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INFRARED STABILITY AND GLOBAL SYMMETRY

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1. INTRODUCTION

The prevailing philosophy of grand unification is based on the fact that symmetry will increase at high energies. This finds its verification in gauge sector where the gauge couplings join together in the ultraviolet region. However, for Yukawa and scalar quartic couplings things are different. Here, as a rule, symmetry tends to increase in the infrared region. This fact was known earlier [1,2], however, with the appearance of the idea of grand unification and hopes to construct a unified theory of all types of interactions without divergences it attracts now new attention [3].

The aim of the present paper is to demonstrate the behaviour of Yukawa and scalar quartic couplings in gauge theories and to show that global symmetry has a tendency to increase in the infrared region. This is not an occasional play of numbers and is a characteristic feature of the given type of interaction. Finally we briefly discuss some consequences on this fact for the model building of particle interactions.

We shall consider the lagrangian density invariant in a given energy region under some group G being a product of local and global groups

$$G = G_{gauge} \otimes G_{global} \; .$$

We shall be interested in a global symmetry group connecting Yukawa and/or scalar quartic couplings. It may be, e.g., a horizontal group of generations or supersymmetry.

2. YUKAWA COUPLINGS. HORIZONTAL SYMMETRY

Let us consider the renormalization group equations describing the evolution of Yukawa couplings in a theory with the horizontal symmetry. To the one loop they are

$$\frac{dY_i}{dL} = Y_i \, [A_i Y_i^2 + \sum_{j \neq i} B_{ij} Y_j^2 - D g^2] \, ,$$
$$\frac{dg}{dL} = -C g^3 \, , \qquad (1)$$

where Y_i and g are Yukawa and gauge couplings, respectively, and $L \equiv \ln Q^2/\mu^2$. We limit ourselves only to one gauge coupling essential in a given energy region. The horizontal group manifests itself in the equality of different Yukawa couplings. Numerical values of coefficients A_i, B_{ij}, C, and D are unessential here. What is important is that A, B, and D are positive (that is always so for Yukawa interactions) and that $A_i > B_{ki}$. The last inequality is also true as far as the contribution of a given charge to self-renormalization is larger than that to the renormalization of another charge.

To solve eqs. (1), it is useful to introduce new variables $u_i = Y_i^2/g^2$. Equations for u_i are

$$\frac{d u_i}{d L} = 2u_i[A_i u_i + \sum_{j \neq i} B_{ij} u_j - (D-C)] g^2$$

or $\qquad$ (2)

$$\dot{u}_i = -u_i [A_i u_i + \sum_{j \neq i} B_{ij} u_j - (D-C)].$$

where $\cdot \equiv Cg^2 \frac{d}{dg^2}$. If $D > C$, just this case is of interest /3,4/, eqs. (2) have a set of fixed points of the type $u_i = \text{const} > 0$ or $Y_i^2 = \text{const} \cdot g^2$. To find them, one has to equal the r.h.s. of eqs. (2) to zero. We get the following hierarchy of fixed points /5/:

1: $u_i = 0, \ \forall_i = 1,2,\ldots,N$,

2: $u_1 \neq 0, \ u_{i \neq 1} = 0$,

3: $u_{1,2} \neq 0, \ u_{i \neq 1,2} = 0$,

. . . .

N-1: $u_{i \neq N} \neq 0, \ u_N = 0$,

N: $u_i \neq 0, \ \forall_i$.

In order to establish the type of a fixed point, that is, to determine whether it is stable or unstable or a saddle-point, we use a standard method. Despite the nonlinearity of eqs. (2) they can be linearized in a vicinity of a fixed point. For this purpose we introduce infinitesimal deviations from the fixed point $u_i = \bar{u}_i + \delta u_i$, where $\bar{u}_i$ is a fixed point. Then we have the linear homogeneous system of eqs. for small deviations

$$\delta \dot{u}_i = S_{ij}(\bar{u}) \delta u_j, \qquad (3)$$

where matrix S depends on the type of s fixed point. The criterion of infrared (IR) stability now is the negative definiteness of solutions to the characteristic equation

$$\mathrm{Det}(S - \Lambda E) = 0,$$

where E is a unit matrix.

IR stability: $\Lambda_i < 0$.

Note that the sign of coefficient C is unessential here.

Applying this criterion to the analysis of eqs. (2), we obtain the following results:

i) The fixed point of I type, $u_i = 0$, $\forall_i$ is absolutely IR unstable and absolutely UV stable. In this case $S = (D-C)E$ and all $\Lambda_i = D - C > 0$.

ii) The fixed point of N-th type, $u_i \neq 0$, $\forall_i$ is absolutely IR stable and absolutely UV unstable if $\mathrm{Det} S^{(N)} > 0$, where $S^{(N)}$ is the matrix

$$S^{(N)} = \begin{pmatrix} A_1 & & B_{ij} \\ & \ddots & \\ B_{ij} & & A_N \end{pmatrix}.$$

This condition is practically satisfied if $A_i > B_{ki}$ and if fixed points of all types exist and $\bar{u}_i > 0$.

iii) All other fixed points are saddle-points. They are stable along some directions in phase space and unstable along the others. The number of stable directions is equal to the number of negative Λ_i and increases with the number of the type of a fixed point in our classification.

We illustrate the above analysis by an example of the real interaction /5/. Consider the Yukawa type interaction of quarks with Higgs scalars in a standard model with $SU_C(3) \times SU_L(2) \times U_Y(1)$ gauge symmetry. Ignoring weak and electromagnetic interactions and quark flavour mixing, we get the following renormalization group equations

$$\frac{d Y_i}{d L} = Y_i [\frac{9}{2} Y_i^2 + \sum_{j \neq i} 3 Y_j^2 - 8 g^2],$$

$$\frac{d g}{d L} = -(11 - \frac{2}{3} n) g^3,$$

or

$$\dot u_i = -u_i[\frac{9}{2}u_i + \sum_{j\neq i} 3u_j - (\frac{2}{3}n - 3)], \qquad (4)$$

where n is the number of flavours in a given energy region.

We present the solutions of eqs. (4) graphically in phase diagrams. Arrows show the direction of decreasing $L = \ln Q^2/\mu^2$.

2.1. $n = \text{const} = 6$. Eqs. (4) become

$$\dot u_i = -u_i[\frac{9}{2}u_i + \sum_{j\neq i} 3u_j - 1], \quad i = 1,2,\ldots,6$$

and have fixed points of all types.

2.1.1. $u_1 \neq 0$, $u_{j\neq 1} = 0$; $\dot u = -u[9/2\ u - 1]$.
The solution is shown in fig. 1. The fixed point $u = 2/9$ is IR stable if $u_{j\neq 1} = 0$ and is a saddle-point in the whole phase space.

2.1.2. $u_{1,2} \neq 0$, $u_{j\neq 1,2} = 0$; $\dot u_1 = -u_1[9/2\, u_1 + 3u_2 - 1]$,
$\dot u_2 = -u_2[3u_1 + 9/2\, u_2 - 1]$.

The solution is shown in fig. 2. The point $u_1 = u_2 = 2/15$ is IR stable, points $u_1 = 0$, $u_2 = 2/9$ and $u_1 = 2/9$, $u_2 = 0$ are saddle-points and the point $u_1 = u_2 = 0$ is IR unstable. There exist solutions connecting different fixed points. They are boundary solutions between different sectors of phase space. The singular solution $u_1 = u_2$ is IR stable.

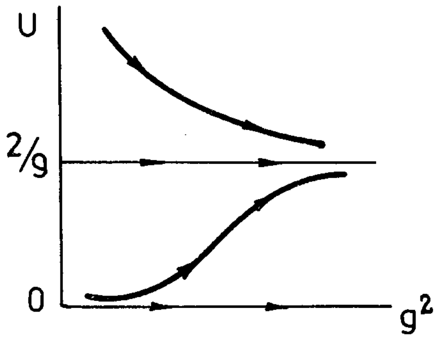

Fig. 1

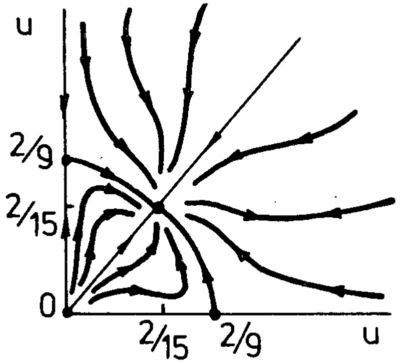

Fig. 2

2.1.3. $u_{1,2,3} \neq 0$, $u_{j\neq 1,2,3} = 0$;
$\dot u_1 = -u_1[9/2\, u_1 + 3u_2 + 3u_3 - 1]$,
$\dot u_2 = -u_2[3u_1 + 9/2\, u_2 + 3u_3 - 1]$,
$\dot u_3 = -u_3[3u_1 + 3u_2 + 9/2\, u_3 - 1]$.

The solution is shown in fig. 3. Singular solutions connecting fixed points form something like an umbrella standing in the corner. IR stable point is $u_1 = u_2 = u_3 = 2/21$, other points are saddle-points and $u_i = 0$ is absolutely IR stable. There is a hierarhy of stability. Absolutely unstable is the point $u_{1,2,3} = 0$, then $u_{1,2} = 0$, $u_3 \neq 0$; $u_1 = 0$, $u_{2,3} \neq 0$ and finally $u_{1,2,3} \neq 0$ (and all permutations of indices). The surface of the "umbrella" is also IR stable and UV unstable. All solutions inside the "umbrella" and on its surface are asymptotically free in the UV region and outside the "umbrella" exhibit a zero-charge behaviour. On the surface of the "umbrella" solutions tend to the most symmetric configurations in the IR region.

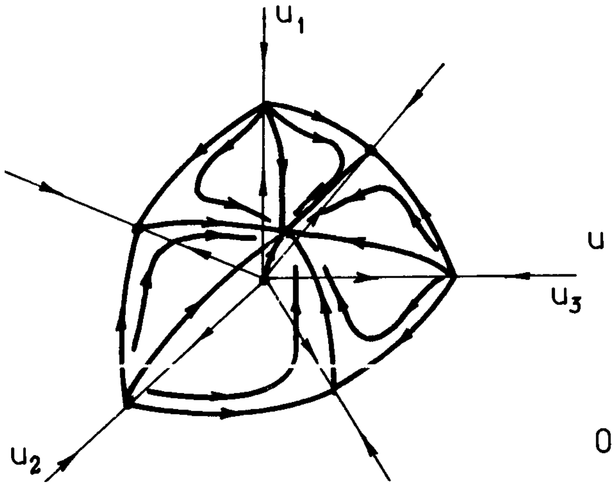

Fig. 3

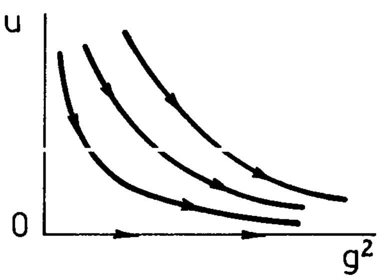

Fig. 4

Hence in the IR regime the most symmetric configuration in the given phase space is realized. Appearance of the new degree of freedom transforms the stable point into the saddle-point and a new stable point arises with a higher symmetry.

2.2. $n = \text{const} = 3$. Here we have

$$\dot u_i = -u_i[9/2\, u_i + \sum_{j\neq i} 3u_j + 1]$$

and there are no fixed points but $u_i = 0$. However, symmetric solutions $u_i = u_j$ and $u_1 = u_2 = u_3$ do exist and are IR stable.

2.2.1. $u_1 \neq 0$, $u_{2,3} = 0$; $\dot u = -u[9/2\, u + 1]$.
The solution is shown in fig. 4.

2.2.2. $u_{1,2} \neq 0$, $u_3 = 0$; $\dot u_1 = -u_1[9/2\, u_1 + 3u_1 + 1]$,
$\dot u_2 = -u_2[3u_1 + 9/2\, u_2 + 1]$.
The solution is shown in fig. 5.

Hence, even in the absence of fixed points (the surface of the "umbrella" reduces to the point) solutions are driven to the most symmetric configuration in the IR regime. In the UV regime we always have a zero-charge behaviour.

The horizontal symmetry - quark flavour permutation symmetry in this case - increases at low energies /5/.

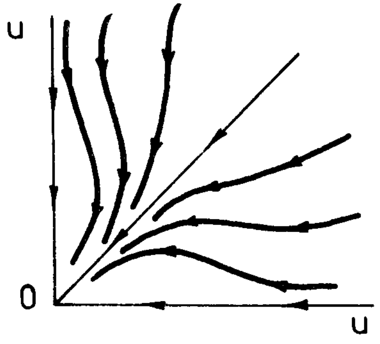

Fig. 5

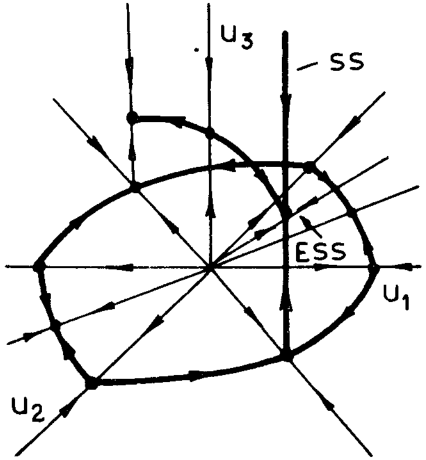

Fig. 6

3. YUKAWA COUPLINGS. SUPERSYMMETRY

Eqs. (1) describing the evolution of Yukawa couplings are not the general ones. In some cases, e.g., for the set of couplings and fields, where the supersymmetry can be realized, there are possible also nondiagonal terms of the type $Y_i Y_k^2$. We shall not give the general analysis of this situation. The characteristic feature of IR and UV behaviour are the same. Instead, we consider an example where supersymmetry is realized as an IR fixed point of renormalization group equations.

3.1. Let $Y_i = u_i g$, where g is the gauge and Y_i are Yukawa couplings. One loop equations for u_i are

$$\dot{u}_1 = -u_1[2u_1^2 + u_2^2 + u_3^2 - 3] + u_2 u_2^2,$$

$$\dot{u}_2 = -u_2[u_1^2 + 2u_2^2 + u_3^2 - 3] + u_1 u_3^2, \qquad (5)$$

$$\dot{u}_3 = -u_3[\frac{1}{2}u_1^2 + \frac{1}{2}u_2^2 + 2u_3^2 - 3].$$

Supersymmetry is realized here as a fixed point SS: $u_1 = u_2 = 1$ and an extended supersymmetry as a fixed point ESS: $u_1 = u_2 = u_3 = 1$. The phase diagram for eqs. (5) is shown in fig. 6. There are also some fixed points of other types. The fixed line ($u_1 = u_2 = 1$, u_3 - arbitrary) is IR stable and on the line and in the whole phase subspace $u_{1,2,3} \geq 0$ absolutely IR stable is the point $u_1 = u_2 = u_3 = 1$. Hence supersymmetry is realized as an IR stable line and the extended supersymmetry as an absolutely IR stable fixed point.

The situation considered in this example is general. Supersymmetry as well as the extended supersymmetry are always IR stable. Consider supersymmetric theory which contains one arbitrary Yukawa coupling (or several couplings equal to each other). When it is equal to the gauge coupling we get the extended supersymmetry. The one-loop equation is

$$\frac{dY}{dL} = aY^3 - bYg^2 , \quad a > 0 .$$

Then for $u = Y^2/g^2$ we have

$$\dot{u} = -2au[u - \frac{b-c}{a}].$$

Without any limitation we put $\frac{b-c}{a} = 1$. We have two fixed points $\bar{u}_1 = 0$ and $\bar{u}_2 = 1$. The solution is shown in fig. 7. The property of IR stability of the fixed point $\bar{u}_2 = 1$ corresponding to the extended supersymmetry follows from the positivity of coefficient **a** , that is a characteristic feature of the Yukawa coupling. Hence the extended supersymmetry relative to the ordinary supersymmetry is always IR stable and hence UV unstable.

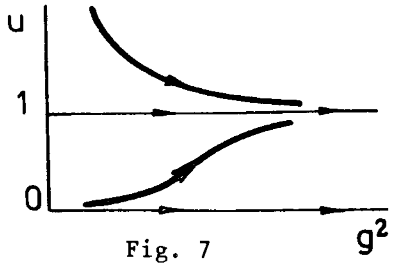

Fig. 7

4. SCALAR COUPLINGS

The one-loop equations for Yukawa couplings considered above are independent of the scalar one. As for the scalar couplings their behaviour depends strongly on the choice for Yukawa couplings. The general analysis of scalar interactions is too complicated. However, when some kind of symmetry like supersymmetry connecting the scalar coupling with gauge or Yukawa couplings is realized, equations for scalar charges have common features which we shall discuss now.

Let us consider an example of one scalar field noninteracting directly with the gauge field. Then the equation of evolution in one-loop order is

$$\frac{dh}{dL} = \alpha h^2 + \beta h \sum_i Y_i^2 - \gamma \sum_i Y_i^4, \qquad (6)$$

where $\alpha, \beta, \gamma > 0$ that is a characteristic feature of the scalar interaction. Solving this equation together with eq. (1) and substituting $h = \nu g^2$ we get the following equation for ν:

$$\dot{\nu} = -[\alpha \nu^2 + \beta \nu \sum_i u_i - \gamma \sum_i u_i^2 + C\nu]. \qquad (7)$$

For appropriate values of coefficients and the choice of u_i the fixed points $\nu_\pm$ = const can exist so that $\nu_+ > \nu_-$ and $\nu_- \leq 0$. From the positivity of α it immediately follows that ν_+ is IR stable and ν_- is IR unstable. No conclusion follows about the existence of any kind of symmetry and its IR stability or unstability. However, usually $\nu_+ \geq 0$ and symmetric solution corresponds to a positively defined scalar potential. This means that among ν_+ and ν_- only ν_+ can be associated with the symmetric solution. It is just so in supersymmetric models. Supersymmetry is IR stable not only for Yukawa couplings but also for scalar ones, i.e., it is an absolutely IR stable fixed point in the whole phase space.

To illustrate the behaviour of the scalar coupling in phase space we consider again the quark coupling with the Higgs field and Higgs self-interaction. In addition to eq. (4) from eq. (7) we have

$$\dot{\nu} = -[4\nu^2 + 12\nu \sum_i u_i - 36 \sum_i u_i^2 + 2(11 - \tfrac{2}{3}n)\nu]. \qquad (8)$$

4.1. n=const=6. Eq. (8) becomes

$$\dot{\nu} = -[4\nu^2 + 12\nu \sum_i u_i - 36 \sum_i u_i^2 + 14\nu]$$

and have two fixed points $\nu_\pm$ = const.

4.1.1. $u_i = 0$; $\dot{\nu} = -[4\nu^2 + 14\nu]$.

The solution is shown in fig. 8. IR stable fixed point $\nu_+ = 0$.

4.1.2. $u_1 \neq 0$, $u_{i \neq 1} = 0$; $\dot{\nu} = -[4\nu^2 + 12\nu u - 36u^2 + 14\nu]$,
$\dot{u} = -u[9/2u - 1]$.

The solution is shown in fig. 9. An absolutely IR stable point is $u = 9/2$, $\nu \simeq 0.1$, an absolutely IR unstable point is $u=0$, $\nu \simeq -3.5$. All other points are saddle-points. Along the ν axis

the phase space is divided into two parts and only the upper one is the region of attraction of the IR stable point. The regime of asymptotical freedom is possible only in the area $0 \leq u \leq 2/9$ bounded by the singular solution connecting points $\nu = 0$ and $\nu \sim 0.1$. Only on this solution it is consistent with positivity of the scalar potential.

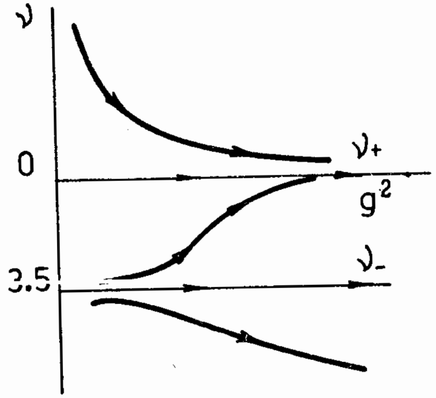

Fig. 8

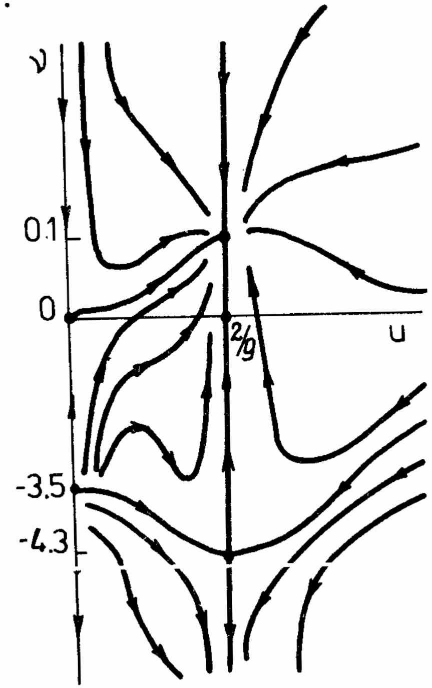

Fig. 9

4.1.3. $u_{1,2} \neq 0$, $u_{i \neq 1,2} = 0$;
$\dot{\nu} = -[4\nu^2 + 12\nu(u_1 + u_2) - 36(u_1^2 + u_2^2) + 14\nu]$,
$\dot{u}_1 = -u_1[9/2\, u_1 + 3u_2 - 1]$,
$\dot{u}_2 = -u_2[3u_1 + 9/2\, u_2 - 1]$.

The solution is shown in fig. 10. IR stable is a thick point on the upper "wing". The upper "wing" is also an AF region with the positive potential. It is IR stable and UV unstable.

4.2. n=const=3. Here there are no fixed points for u_i.

4.2.1. $u_i = 0$. This case is the same as 4.1.1.

4.2.2. $u_1 \neq 0$, $u_{i \neq 1} = 0$; $\dot{\nu} = -[4\nu^2 + 18\nu + 12\nu u - 36u^2]$,
$\dot{u} = -u[9/2\, u - 1]$.

The solution is shown in fig. 11. IR stable is the point $u = \nu = 0$. There is no AF regime here.

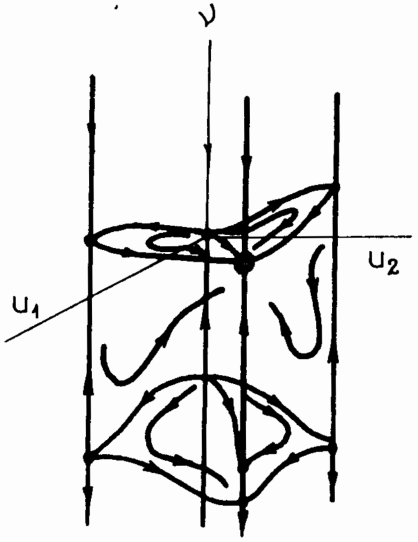

Fig. 10

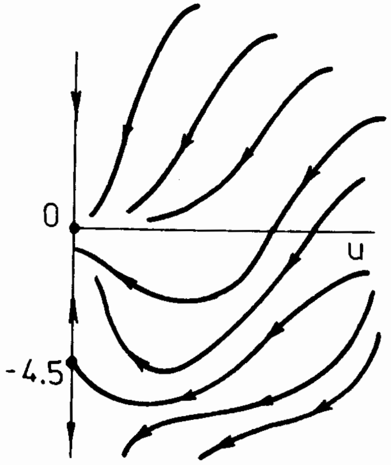

Fig. 11

5. EQUATIONS WITH VARYING COEFFICIENTS

Examples of the charge evolution considered above are valid only in a limited energy interval. When considering the whole energetic scale, some coefficients become variables due to the change of the effective number of particles taking part in the interaction. For example, the number of flavours n effectively depends on Q^2. If we assume $n \neq$ const and increases with energy when we pass through the threshold of creation of new particles, we come to the coupling-constant behaviour which is a combination of that depicted in figs. (1-3) and (4-5).

5.1. $n \neq$ const. The evolution of Yukawa couplings is described by eq. (4).

5.1.1. $u_1 \neq 0$, $u_{i \neq 1} = 0$. The solution is shown in fig. 12. The singular solution disappears in the IR region remaining stable.

5.1.2. $u_{1,2} \neq 0$, $u_{i \neq 1,2} = 0$. Instead of fig. 2 we have the behaviour shown in fig. 13. The fixed point tends to zero remaining IR stable.

5.1.3. $u_{1,2,3} \neq 0$, $u_{i \neq 1,2,3} = 0$. Instead of fig. 3 we have the behaviour shown in fig. 14. The center of the "umbrella" is likely to sweep inside.

Some deformation of the picture will be caused also by taking into account higher loop corrections in the renormalization group equations. The fixed points will not disappear but will take the form $Y_i^2 = u_i g^2 + v_i g^4 + \ldots$ It will not be essential in

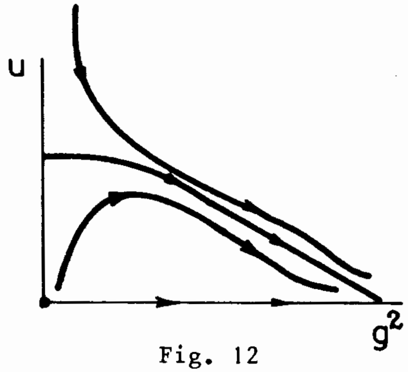

Fig. 12

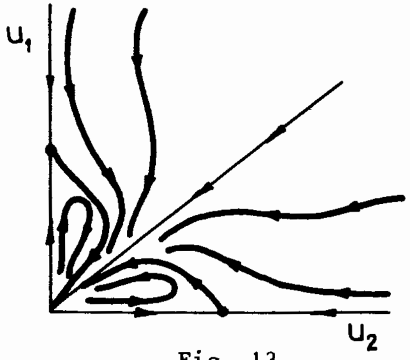

Fig. 13

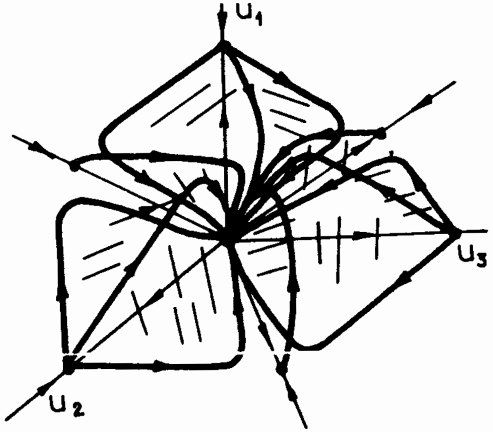

Fig. 14

the UV region due to the AF of the gauge coupling, but will change the quantitative behaviour in the IR region. However, symmetric solutions will exist and will be UV unstable and IR stable. In the case of exact symmetry like supersymmetry, the fixed points will not change their form even when allowing for higher loop corrections.

6. DISCUSSION

The general conclusion of the above analysis is that the global symmetry increases in the infrared region. Effective Yukawa and scalar quartic couplings tend towards the most symmetric configuration in the IR regime. If the fixed points exist, the trivial one is always UV stable and the IR stable point $u_i \neq 0$ is the most symmetric one. All other fixed points are saddle-points.

This leads to constraints on the model construction in the theories of grand unification. If we assume the equality or approximate equality of Yukawa and/or scalar quartic couplings in the far UV region, at the "unification point", they will not go away from each other in the IR region, in the region of modern energies. If quarks obtain their masses via the Higgs mechanism, these masses are proportional to the Yukawa couplings. Hence, if the quark masses are close or equal at the unification points, they will not give the spectra observed at modern energies in the simplest cases with one or two Higgs doublets.

Present hopes to construct a unified model of all types of interaction are associated with the extended supersymmetry. Supersymmetry is of a special importance when constructing a theory without divergences. It is usually assumed that this symmetry is exact only at very high energies (10^{15}-10^{19} GeV) and for lower energies the extended supersymmetry is broken. However, we have seen that both the supersymmetry and extended supersymmetry are UV unstable. Hence if the symmetry is broken strongly, i.e., by the coupling constants, we shall not come to it at high energies. This fact leads to strong constraints on the pattern of the (extended) supersymmetry breaking, which should be clearly broken to get the observed spectra of elementary particles. The symmetry breaking should be only "soft", i.e., by operators of dimension less than four, e.g., ϕ^2 or ϕ^3.

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Инфракрасная устойчивость и глобальная симметрия

Рассматривается поведение юкавских и скалярных четверных эффективных зарядов в калибровочных теориях. Демонстрируется наличие фиксированных точек, в которых названные заряды пропорциональны калибровочному. Показано, что фиксированные точки, отвечающие наиболее симметричным конфигурациям, инфракрасно устойчивы и ультрафиолетово неустойчивы. Суперсимметрия и расширенная суперсимметрия также реализуются,как ИК устойчивые фиксированные точки. Отмечается общая тенденция возрастания глобальной симметрии в инфракрасной области. Обсуждаются следствия этого факта для построения моделей большого объединения взаимодействий.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Infrared Stability and Global Symmetry

The behaviour of Yukawa and scalar quartic couplings in gauge theories is examined. The existence of fixed points where these couplings are proportional to the gauge one is demonstrated. It is shown that fixed points corresponding to the most symmetric configurations are infrared stable and ultraviolet unstable. Supersymmetry and extended supersymmetry are also realized as IR stable fixed points. We note the general tendency of increasing global symmetry in the IR region. Consequences of this fact for the grand unification model building are briefly discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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