first correction to the asymptotics

To calculate the first correction to the leading approximation of the asymptotic formula for disc $G^{(4)}_{in}(y_n;-g)$, we have to retain the term proportional to g in the expansion of $\Phi[\psi]$ (see eq. (13)) in coupling constant. The integrals contributing to corrections to asymptotic formula (4) can be pictured in the form of diagrams.

The Feynman rules in the case N=1 are represented in fig. 1, $G_0(p)$ is the free propagator of the initial theory (6). The diagrams depicted in Fig. 2 a-e contribute to the first correction a_1. Moreover, the diagrams F_1, F_2, F_3 arise from the expansion of ΔS in (13). The double line in Fig. 2 d-e corresponds to the functions arising from the expansion of det$(\delta_{\rho\sigma}+\sqrt{g}\,a_{\rho\sigma}[\psi])$ and the product of $\psi(\varkappa_n)$ in (13). The counterterms must also be represented as a series in g . All the contributions are computed with the help of the formulas given in /14/ and /15/.

Let us consider the calculation of F_3 in detail. We did not succeed in direct calculation of the integral

$$\int d\Omega_d\, d\Omega'_d\,[G(p(x,x'))]^3.$$

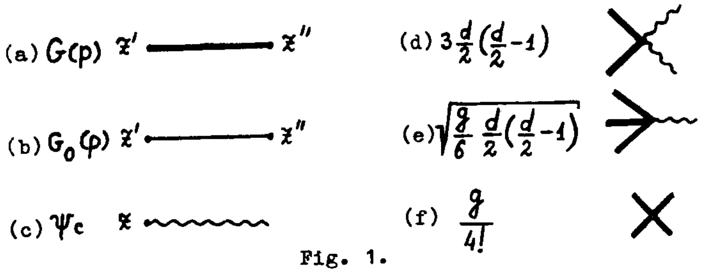

Fig. 1.

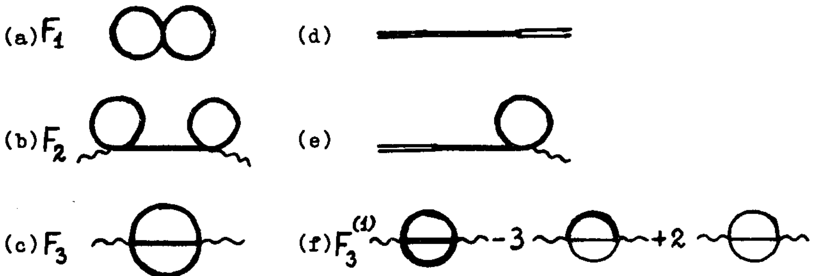

Fig. 2.

That is why we represent F_3 in the form $F_3=F_3^{(1)}+F_3^{(2)}$, where $F_3^{(1)}$ corresponds to the diagrams in Fig. 2,f. It can be easily shown by defining on the sphere the free propagator $G_0(p)$ of the initial theory

$$G_0(p)=\frac{\Gamma(\frac{d-2}{2})}{2^{\frac{d+2}{2}}\pi^{d/2}}\,\frac{1}{(1-p)^{d/2-1}}$$

that $F_3^{(1)}$ is finite for $\varepsilon=0$ and can be calculated with another regularization. For example, we may write the integral in the form

$$\int d\Omega_d\, d\Omega'_d\ldots=\Omega_d\Omega_{d-1}\int_{-1}^{1}(1-p^2)^{d/2-1}dp\ldots=\Omega_d\Omega_{d-1}\lim_{\alpha\to0}\int_{-1}^{1-\alpha}(1-p^2)^{d/2-1}dp\ldots$$

and then take the limit $\varepsilon=0$. Finally we obtained the following values of the constants in asymptotic formulas (2) and (4):

$$\alpha=N/2+3\;;\;A=16\pi^2\;;$$

$$C=\frac{N+8}{9}\,\frac{\Gamma(5/2)}{\Gamma(\frac{N+4}{2})}\,2^{\frac{40-N}{2}}\,3^{\frac{2N+7}{6}}\,\pi^{-\frac{N-1}{6}}\,W\cdot$$

$$\exp\left[\frac{N+2}{\pi^2}\zeta'(2)-\frac{N+2}{6}C_E-\frac{N+14}{4}\right].$$

Here $C_E = 0.577216\ldots$ is the Euler constant, $\zeta'(2) = -0.937\ldots$ is the derivative of the Riemannian zeta-function, the factor W is

$$W = \int_0^\infty dx \, K_1^4(x)\, x^{3+(N+8)/3} ,$$

where $K_1(x)$ is the modified Bessel function. The expression for C coincides with that obtained in /1,2/ (the only difference is in the form of writing). Our main result is the value of the coefficient a_1 in (4):

$$a_1 = -\frac{N^2+10N+36}{6} C_E + \frac{3N^2+28N+104}{36} \ln\frac{3}{4} - \frac{N+8}{6}\,\frac{7}{36}\,\pi^2 +$$

$$\frac{5N+22}{9}\,\mathcal{T} + \frac{73N^2-228N-2000}{360} - \frac{N^2+10N+36}{2(N+8)} + I_1 + I_2 + I_3 ;$$

$$I_1 = -\frac{1}{W}\,\frac{N^2+10N+36}{6} \int_0^\infty dx \, x^{3+(N+8)/3} K_1^4(x) \ln(x/2) ;$$

$$I_2 = -\frac{N+8}{9}\,\frac{3}{\pi^2 W} \int_0^\infty dx \, x^{4+(N+8)/3} K_1^3(x) \int d^4y \, \frac{e^{ix(\nu y)}}{1+y^2} \cdot$$

$$\left[\frac{y^2+14}{60(1+y^2)} - \frac{5}{12} + \frac{\ln(1+y^2)}{1+y^2} \right] ;$$

$$I_3 = \frac{1}{W}\,\frac{1}{2\pi^2} \int_0^\infty dx \, x^{5+(N+8)/3} K_1^2(x) \int d^4y_1 \, d^4y_2 \cdot$$

$$\cdot \frac{\exp\left(ix(\nu_1 y_1) + ix(\nu_2 y_2)\right)}{(1+y_1^2)\,(1+y_2^2)} \left[G^L(\rho(z_1,z_2)) + \frac{N-1}{3} G^T(\rho(z_1,z_2)) \right],$$

where the factor $\mathcal{T} = 0.749\ldots$ arises from the two-loop counterterm (see in Appendix), the points z_1 and z_2 on the sphere S^4 correspond to the points y_1 and y_2 in the four-dimensional Euclidean space, the 4-vectors ν, ν_1, ν_2 satisfy the relations $|\nu| = |\nu_1| = |\nu_2| = 1$, $\cos(\widehat{\nu_1 \nu_2}) = -\frac{1}{3}$. After simplifications the integrals I_1, I_2, I_3 may be computer - calculated numerically. The Table presents the numerical values of the constants C, a_1, b_1 (see formulas (2) and (4)) for certain N.

Table

The values of the factor C and coefficients a_1, b_1 in the asymptotic formulas (4) and (2)

Isotopic Number N	1	2	3	4	7	10
C	1.097	0.543	0.249	0.107	0.00644	$0.284\cdot 10^{-3}$
a_1	-12.6	-14.8	-17.1	-19.2	-26.5	-34.4
b_1	-4.7	-4.82	-4.76	-4.2	-2.1	1.6

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Appendix

To integrate over the rotational variables u_i , we shall introduce N-1 values v_s :

$$u_s = \frac{2v_s}{1+v^2} , \quad u_N = \frac{1-v^2}{1+v^2} \qquad (s = 1, 2, \ldots, N-1) .$$

We substitute the following representation of unity into eq. (8):

$$1 = \int_{-\infty}^{\infty} d\lambda \int_{-\infty}^{\infty} d^d x_0 \int_{-\infty}^{\infty} d^{N-1} v \; \delta^{(d)}\Big(\int dx \, \lambda^{d/2-2} \varphi_i(\lambda x + x_0) u_i \frac{2x_\mu}{(1+x^2)^{d/2+2}} \Big) \cdot$$

$$\delta\Big(\int dx \, \varphi_i(\lambda x + x_0) u_i \frac{x^2-1}{(1+x^2)^{d/2+2}} \Big) \, \delta^{(N-1)}\Big(\frac{4}{d+2} \int dx \, \lambda^{d/2-1} \cdot$$

$$\varphi_i(\lambda x + x_0) V_{i,s} \frac{1}{(1+x^2)^{d/2+1}} \Big) \det B[\varphi; \lambda, x_0, v] ;$$

$$V_{t,s}(v) = \delta_{ts} - u_t u_s / (1+u_N) ; \quad V_{N,s}(v) = -u_s ;$$

$$s, t = 1, 2, \ldots, N-1 .$$

Here λ is the scale and x_0 is the translational parameter , B is the matrix of the size $(d+N)\times(d+N)$. Its components may be obtained by differentiation of the arguments of δ -functions with respect to x_0, λ and v .

The constants in counterterms (7) in our renormalization scheme are as follows:

$$\alpha_2(\mu) = -\frac{N+8}{6}\frac{1}{(2\pi)^d}\int\frac{d^dp}{p^2(q_\mu-p)^2} = -\frac{N+8}{6}\frac{\Gamma(2-d/2)}{\Gamma(d-2)}\frac{1}{16}\frac{\Gamma^2(d/2-1)}{\pi^{d/2}}\cdot\left(\frac{4}{q_\mu^2}\right)^{2-d/2};$$

$$\alpha_3(\mu) = \alpha_3^{(1)}(\mu) + \alpha_3^{(2)}(\mu);$$

$$\alpha_3^{(1)}(\mu) = \frac{N^2+26N+108}{36}\left[\frac{1}{(2\pi)^d}\int\frac{d^dp}{p^2(q_\mu-p)^2}\right]^2 = \frac{N^2+26N+108}{(N+8)^2}(\alpha_2(\mu))^2;$$

$$\alpha_3^{(2)}(\mu) = -\frac{4(5N+22)}{36}\frac{1}{(2\pi)^{2d}}\int\frac{d^dp\,d^dk}{p^2k^2(q_{1\mu}-p-k)^2(q_\mu-k)^2} =$$

$$= -\frac{4(5N+22)}{36}\frac{1}{2^8}\left[\frac{1}{2\pi^{4-2\varepsilon}}\frac{\Gamma^3(1-\varepsilon)\Gamma(1+2\varepsilon)\Gamma(1-2\varepsilon)}{\Gamma(2-2\varepsilon)\Gamma(2-3\varepsilon)}\frac{1}{\varepsilon^2}\left(\frac{4}{q_\mu^2}\right)^{2\varepsilon} + \frac{\mathcal{T}}{\pi^4}\right];$$

$$c_2(\mu) = \frac{N+2}{18}\frac{1}{(2\pi)^{2d}}\frac{1}{q_{1\mu}^2}\int\frac{d^dp_1\,d^dp_2}{p_1^2p_2^2(q_{1\mu}-p_1-p_2)^2} =$$

$$= \frac{N+2}{18}\frac{1}{2^9\pi^{4-2\varepsilon}}\left(\frac{4}{q_{1\mu}^2}\right)^{2\varepsilon}\frac{\Gamma^3(1-\varepsilon)\Gamma(1+2\varepsilon)}{(1-2\varepsilon)\Gamma(3-2\varepsilon)}\frac{1}{\varepsilon};$$

$$d_i(\mu) = 0.$$

Here q_μ and $q_{1\mu}$ are the momentum subtraction points, $q_\mu^2 = \mu^2$, $q_{1\mu}^2 = 3\mu^2/4$, factor $\mathcal{T}$ is

$$\mathcal{T} = \frac{1}{\pi^2}\int\frac{d^4p}{p^2(q_\mu-p)^2}\ln\frac{(q_{1\mu}-p)^2}{p^2} = 0.749\ldots \quad [7].$$

The constants $\mathcal{D}$ and $A(\varepsilon)$ in (12) are:

$$\mathcal{D} = \left[-\sqrt{3\Omega_d\,d(d/2-1)}\right]^{d+N}\left(\frac{d-2}{2\sqrt{d+1}}\right)^{d+1}\left[3d(d/2-1)\right]^2;$$

$$\Omega_d = 2\pi^{\frac{d+1}{2}}/\Gamma\left(\frac{d+1}{2}\right);$$

$$A(\varepsilon) = \frac{3}{2}\Omega_d\left[\frac{d}{2}\left(\frac{d}{2}-1\right)\right]^2.$$

Ω_d is the surface of the sphere S^d, $A(\varepsilon)$ characterizes the action calculated on the classical solution φ_c ($S[\varphi_c] - S'[\varphi_c] = A(\varepsilon)/g$).

The generalized Dugoll equality is

$$\sum_{n=0}^{\infty}\frac{(2n+2\nu)(-1)^n}{(n-\mu)(n+2\nu+\mu)}\frac{\Gamma(n+2\nu)}{\Gamma(n+1)}\cos(n+\nu)\vartheta = -\frac{\pi}{\sin\pi\mu}\cos(\mu+\nu)\vartheta\frac{\Gamma(\mu+2\nu)}{\Gamma(\mu+1)}.$$

Let us consider the integral along the circumference of the radius $N+1/2$ (N is integer) in order to derive this equality:

$$\int_{C_{(N+\frac{1}{2})}}\frac{\cos(z+\nu)\vartheta}{z-\mu}\frac{1}{\sin\pi z}\frac{\Gamma(z+2\nu)}{\Gamma(z+1)}\,dz.$$

We may evaluate it with the help of the theorem of residues and then take the limit $N\to\infty$ and obtain the previous equality. For $\nu = 1/2$ this equality is given in /14/.

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Поправки к асимптотической формуле для высоких порядков теории возмущений

Разработана техника вычисления поправок по $1/n$ к асимптотической формуле для n-го члена ряда теории возмущений на примере скалярной безмассовой модели $\phi^4_{(4)}$ с внутренней симметрией O(N). Получена первая поправка для β -функции.

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Kubyshin Yu.A. E2-82-915

Corrections to the Asymptotic Formula for High Orders of Perturbation Theory

The technique for calculating the corrections in $1/n$ to the asymptotic formula of the n-th term of the perturbation expansions is developed using the example of the scalar massless $\phi^4_{(4)}$ model with internal symmetry O(N). The first correction of the β-function has been obtained.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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