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INTRINSIC GEOMETRY OF THE N=1 SUPERSYMMETRIC YANG-MILLS THEORY

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1. Introduction

In superspace, supersymmetric Yang-Mills and supergravity theories reveal quite different geometric structures as compared with those seen at the component level. Standard gauge potentials in superspace carry too much degrees of freedom even in the fixed gauge and for this reason cannot serve as the fundamental quantities. In any self-contained superfield gauge theory they appear as composite objects constructed from a lesser number of unconstrained superfields, prepotentials. The prepotentials proved to be very useful concept. Being directly related to the physical field content of a given theory, they provide an adequate realization of its minimal invariance group and hence can be considered as natural carriers of the corresponding intrinsic superspace geometry.

At present, the complete prepotential formulations exist for the N=1 Yang-Mills [1,2] and supergravity[3,4,5,6] theories and, at the linearized level, for their N=2 counterparts [7,8] *). The standard strategy to search for prepotentials is as follows. One starts with the ordinary differential geometry in superspace[10] and then solves proper constraints on covariant strengths, curvatures, torsions, etc. It is not so easy to guess what are the adequate constraints in one or another specific case, because of lack of general procedure .

Another approach, which seems to be more universal proceeds directly from exposing the minimal invariance group and intrinsic superspace geometry of a given theory. Once these are established relevant prepotentials are expected to naturally arise within this framework as objects with a clear group and geometric meaning. Such a program has been carried out through only for the N=1 supergravity as yet. Ogievetsky and Sokatchev [4,5] have shown that the underlying geometry of minimal N=1 supergravity is the complex geometry of real superspace $\mathbb{R}^{4|4} = \{x^m, \theta^\mu, \bar{\theta}^{\dot{\mu}}\}$ embedded

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*) The quantities suggested in[9] as the prepotentials of complete N=2 supergravity seem not to be true ones as they are still subjected to certain constraints.

as a hypersurface into the complex chiral superspace $\mathbb{C}^{4|2} = \{x_L^m, \theta_L^\mu\}$. The geometric role of corresponding prepotential $H^m(x,\theta,\bar\theta)$ is to specify this embedding:

$$\operatorname{Im} x_L^m = H^m(x,\theta,\bar\theta),\ \operatorname{Re} x_L^m = x^m,\ \theta_L^\mu = \theta^\mu,\ (\theta_L^\mu)^\dagger \equiv \bar\theta_R^{\dot\mu} = \bar\theta^{\dot\mu}. \tag{1.1}$$

The minimal invariance group of N=1 supergravity is the supergroup of general analytic coordinate transformations of $\mathbb{C}^{4|2}$ (its divergenceless subgroup in the Einstein case /4/). By the identification (1.1), it has a natural realization on $\{x^m, \theta^\mu, \bar\theta^{\dot\mu}, H^n(x,\theta,\bar\theta)\}$. In the case of nonminimal N=1 supergravities $\mathbb{R}^{4|4}$ is embedded into a larger complex superspace $\mathbb{C}^{4|4}$ having two spinor dimensions in addition /9/. These extra spinor coordinates, being restricted to $\mathbb{R}^{4|4}$ constitute, together with $H^m(x,\theta,\bar\theta)$, the full set of relevant prepotentials /6/. It is essential that in both cases, minimal and nonminimal, the prepotentials appear primarily as the coordinates of certain complex superspaces.

It is unknown which superfields play the role of prepotentials in gauge theories with N = 2 (except for N=2 electrodynamics) and which geometries are associated with them. No clear geometric interpretation exists even for the N=1 Yang-Mills prepotential $V^i(x,\theta,\bar\theta)$ (i is the index of adjoint representation of the gauge group). At the same time, in order to unmask the minimal geometric structure of higher N gauge theories (and supergravities as well) it is necessary before to clearly understand the geometry of the text-book N=1 case.

This analysis is performed in the present paper. We demonstrate that the intrinsic superspace geometry of the N=1 Yang-Mills theory reveals a close similarity to that of minimal N=1 supergravity. We start with the complexification G^c of the gauge group G and define the extended chiral superspace $\mathbb{C}^{4+M|2} = \{x_L^m, \theta_L^\mu, \varphi_L^i\}$ $(i=1,\dots M)$, where $M = \dim G$ and φ_L^i are local complex coordinates on G^c. The N=1 Yang-Mills theory turns out to be associated with the dynamics of embedding of $\mathbb{R}^{4|4}$ into $\mathbb{C}^{4+M|2}$. The N=1 prepotential $V^i(x,\theta,\bar\theta)$ coincides with $\operatorname{Im}\varphi_L^i$ restricted to $\mathbb{R}^{4|4}$. It is introduced by the equation (2.11a) analogous to the first of eqs. (1.1) and has a simple meaning: it parametrizes the coset space G^c/G. $\operatorname{Re}\varphi_L^i$ remains arbitrary and does not influence the dynamics, so that $\mathbb{C}^{4+M|2}$ actually reduces to the quotient $\mathbb{C}^{4+M|2}/G$. The fact that $V^i(x,\theta,\bar\theta)$ takes values in the coset G^c/G allows the N=1 Yang-Mills theory to be interpreted as a

generalized nonlinear σ-model. Hence, the powerful method of Cartan differential forms /11-14/ may be applied to the construction of relevant invariants and other geometric objects.

The organization of paper is as follows. In Sect. 2 we present the geometric derivation of the N=1 Yang-Mills prepotential. It goes in the same way for the cases of rigid and local supersymmetries*). In Sect. 3 the corresponding Cartan forms are defined and it is explained how to construct from them the standard geometric characteristics of the N=1 Yang-Mills theory which automatically respect the conventional kinematic constraints /15/. We begin with the case of flat geometry on $\mathbb{R}^{4|4}$ and then extend our study to the case of couplings with N=1 supergravity. In Conclusion we indicate some consequences of the proposed geometric picture and make an attempt to realize what would be the analog of the complex group G^c in the case of N=2 Yang-Mills theory.

2. The geometric derivation of the N=1 Yang-Mills prepotential

1. We begin with definition of the complex group G^c.

Let G be a compact M-dimensional group with generators T^i $(i=1,\dots M)$. In the basis where T^i are hermitean they satisfy the commutation relations

$$[T^i, T^k] = i c^{ikl} T^l, \tag{2.1}$$

c^{ikl} being real totally skew-symmetric structure constants. G^c is defined as the group with M complex generators T_L^i, which constitute, together with $T_R^i \equiv (T_L^i)^\dagger$ the following Lie algebra:

$$[T_L^i, T_L^k] = i c^{ikl} T_L^l,\quad [T_R^i, T_R^k] = i c^{ikl} T_R^l \tag{2.2a}$$

$$[T_L^i, T_R^k] = 0 \tag{2.2b}$$

or, in terms of hermitean generators

$$\begin{aligned} [T^i, T^k] &= i c^{ikl} T^l \\ [T^i, A^k] &= i c^{ikl} A^l \\ [A^i, A^k] &= -i c^{ikl} T^l, \end{aligned} \tag{2.3}$$

*) Some results of this Section have already been published as a letter /25/. Analogous consideration has independently been given by A.A.Rosly /26/.

where

$$T^i = T_L^i + T_R^i, \quad A^i = i(T_L^i - T_R^i). \tag{2.4}$$

Besides, the group parameters associated with T_L^i, T_R^k are assumed to be mutually conjugated. It is seen from (2.2), (2.3) that G^c has (locally) the structure of the direct product $G^c = G_L \times G_R$ with G_L and $G_R = (G_L)^\dagger$ generated, respectively, by T_L^i and T_R^i. The initial group G with generators T^i appears in this product as a diagonal, the remaining generators A^k span the real M-dimensional symmetric coset space G^c/G.

It is worth noting that the group G^c is noncompact. In particular, if $G = SU(n)$, then $G^c = SL(n,C)$. Due to noncompactness of G^c any its unitary representation is infinite-dimensional, for this reason generators T^i, A^k may be simultaneously chosen hermitean (T_L^i, T_R^i mutually conjugated) only provided they are represented by infinite-dimensional matrices. Our conclusions do not depend on a choice of representation, so in what follows we may consider T^i, A^k hermitean without loss of generality.

Let us treat G^c as a Riemannian manifold and introduce, in a vicinity of its identity element, local coordinates φ_L^i, $\varphi_R^i = (\varphi_L^i)^\dagger$, using for definiteness the exponential parametrization of G^c:

$$g^c(\varphi_L, \varphi_R) = g_L(\varphi_L)\, g_R(\varphi_R) = e^{i\varphi_L^k T_L^k} e^{i\varphi_R^k T_R^k} = e^{i(\mathrm{Re}\,\varphi_L^k T^k + \mathrm{Im}\,\varphi_L^k A^k)}. \tag{2.5}$$

Note that $\mathrm{Re}\,\varphi_L^k$ and $\mathrm{Im}\,\varphi_L^k$ parametrize, respectively, the subgroup G and the coset G^c/G. Now we define the superspace $\mathbb{C}^{4+M|2}$ playing the fundamental role in further consideration. It is the direct sum of ordinary chiral N=1 superspace $\mathbb{C}^{4|2} = \{x_L^m, \theta_L^\mu\}$ and the group G_L regarded as a complex M-dimensional manifold:

$$\mathbb{C}^{4+M|2} = \{x_L^m, \theta_L^\mu, \varphi_L^i\} = \mathbb{C}^{4|2} \oplus G_L. \tag{2.6}$$

Since the left and right superspace coordinates x_L^m, θ_L^μ and $x_R^m = (x_L^m)^\dagger$, $\bar\theta_R^{\dot\mu} = (\theta_L^\mu)^\dagger$ are related by P-parity, it is natural to accept the same convention for φ_L^i, φ_R^i:

$$\varphi_L^i \overset{P}{\leftrightarrow} \varphi_R^i, \ \mathrm{Re}\,\varphi_L^i \xrightarrow{P} \mathrm{Re}\,\varphi_L^i, \ \mathrm{Im}\,\varphi_L^i \xrightarrow{P} -\mathrm{Im}\,\varphi_L^i. \tag{2.7}$$

Correspondingly, if T^i are scalars, A^i must be pseudoscalars:

$$T_L^i \overset{P}{\leftrightarrow} T_R^i, \ T^i \xrightarrow{P} T^i, \ A^i \xrightarrow{P} -A^i. \tag{2.8}$$

Clearly, (2.8) is the automorphism of the algebra (2.2), (2.3).

The next step is to define the action of the group G^c in $\mathbb{C}^{4+M|2}$. G^c can naturally be implemented in this superspace as the group of left nonlinear translations of group coordinates φ_L^i, φ_R^i:

$$\begin{aligned} g_L(\lambda_L)\, g_L(\varphi_L) &= g_L(\varphi_L'(\varphi_L, \lambda_L)) \\ g_R(\lambda_R)\, g_R(\varphi_R) &= g_R(\varphi_R'(\varphi_R, \lambda_R)), \end{aligned} \tag{2.9}$$

where λ_L^i, $\lambda_R^i = (\lambda_L^i)^\dagger$ are group parameters. To promote global G^c transformations to the local ones we assume that λ_L^i are arbitrary analytic functions given over the superspace $\mathbb{C}^{4|2}$:

$$\lambda_L^i = \lambda_L^i(x_L, \theta_L), \ \lambda_R^i = (\lambda_L^i)^\dagger = \lambda_R^i(x_R, \bar\theta_R). \tag{2.10}$$

The gauge group thus defined constitutes a semi-direct product with the supergroup realized on x_L^m, θ_L^μ: the Lie bracket of their two arbitrary transformations is a gauge transformation of the type (2.9). As is implied by the relation (2.2b) the left and right components of the gauge group $G^c_{loc} = G_{L\,loc} \times G_{R\,loc}$ commute with each other so that at the initial stage the "left" and "right" worlds are entirely disjoined (though conjugated).

2. We wish to show that G^c_{loc} is the invariance group of N=1 Yang-Mills theory /1,2/ and that the latter naturally emerges after extracting a special hypersurface in $\mathbb{C}^{4+M|2}$. This hypersurface is the real superspace $\mathbb{R}^{4|4} = \{x^m, \theta^\mu, \bar\theta^{\dot\mu}\}$ just as in the case of N=1 supergravity /4-6/. An essential difference is that it possesses now purely internal degrees of freedom besides those represented by the axial superfield $H^m(x,\theta,\bar\theta)$ (1.1), because of additional bosonic dimensions in $\mathbb{C}^{4+M|2}$. Accordingly, the embedding conditions (1.1) should be supplemented with 2M conditions

$$a)\ \mathrm{Im}\,\varphi_L^i = V^i(x,\theta,\bar\theta), \quad b)\ \mathrm{Re}\,\varphi_L^i = U^i(x,\theta,\bar\theta), \tag{2.11}$$

where V^i and U^i are real pseudoscalar and scalar superfields. Their transformation properties in G^c_{loc} are uniquely determined by those of φ_L^i, φ_R^i given by eqs. (2.9). These superfields span, respectively, the coset space G^c/G and the subgroup G. Hence, they are of the Goldstone type with respect to the corresponding G^c-transformations. We want G to be unbroken; then $U^i(x,\theta,\bar\theta)$ should be made to have no dynamical manifestations. To achieve this, one may proceed as in standard nonlinear σ-models (see, e.g./13,16/) and require the theory to be invariant under the right gauge G-transformations:

$$e^{i(U^kT^k+V^kA^k)} \to e^{i(U^kT^k+V^kA^k)}e^{i\lambda^\ell T^\ell}, \tag{2.12}$$

where $\lambda^\ell=\lambda^\ell(x,\theta,\bar\theta)$ are M real superparameters. Then $U^i(x,\theta,\bar\theta)$ represent purely gauge degrees of freedom. From the geometric point of view, the invariance under (2.12) means that different G-directions in $\mathbb{C}^{4+M|2}$ are indistinguishable; the dynamics is required to depend only on the position of the hypersurface $\mathbb{R}^{4|4}$ with respect to directions spanning the coset space G^c/G. In other words, it is the quotient $\mathbb{C}^{4+M|2}/G$ what does really enters after allowing for the gauge freedom (2.12).

Upon imposing the natural gauge condition

$$U^i(x,\theta,\bar\theta)=0 \tag{2.13}$$

we are left with M pseudoscalar superfields $V^i(x,\theta,\bar\theta)$ which "live" in cosets G^c/G and transform under G^c_{loc} according to the generic formula of nonlinear realizations/11-14/:

$$e^{i(\mathrm{Re}\lambda_L^kT^k+\mathrm{Im}\lambda_L^kA^k)}e^{iV^kA^k}=e^{iV'^kA^k}\,e^{iK^\ell(V,\lambda_L)T^\ell} \tag{2.14}$$

with λ_L^i as in (2.6). The transformation law of matter superfields $\Phi(x,\theta,\bar\theta)$ can be then defined following general prescriptions of refs./11-14/:

$$\Phi'(x,\theta,\bar\theta)=e^{iK^\ell(V,\lambda)\overline{T}^\ell}\,\Phi(x,\theta,\bar\theta), \tag{2.15}$$

where $\overline{T}^\ell$ are a proper matrix representation of G-generators (indices of the representation are suppressed).

Now, let us demonstrate that the law (2.14) is actually equivalent to the standard transformation law of the N=1 Yang-Mills prepotential/1,2/. To this end, we first exploit the automorphism (2.4) of the algebra (2.3) to rewrite (2.14) in another form:

$$e^{i(\mathrm{Re}\lambda_L^kT^k-\mathrm{Im}\lambda_L^kA^k)}e^{-iV^kA^k}=e^{-iV'^kA^k}e^{iK^\ell T^\ell} \tag{2.14'}$$

The next step is to eliminate the factor $\exp\{iK^\ell T^\ell\}$ from eqs. (2.14), (2.14') that yields the one more possible form of the transformation of $V^i(x,\theta,\bar\theta)$:

$$e^{i(\mathrm{Re}\lambda_L^kT^k+\mathrm{Im}\lambda_L^kA^k)}e^{2iV^kA^k}e^{-i(\mathrm{Re}\lambda_L^kT^k-\mathrm{Im}\lambda_L^kA^k)}=e^{2iV'^kA^k}. \tag{2.16}$$

Finally, passing to the complex generators T_L^i, T_R^i (by the formula (2.4)) and taking into account their commutativity we observe that eq. (2.15) is equivalent to the following one:

$$e^{i\lambda_L^kT_L^k}e^{-2V^kT_L^k}e^{-i\lambda_R^kT_L^k}=e^{-2V'^iT_L^i} \tag{2.17}$$

(or with T_R^i instead of T_L^i). But this is just we are aiming at because T_L^i fulfill the same commutation relations as T^i, while the structure of V'^i in (2.14) does not depend on a particular choice of generators and is determined solely by their commutation relations.

In fact, the standard form of the N=1 prepotential transformation law (with T^i in place of T_L^i) is recovered by substituting for A^i in (2.16). its particular representation:

$$A_L^i=i\overline{T}^i \qquad (\overline{T}_L^i=\overline{T}^i,\ \overline{T}_R^i=0). \tag{2.18a}$$

This choice is non-self-conjugated, in accordance with the property that any finite-dimensional representation of the non-compact group G^c is non-unitary. By the identification (2.18a) or the conjugated one

$$\overline{A}_R^i=-i\overline{T}^i \qquad (\overline{T}_L^i=0,\ \overline{T}_R^i=\overline{T}^i) \tag{2.18b}$$

any representation of G can be extended to that of the whole G^c. Then, using the general connection between representations and nonlinear realizations/11/ one may relate any matter superfield with the standard nonlinear transormation law (2.15) to the superfields transforming in G^c_{loc} linearly, according to the representations (2.18a), (2.18b):

$$\begin{aligned}\Phi_L(x,\theta,\bar\theta)&=e^{iV^k\overline{A}_L^k}\,\Phi(x,\theta,\bar\theta)=e^{-V^k\overline{T}^k}\,\Phi(x,\theta,\bar\theta)\\ \Phi_R(x,\theta,\bar\theta)&=e^{iV^k\overline{A}_R^k}\,\Phi(x,\theta,\bar\theta)=e^{V^k\overline{T}^k}\,\Phi(x,\theta,\bar\theta)=\\ &=e^{2V^k\overline{T}^k}\,\Phi_L(x,\theta,\bar\theta)\end{aligned} \tag{2.19}$$

$$\Phi'_L(x,\theta,\bar\theta)=e^{i\lambda^k_L T^k}\Phi_L(x,\theta,\bar\theta),\ \Phi'_R(x,\theta,\bar\theta)=e^{i\lambda^k_R \bar T^k}\Phi_R(x,\theta,\bar\theta).$$

These relations can be interpreted as describing the transition from the real basis in the group space of G^c to its complex left and right bases, in a perfect analogy with the connection between real and complex bases in superspace*). The relations (2.19) were known earlier [6,17], but our consideration renders to them a clear group-theoretical meaning. Note that the substitution of (2.18a) or (2.18b) in the basic law (2.14) yields the transformations of the N=1 Yang-Mills prepotential in the form given by Siegel and Gates [6]:

$$e^{i\lambda^k_L \bar T^k}e^{-V^k\bar T^k}=e^{-V'^k\bar T^k}e^{iK^\ell T^\ell}$$
$$e^{i\lambda^k_R \bar T^k}e^{V^k\bar T^k}=e^{V'^k\bar T^k}e^{iK^\ell \bar T^\ell}. \qquad (2.20)$$

Also, the invariance under the right gauge G-transformations (2.12) reduces to the well-known freedom of complexifying the prepotential:

$$e^{-V^i\bar T^i}\to e^{-W^i\bar T^i}=e^{-V^i\bar T^i}e^{i\lambda^k\bar T^k}, \qquad (2.21)$$
$$e^{2V^i\bar T^i}=e^{W^{\dagger k}\bar T^k}\,e^{W^k\bar T^k}. \qquad (2.22)$$

To summarize, we have derived the N=1 Yang-Mills prepotential $V^i(x,\theta,\bar\theta)$ from simple geometric and group principles similar to those constituting the basis of the Ogievetsky-Sokatchev formulation of minimal N=1 supergravity [4,5]. In previous studies, the transformation rule of V^i and V^i itself either were simply postulated [1,2] or appeared as a solution of proper constraints on covariant strengths [15]. The underlying complex group structure of the N=1 Yang-Mills remained implicit because the generators of G^c appeared always in their particular form (2.18). Finally, we notice that the noncompactness of G^c has no explicit dynamical manifestations at the level of physical components. This is because the G^c symmetry is spontaneously broken from the beginning to the compact symmetry with respect to

*) To avoid a possible confusing, we note that no correlation exists between choices of bases in G^c and in superspace.

G and, besides, the Goldstone fields associated with this breaking are purely gauge degrees of freedom (they are contained in the superspin zero part of V^i). In the W.Z. gauge, the G^c/G -transformations have the form of ordinary gauge G -transformations and so are completely hidden. On the other hand, in any supersymmetric gauge they appear independently. Thus, the G^c/G-invariance can be thought of as the consistency condition between ordinary gauge invariance and manifest supersymmetry.

3. The Cartan form analysis of the N=1 Yang-Mills theory

1. We have shown above that the N=1 Yang-Mills theory, from the group-theoretic point of view, is a kind of the generalized nonlinear G model*). Indeed, $V^i(x,\theta,\bar\theta)$ takes the values in the coset G^c/G and hence is the Goldstone superfield ($\exp\{2iV^kA^k\}$ is nothing but the corresponding "chiral field"). Therefore, relevant invariants and other geometric objects should have an adequate expression in the universal language of Cartan differential forms which is of common use in theories with the nonlinearly realized symmetry [12-14]. In the present Section we construct the Cartan forms of the N=1 Yang-Mills theory and show that they provide a convenient general basis for analyzing the dynamical structure of this theory.

The basic forms in the present case are spinorial ones, they are introduced by the relations

$$e^{-iV^kA^k}(\mathcal{D}_\alpha+i\vartheta^L_\alpha)e^{iV^kA^k}=i(\omega^\ell_\alpha A^\ell+\Omega^\ell_\alpha T^\ell)\equiv i\Omega_\alpha \qquad (3.1)$$
$$e^{-iV^kA^k}(\bar{\mathcal{D}}_{\dot\alpha}+i\bar\vartheta^R_{\dot\alpha})e^{iV^kA^k}=i(\bar\omega^\ell_{\dot\alpha}A^\ell+\bar\Omega^\ell_{\dot\alpha}T^\ell)\equiv i\bar\Omega_{\dot\alpha}.$$

Here, $\mathcal{D}_\alpha$, $\bar{\mathcal{D}}_{\dot\alpha}$ are ordinary covariant spinor derivatives (they may correspond to the flat as well as curved geometries on $\mathbb{R}^{4|4}$) and $\vartheta^L_\alpha\equiv\vartheta^{Li}_\alpha T^i_L$, $\bar\vartheta^R_{\dot\alpha}=(\vartheta^L_\alpha)^\dagger$ are spinor connections on the group G^c_{loc}:

*) An analogous fact for ordinary gauge theories has been established in [18]. In the supercase, the similarity with nonlinear G models is even more transparent and striking.

$$\vartheta_\alpha^{L\prime}=e^{i\lambda_L^k T_L^k}\,\vartheta_\alpha^L\, e^{-i\lambda_L^k T_L^k}+\frac{1}{i}e^{i\lambda_L^k T_L^k}\,\mathcal{D}_\alpha\, e^{-i\lambda_L^k T_L^k} \qquad (3.2)$$
$$\bar\vartheta_{\dot\alpha}^{R\prime}=e^{i\lambda_R^k T_R^k}\,\bar\vartheta_{\dot\alpha}^R\, e^{-i\lambda_R^k T_R^k}+\frac{1}{i}e^{i\lambda_R^k T_R^k}\,\bar{\mathcal{D}}_{\dot\alpha}\, e^{-i\lambda_R^k T_R^k}.$$

Their role is to compensate the necommutativity of differential operators $\mathcal{D}_\alpha$, $\bar{\mathcal{D}}_{\dot\alpha}$ in the l.h.s. of (3.1) with elements of gauge groups $G_{L\,loc}$, $G_{R\,loc}$, respectively. We shall see below that ϑ_α^L, $\bar\vartheta_{\dot\alpha}^R$ can be constructed from $V^i(x,\theta,\bar\theta)$ alone.

It is easy to check that under the gauge group (2.14), (3.2) the object ω_α^i, Ω_α^i and their conjugates display the standard transformation properties of Cartan forms:

$$\Omega_\alpha'=e^{iK^\ell T^\ell}\,\Omega_\alpha\, e^{-iK^\ell T^\ell}+\frac{1}{i}e^{iK^\ell T^\ell}\,\mathcal{D}_\alpha\, e^{-iK^\ell T^\ell} \qquad (3.3)$$
$$\bar\Omega_{\dot\alpha}'=e^{iK^\ell T^\ell}\,\bar\Omega_{\dot\alpha}\, e^{-iK^\ell T^\ell}+\frac{1}{i}e^{iK^\ell T^\ell}\,\bar{\mathcal{D}}_{\dot\alpha}\, e^{-iK^\ell T^\ell}.$$

As follows from (3.3) and the commutation relations (2.3), ω_α^k, $\bar\omega_{\dot\alpha}^k$ transform homogeneously:

$$\omega_\alpha^{t\prime}A^t=e^{iK^\ell T^\ell}\,\omega_\alpha^t A^t\, e^{-iK^\ell T^\ell} \qquad (3.4)$$
$$\bar\omega_{\dot\alpha}^{t\prime}A^t=e^{iK^\ell T^\ell}\,\bar\omega_{\dot\alpha}^t A^t\, e^{-iK^\ell T^\ell}$$

and can be interpreted as gauge-covariant spinor derivatives of the prepotential $V^i(x,\theta,\bar\theta)$. The remaining forms Ω_α^i, $\bar\Omega_{\dot\alpha}^i$ are the connections on the coset space G^c/G : they transform according to the inhomogeneous law (3.3). These forms define the gauge-covariant spinor derivatives of matter superfields $\Phi(x,\theta,\bar\theta)$:

$$\nabla_\alpha\Phi(x,\theta,\bar\theta)=(\mathcal{D}_\alpha+i\,\Omega_\alpha^\ell\,\overline{T}^\ell)\,\Phi(x,\theta,\bar\theta), \qquad (3.5)$$
$$\bar\nabla_{\dot\alpha}\Phi(x,\theta,\bar\theta)=(\bar{\mathcal{D}}_{\dot\alpha}+i\,\bar\Omega_{\dot\alpha}^\ell\,\overline{T}^\ell)\,\Phi(x,\theta,\bar\theta).$$

Now, let us come back to the discussion of the status of gauge superpotentials ϑ_α^L, $\bar\vartheta_{\dot\alpha}^R$. Fortunately, there is no need to associate with them independent degrees of freedom. These superfields can be taken composite by imposing the manifestly covariant constraints of the inverse Higgs phenomenon [19]:

$$\omega_\alpha^i=\bar\omega_{\dot\alpha}^i=0. \qquad (3.6)$$

The equations (3.6) are algebraic with respect to ϑ_α^L, $\bar\vartheta_{\dot\alpha}^R$, therefore they can easily be solved to give

$$\vartheta_\alpha^L=\frac{1}{i}e^{-2V^tT_L^t}\,\mathcal{D}_\alpha\, e^{2V^tT_L^t} \qquad (3.7a)$$
$$\bar\vartheta_{\dot\alpha}^R=\frac{1}{i}e^{2V^tT_R^t}\,\bar{\mathcal{D}}_{\dot\alpha}\, e^{-2V^tT_R^t} \qquad (3.7b)$$

(in deriving (3.7), we have taken advantage of the automorphism (2.8)). After substituting (3.7) back into the basic relation (3.1) we are left with the spinor connections on the coset G^c/G:

$$\Omega_\alpha=\Omega_\alpha^tT^t=\frac{1}{i}e^{-V^kT^k}\,\mathcal{D}_\alpha\, e^{V^kT^k} \qquad (3.8)$$
$$\bar\Omega_{\dot\alpha}=\bar\Omega_{\dot\alpha}^tT^t=\frac{1}{i}e^{V^kT^k}\,\bar{\mathcal{D}}_{\dot\alpha}\, e^{-V^kT^k}.$$

These are the fundamental quantities, of which all the geometric characteristics of the theory can be built up: invariants, covariant strengths, etc. This can be done following the standard procedure of refs. [10,15]. We find it instructive to repeat the derivation in the context of the proposed geometric interpretation of N=1 Yang-Mills theory.

2. Till this point our consideration proceeded in the same way both for rigid and local supersymmetries. Now, we need the explicit form of spinor derivatives $\mathcal{D}_\alpha$, $\bar{\mathcal{D}}_{\dot\alpha}$. We begin with the flat case and choose the real basis in superspace $\mathbb{R}^{4|4}=\{x^m,\theta^\alpha,\bar\theta^{\dot\alpha}\}$. In this basis:

$$\mathcal{D}_\alpha=\frac{\partial}{\partial\theta^\alpha}-i(\not\partial\bar\theta)_\alpha\,,\quad \bar{\mathcal{D}}_{\dot\alpha}=-\frac{\partial}{\partial\bar\theta^{\dot\alpha}}+i(\theta\not\partial)_{\dot\alpha} \qquad (3.9)$$
$$\{\mathcal{D}_\alpha,\mathcal{D}_\beta\}=\{\bar{\mathcal{D}}_{\dot\alpha},\bar{\mathcal{D}}_{\dot\beta}\}=0 \qquad (3.10a)$$
$$\{\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot\beta}\}=2i(\not\partial)_{\alpha\dot\beta}\equiv 2i\,\sigma^m_{\alpha\dot\beta}\,\partial_m. \qquad (3.10b)$$

Then, the gauge-covariant spinor derivatives ∇_α, $\bar\nabla_{\dot\alpha}$ (3.5) satisfy the commutation relations

$$\{\nabla_\alpha,\nabla_\beta\}=i\,F_{\alpha\beta} \qquad (3.11a)$$
$$\{\bar\nabla_{\dot\alpha},\bar\nabla_{\dot\beta}\}=i\,\bar F_{\dot\alpha\dot\beta} \qquad (3.11b)$$
$$\{\nabla_\alpha,\bar\nabla_{\dot\beta}\}=2i(\not\partial+\Omega)_{\alpha\dot\beta}\equiv 2i\,\sigma^m_{\alpha\dot\beta}\,\nabla_m. \qquad (3.11c)$$

where $F_{\alpha\beta}$, $\bar{F}_{\dot\alpha\dot\beta}$, $\Omega_{\alpha\dot\beta}$ are the G-algebra valued differential 2-superforms

$$F_{\alpha\beta} = \mathcal{D}_\alpha \Omega_\beta + \mathcal{D}_\beta \Omega_\alpha + i\{\Omega_\alpha, \Omega_\beta\} \tag{3.12}$$

$$\bar{F}_{\dot\alpha\dot\beta} = \bar{\mathcal{D}}_{\dot\alpha}\bar{\Omega}_{\dot\beta} + \bar{\mathcal{D}}_{\dot\beta}\bar{\Omega}_{\dot\alpha} + i\{\bar{\Omega}_{\dot\alpha}, \bar{\Omega}_{\dot\beta}\} \tag{3.13}$$

$$\Omega_{\alpha\dot\beta} = \frac{1}{2}\left(\mathcal{D}_\alpha \bar{\Omega}_{\dot\beta} + \bar{\mathcal{D}}_{\dot\beta}\Omega_\alpha + i\{\Omega_\alpha, \bar{\Omega}_{\dot\beta}\}\right). \tag{3.14}$$

The covariant strengths $F_{\alpha\beta}$, $\bar{F}_{\dot\alpha\dot\beta}$ transform in G^c_{loc} homogeneously, by the law similar to (3.4). Substituting the explicit expressions (3.8) for the forms Ω_α, $\bar{\Omega}_{\dot\beta}$ into (3.12), (3.13) yields

$$F_{\alpha\beta} = \bar{F}_{\dot\alpha\dot\beta} = 0 \tag{3.15}$$

that are just the constraints placed on the strengths in the traditional approach starting from the gauge potentials in superspace /15/. The quantity $\Omega_{\alpha\dot\beta}$ is the vector connection:

$$\Omega_{\alpha\dot\beta} \equiv i\sigma^m_{\alpha\dot\beta}\Omega_m = e^{-iV^tA^t}(\partial + \vartheta)_{\alpha\dot\beta}\, e^{iV^tA^t}, \tag{3.16}$$

where

$$\vartheta_{\alpha\dot\beta} \equiv i\sigma^m_{\alpha\dot\beta}\vartheta_m = \frac{1}{2}\left(\mathcal{D}_\alpha \bar{v}^R_{\dot\beta} + \bar{\mathcal{D}}_{\dot\beta} v^L_\alpha + i\{v^L_\alpha, \bar{v}^R_{\dot\beta}\}\right) = \frac{1}{2}\left(\mathcal{D}_\alpha \bar{v}^R_{\dot\beta} + \bar{\mathcal{D}}_{\dot\beta} v^L_\alpha\right). \tag{3.17}$$

The composite gauge superfield $\vartheta_m(x,\theta,\bar\theta)$ transforms under G^c_{loc} according to

$$\vartheta_m' = g^c(\lambda_L,\lambda_R)\,\vartheta_m\, g^{c\,-1}(\lambda_L,\lambda_R) + \frac{1}{i} g^c(\lambda_L,\lambda_R)\,\partial_m\, g^{c\,-1}(\lambda_L,\lambda_R) \tag{3.18}$$

thereby ensuring the standard transformation law for Ω_m:

$$\Omega_m' = e^{iK^tT^t}\Omega_m e^{-iK^tT^t} + \frac{1}{i} e^{iK^tT^t}\partial_m e^{-iK^tT^t} \tag{3.19}$$

which is quite similar to the laws (3.3). Note that one more conventional constraint, on the strength with mixed indicies /15/:

$$F_{\alpha\dot\beta} = \mathcal{D}_\alpha \bar{\Omega}_{\dot\beta} + \bar{\mathcal{D}}_{\dot\beta}\Omega_\alpha + i\{\Omega_\alpha, \bar{\Omega}_{\dot\beta}\} - 2i\sigma^m_{\alpha\dot\beta}\Omega_m = 0 \tag{3.20}$$

is fulfilled in the present approach identically, by the definition (3.14).

Let us now define the three-index form

$$F_{\beta\rho\dot\alpha} = \nabla_\beta \Omega_{\rho\dot\alpha} - i(\partial)_{\rho\dot\alpha}\Omega_\beta = i\sigma^m_{\rho\dot\alpha}(\nabla_\beta\Omega_m - \partial_m\Omega_\beta) \equiv i\sigma^m_{\rho\dot\alpha}F_{\beta m}, \tag{3.21}$$

where ∇_β is the spinor gauge-covariant derivative in the adjoint representation of G:

$$\nabla_\beta = \mathcal{D}_\beta + i[\Omega_\beta, \ \ ] \tag{3.22}$$

and the commutator or anticommutator is chosen depending on whether even or odd is the form on which ∇_β acts. It is easy to see that under the transformations (3.3), (3.19) the strength (3.21) undergoes the homogeneous transformation:

$$F_{\beta m}' = e^{iK^tT^t} F_{\beta m} e^{-iK^tT^t}. \tag{3.23}$$

This strength and its conjugate $\bar{F}_{\dot\beta m}$ naturally arise when spinor gauge-covariant derivatives ∇_β, $\bar\nabla_{\dot\beta}$ are commuted with the vector one ∇_m (3.11c):

$$[\nabla_\alpha, \nabla_m] = i F_{\alpha m} \tag{3.24}$$

$$[\bar\nabla_{\dot\alpha}, \nabla_m] = i \bar{F}_{\dot\alpha m}.$$

Using the relations (3.15), one may check that $F_{\alpha\beta\dot\rho}$ satisfy the equations

$$F_{\alpha\beta\dot\rho} = -F_{\beta\alpha\dot\rho} \tag{3.25}$$

$$\nabla_\alpha F_{\beta\nu\dot\rho} + \nabla_\beta F_{\alpha\nu\dot\rho} = 0. \tag{3.26}$$

The first one implies that $F_{\alpha\beta\dot\rho}$ can be represented as

$$F_{\alpha\beta\dot\rho} = \frac{1}{2}\epsilon_{\alpha\beta}\bar{W}_{\dot\rho} = \frac{1}{2}\epsilon_{\alpha\beta}\left[\nabla^\gamma \Omega_{\gamma\dot\rho} - i(\partial)_{\gamma\dot\rho}\Omega^\gamma\right]. \tag{3.27}$$

Then, the second equation yields

$$\nabla_\alpha \bar{W}_{\dot\beta} = 0. \tag{3.28}$$

By passing to the right basis in the group space of G^c according to the second of formulas (2.18)

$$\overline{W}^{R}_{\dot\beta} = e^{V^k T^k}\, \overline{W}_{\dot\beta}\, e^{-V^k T^k} \qquad (3.29)$$

this condition is reduced to the ordinary chirality condition

$$\mathcal{D}_\alpha \overline{W}^{R}_{\dot\beta} = e^{V^k T^k}\, \nabla_\alpha \overline{W}_{\dot\beta}\, e^{-V^k T^k} = 0. \qquad (3.30)$$

A direct calculation utilizing the explicit expression for the form Ω_α (3.8) gives

$$\overline{W}^{R}_{\dot\beta} = \frac{1}{2i} \mathcal{D}^\alpha \mathcal{D}_\alpha \left(e^{2V^k T^k}\, \overline{\mathcal{D}}_{\dot\beta}\, e^{-2V^k T^k} \right). \qquad (3.31)$$

This coincides with the standard expression for the covariant spinor strength of the N=1 Yang-Mills theory[1,2]. Using the connection (3.29), one easily establishes also the form of $\overline{W}_{\dot\beta}$:

$$\overline{W}_{\dot\beta} = \frac{1}{2i} \nabla^\alpha \nabla_\alpha \left[\overline{\nabla}_{\dot\beta}\, e^{-V^k T^k}\, e^{V^k T^k} \right]. \qquad (3.32)$$

Now, we discuss the couplings to matter. In ordinary nonlinear realizations[11-14] interactions of matter fields with the Goldstone fields are introduced as follows. One starts with a Lagrangian invariant under the vacuum stability subgroup and then replaces the ordinary derivatives by the covariant ones. In our case, the vacuum stability subgroup is the group of rigid G transformations. Therefore, in order to implement the couplings between matter superfields themselves and with the prepotential $V^i(x,\theta,\bar\theta)$ in the manner invariant under the whole group G^c_{loc} it is sufficient to make the change $\{\mathcal{D}_\alpha, \overline{\mathcal{D}}_{\dot\alpha}, \partial^m\} \to \{\nabla_\alpha, \overline{\nabla}_{\dot\alpha}, \nabla^m\}$ in some superfield Lagrangian having global G symmetry. However, sometimes it is more convenient, to bring beforehand superfields into the right or left G -bases according to the relations (2.19). All the geometric characteristics constructed above can be recast into these bases by formulas of the type (2.19):

$$\{\nabla^L_\alpha, \overline{\nabla}^L_{\dot\alpha}, \nabla^L_m\} = e^{-V^k \overline{T}^k} \{\nabla_\alpha, \overline{\nabla}_{\dot\alpha}, \nabla_m\} e^{V^k \overline{T}^k} \qquad (3.33a)$$

$$\{\nabla^R_\alpha, \overline{\nabla}^R_{\dot\alpha}, \nabla^R_m\} = e^{V^k \overline{T}^k} \{\nabla_\alpha, \overline{\nabla}_{\dot\alpha}, \nabla_m\} e^{-V^k \overline{T}^k} \qquad (3.33b)$$

(here, the differential operators are assumed to act on everything to the right of them). The explicit form of covariant derivatives in the left basis is as follows

$$\nabla^L_\alpha = \mathcal{D}_\alpha + i\, \mathcal{V}^{Li}_\alpha \overline{T}^i, \quad \overline{\nabla}^L_{\dot\alpha} = \overline{\mathcal{D}}_{\dot\alpha}, \quad \overline{\nabla}^{Lm} = \partial^m + i\, \tilde\sigma^{m\dot\beta\alpha} \overline{\mathcal{D}}_{\dot\beta} \mathcal{V}^{Li}_\alpha \overline{T}^i \qquad (3.34)$$

with $\mathcal{V}^{Li}_\alpha$ given by eq. (3.7a). These operators are related to the corresponding quantities in the right basis by complex conjugation. The covariant strengths in the complex bases can be obtained by commuting relevant covariant derivatives between themselves; they all are expressed through $\mathcal{V}^L_\alpha$, $\overline{\mathcal{V}}^R_{\dot\alpha}$ (3.7) and have a more simple appearance as compared with those in the real basis (cf. expressions (3.32) and (3.33)). Covariant derivatives $\overline{\nabla}^L_{\dot\alpha}$, ∇^R_α do not contain dependence on $V^i(x,\theta,\bar\theta)$ so one may impose on Φ_L, Φ_R ordinary chirality conditions[6,7]:

$$\overline{\mathcal{D}}_{\dot\alpha} \Phi^{I}_L = 0 \rightarrow \Phi^{I}_L = \varphi^{I}_L(x_L, \theta_L)$$
$$\mathcal{D}_\alpha \Phi^{\overline{II}}_R = 0 \rightarrow \Phi^{\overline{II}}_R = \varphi^{\overline{II}}_R(x_R, \bar\theta_R). \qquad (3.35)$$

In the real basis, these constraints look more complicated:

$$\overline{\nabla}_{\dot\alpha} \Phi^{I} = 0 \rightarrow \Phi^{I} = e^{V^i \overline{T}^i} \varphi^{I}_L$$
$$\nabla_\alpha \Phi^{\overline{II}} = 0 \rightarrow \Phi^{\overline{II}} = e^{-V^i \overline{T}^i} \varphi^{\overline{II}}_R \qquad (3.36)$$

3. Now, let us discuss in short the case of curved geometry on $\mathbb{R}^{4|4}$. We restrict our consideration to the standard minimal Einstein N=1 supergravity[4-6]. To repeat the above analysis, one needs the following commutation relations between curved counterparts $\widetilde{\mathcal{D}}_\alpha$, $\overline{\widetilde{\mathcal{D}}}_{\dot\alpha}$, $\widetilde{\mathcal{D}}_a$ of flat superspace derivatives[5,20,21]:

$$\{\widetilde{\mathcal{D}}_\alpha, \widetilde{\mathcal{D}}_\beta\} = -R_{\alpha\beta}$$
$$\{\widetilde{\mathcal{D}}_\alpha, \overline{\widetilde{\mathcal{D}}}_{\dot\beta}\} = 2i\, \sigma^a_{\alpha\dot\beta} \widetilde{\mathcal{D}}_a \equiv 2i\, \widetilde{\mathcal{D}}_{\alpha\dot\beta} \qquad (3.37)$$
$$[\widetilde{\mathcal{D}}_\alpha, \widetilde{\mathcal{D}}_{\beta\dot\beta}] = -T^{\gamma}_{\alpha,\beta\dot\beta} \widetilde{\mathcal{D}}_\gamma - T^{\dot\gamma}_{\alpha,\beta\dot\beta} \overline{\widetilde{\mathcal{D}}}_{\dot\gamma} - R_{\alpha,\beta\dot\beta},$$

By passing to the right basis in the group space of G^c according to the second of formulas (2.18)

$$\overline{W}^{R}_{\dot\beta} = e^{V^kT^k}\,\overline{W}_{\dot\beta}\,e^{-V^kT^k} \tag{3.29}$$

this condition is reduced to the ordinary chirality condition

$$\mathcal{D}_\alpha \overline{W}^{R}_{\dot\beta} = e^{V^kT^k}\,\nabla_\alpha \overline{W}_{\dot\beta}\,e^{-V^kT^k} = 0. \tag{3.30}$$

A direct calculation utilizing the explicit expression for the form Ω_α (3.8) gives

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$$\{\nabla^L_\alpha, \overline{\nabla}^L_{\dot\alpha}, \nabla^L_m\} = e^{-V^k\overline{T}^k}\{\nabla_\alpha, \overline{\nabla}_{\dot\alpha}, \nabla_m\}\,e^{V^k\overline{T}^k} \tag{3.33a}$$

$$\{\nabla^R_\alpha, \overline{\nabla}^R_{\dot\alpha}, \nabla^R_m\} = e^{V^k\overline{T}^k}\{\nabla_\alpha, \overline{\nabla}_{\dot\alpha}, \nabla_m\}\,e^{-V^k\overline{T}^k} \tag{3.33b}$$

(here, the differential operators are assumed to act on everything to the right of them). The explicit form of covariant derivatives in the left basis is as follows

$$\nabla^L_\alpha = \mathcal{D}_\alpha + i\,v^{Li}_\alpha \overline{T}^i,\quad \overline{\nabla}^L_{\dot\alpha} = \overline{\mathcal{D}}_{\dot\alpha},\quad \overline{\nabla}^{Lm} = \partial^m + i\,\tilde\sigma^{m\dot\beta\alpha}\,\overline{\mathcal{D}}_{\dot\beta}\,v^{Li}_\alpha \overline{T}^i \tag{3.34}$$

with v^{Li}_α given by eq. (3.7a). These operators are related to the corresponding quantities in the right basis by complex conjugation. The covariant strengths in the complex bases can be obtained by commuting relevant covariant derivatives between themselves; they all are expressed through v^L_α, $\bar v^R_{\dot\alpha}$ (3.7) and have a more simple appearance as compared with those in the real basis (cf. expressions (3.32) and (3.33)). Covariant derivatives $\overline{\nabla}^L_{\dot\alpha}$, ∇^R_α do not contain dependence on $V^i(x,\theta,\bar\theta)$ so one may impose on Φ_L, Φ_R ordinary chirality conditions[6,7];

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In the real basis, these constraints look more complicated:

$$\begin{aligned} &\overline{\nabla}_{\dot\alpha}\Phi^{I} = 0 \to \Phi^{I} = e^{V^i\overline{T}^i}\varphi^{I}_L \\ &\nabla_\alpha \Phi^{\overline{II}} = 0 \to \Phi^{\overline{II}} = e^{-V^i\overline{T}^i}\varphi^{\overline{II}}_R \end{aligned} \tag{3.36}$$

3. Now, let us discuss in short the case of curved geometry on $\mathbb{R}^{4|4}$. We restrict our consideration to the standard minimal Einstein N=1 supergravity[4-6]. To repeat the above analysis, one needs the following commutation relations between curved counterparts $\widetilde{\mathcal{D}}_\alpha$, $\overline{\widetilde{\mathcal{D}}}_{\dot\alpha}$, $\widetilde{\mathcal{D}}_a$ of flat superspace derivatives[5,20,21];

$$\begin{aligned} &\{\widetilde{\mathcal{D}}_\alpha, \widetilde{\mathcal{D}}_\beta\} = -R_{\alpha\beta} \\ &\{\widetilde{\mathcal{D}}_\alpha, \overline{\widetilde{\mathcal{D}}}_{\dot\beta}\} = 2i\,\sigma^a_{\alpha\dot\beta}\,\widetilde{\mathcal{D}}_a \equiv 2i\,\widetilde{\mathcal{D}}_{\alpha\dot\beta} \\ &[\widetilde{\mathcal{D}}_\alpha, \widetilde{\mathcal{D}}_{\beta\dot\beta}] = -T^{\gamma}_{\alpha,\beta\dot\beta}\widetilde{\mathcal{D}}_\gamma - T^{\dot\gamma}_{\alpha,\beta\dot\beta}\overline{\widetilde{\mathcal{D}}}_{\dot\gamma} - R_{\alpha,\beta\dot\beta}, \end{aligned} \tag{3.37}$$

where the symbols T, R denote components of torsion and curvature (the latter takes values in the algebra of $SL(2,C)$) and the conventional constraints[5,20,21] are taken into account (we basically use the notation of Ogievetsky and Sokatchev[5]). For our purpose, it is necessary to know explicit expressions for the components $R_{\alpha\beta,\gamma\delta}$, $T^{\dot\gamma}_{\alpha,\beta\dot\beta}$ [5,20,21]:

$$R_{\alpha\beta,\gamma\delta} = -\frac{1}{2}\left(\epsilon_{\alpha\gamma}\epsilon_{\beta\delta} + \epsilon_{\alpha\delta}\epsilon_{\beta\gamma}\right)\bar{R} \tag{3.38}$$

$$T^{\dot\gamma}_{\alpha,\beta\dot\beta} = -\frac{i}{4}\epsilon_{\alpha\beta}\,\delta^{\dot\gamma}_{\dot\beta}\bar{R}, \qquad (\tilde{\mathcal{D}}_\alpha \bar{R} = 0),$$

where $\bar{R}$ is one of the basic superfields of minimal N=1 supergravity. Also, we will use the Bianchi identity [21]:

$$R_{\alpha,\beta\dot\beta,\gamma\delta} + R_{\gamma,\beta\dot\beta,\alpha\delta} = -\tilde{\mathcal{D}}_\alpha T_{\gamma,\beta\dot\beta,\delta} - \tilde{\mathcal{D}}_\gamma T_{\alpha,\beta\dot\beta,\delta}. \tag{3.39}$$

All the basic gauge-covariant quantities of the flat case, except for $F_{\beta\alpha\dot\alpha}$ (3.21), are generalized to the curved superspace simply by means of the change $\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}, \partial_a \to \tilde{\mathcal{D}}_\alpha, \tilde{\bar{\mathcal{D}}}_{\dot\alpha}, \tilde{\mathcal{D}}_a$ in corresponding formulas. The strength $F_{\beta\alpha\dot\alpha}$ gets a minor modification:

$$\tilde{F}_{\beta\alpha\dot\alpha} = \tilde{\nabla}_\beta \tilde{\Omega}_{\alpha\dot\alpha} - i\tilde{\mathcal{D}}_{\alpha\dot\alpha}\tilde{\Omega}_\beta + iT^{\gamma}_{\beta\alpha\dot\alpha}\tilde{\Omega}_\gamma + iT^{\dot\gamma}_{\beta,\alpha\dot\alpha}\bar{\tilde{\Omega}}_{\dot\gamma}. \tag{3.40}$$

Using the relations (3.37)-(3.39), one may be convinced that $\tilde{F}_{\beta\alpha\dot\alpha}$ enjoys the same properties (3.25), (3.26), (3.28) as $F_{\beta\alpha\dot\alpha}$ in the flat case. A simple calculation yields for $\bar{\tilde{W}}_{\dot\alpha} = \epsilon^{\alpha\beta}\tilde{F}_{\beta\alpha\dot\alpha}$ the well-known expression

$$\bar{\tilde{W}}_{\dot\alpha} = \frac{1}{2i}\left(\tilde{\nabla}^\alpha\tilde{\nabla}_\alpha + \bar{R}\right)\left(\bar{\tilde{\nabla}}_{\dot\alpha} e^{-V^kT^k} e^{V^kT^k}\right) \tag{3.41}$$

which simplifies in the right G^c-basis to

$$\bar{\tilde{W}}^R_{\dot\alpha} = e^{V^kT^k}\,\bar{\tilde{W}}_{\dot\alpha}\,e^{-V^kT^k} = \frac{1}{2i}\left(\tilde{\mathcal{D}}^\alpha\tilde{\mathcal{D}}_\alpha + \bar{R}\right)\left(e^{2V^kT^k}\bar{\tilde{\mathcal{D}}}_{\dot\alpha} e^{-2V^kT^k}\right) \tag{3.42}$$

$$\left(\tilde{\mathcal{D}}_\alpha \bar{\tilde{W}}^R_{\dot\alpha} = e^{V^kT^k}\,\tilde{\nabla}_\alpha\bar{\tilde{W}}_{\dot\alpha}\,e^{-V^kT^k} = 0\right).$$

Thus, we have demonstrated that all the necessary quantities of the N=1 Yang-Mills theory can be obtained algorithmically,

starting solely with the structure relations (2.1) and the standard nonlinear realization formulas (3.1) supplemented by the covariant constraint (3.6). Perhaps, it would be interesting to relate this formalism to the Levi superform approach advocated by Schwarz[22,23] as the most adequate geometric language to deal with hypersurfaces in complex superspaces.

Finally, we note that, with respect to the right gauge group (2.20), all the covariant objects in the real G^c-basis transform just as in G^c_{loc}, but with arbitrary superfunctions $\lambda^i(x,\theta,\bar\theta)$ instead of K^i. The corresponding quantities in the right and left complex G^c-bases are invariant under this gauge group (this property is checked with the help of representation (2.22)).

4. Conclusion

The above consideration suggests several interesting new possibilities for the N=1 Yang-Mills theory. First, the fact that this theory is a kind of nonlinear σ-model on the group G^c raises the problem of constructing the relevant linear σ-model, with G^c as the vacuum invariance group. As any unitary representation of G^c is infinite-dimensional such a σ-model should naturally give rise to infinite-dimensional field multiplets. In fact, using general theorems on the relation between linear representations and nonlinear realization[11], one may construct out of $V^i(x,\theta,\bar\theta)$ alone any representation of G^c including the unitary ones, provided those contain an invariant of the subgroup G. The possibility of constructing such composite linear G^c-multiplets may be considered as the group-theoretical argument in favour of existence of the dynamical phase with unbroken G^c-symmetry in the N=1 Yang-Mills theory. An interesting point is the inevitable presence of G-invariant (i.e.,"colourless") states in these multiplets.

Another line of thinking concerns the geometric analogy between the N=1 Yang-Mills and N=1 supergravity. A natural conjecture is that these theories admit a unification within a larger theory of the Kaluza-Klein type. One may treat $\mathrm{Re}\,\varphi^i_L \equiv \varphi^i$ as an independent coordinate like x^m in eq. (1.1), choose the base real superspace to be $\mathbb{R}^{4+M|4} = \{x^m, \varphi^i, \theta^\mu, \bar\theta^{\dot\mu}\}$ instead of $\mathbb{R}^{4|4}$ and construct a 4+M-dimensional extension of minimal N=1 supergravity by embedding $\mathbb{R}^{4+M|4}$ into $\mathbb{C}^{4+M|2}$. The standard theory is expected to be reproduced as the lowest order in a proper expansion in φ^i.

However, the most exciting task is to extend the geometric picture described here to higher N gauge theories, at least to the case of N=2. The necessity to complexify G in the N=1 case can be related to the fact that the fundamental superspace of N=1 supersymmetry is complex superspace $\mathbb{C}^{4|2}$. Its true N=2 analog seems to be a superspace bosonic coordinates of which form a quaternion /24/. So, in the N=2 case one may, instead of the extension $T^i \to \{T^i, iT^k\}$, try the extension of the type $T^k \to \{T^k, q^i \otimes T^k, \ldots\}$, where q^i ($i =$ 1,2,3) are imaginary quaternion units transforming as a triplet with respect to the automorphism group SU(2) of N=2 superalgebra. The corresponding prepotential should then acquire an additional triplet index. That is just what happens in the N=2 electrodynamics /7/. The elucidation of the minimal geometric structure of the N=2 Yang-Mills theory may essentially help in exposing the analogous structure of N=2 supergravity.

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Внутренняя геометрия N=1-суперсимметричной теории Янга-Миллса

N=1-суперсимметричная теория Янга-Миллса сформулирована аналогично минимальной N=1-супергравитации в подходе Огиевецкого-Сокачева. Показано, что внутренней геометрией N=1-теории Янга-Миллса является комплексная геометрия вложения вещественного суперпространства $R^{4|4}=\{x^m, \theta^\mu, \bar\theta^{\dot\mu}=(\theta^\mu)^+\}$ в расширенное комплексное суперпространство $C^{4+M|2}=\{x^m_L, \theta^\mu_L=\theta^\mu, \phi^i_L\}(i=1,\dots M)$, где ϕ^i_L - локальные координаты на группе G^c (комплексификации калибровочной группы G), $M=\dim G$. Препотенциал N=1-теории отождествляется с $\operatorname{Im}\phi^i_L$, ограниченной на гиперповерхность $R^{4|4}$. Он принимает значения в фактор-пространстве G^c/G, поэтому N=1-теорию Янга-Миллса можно интерпретировать как обобщенную нелинейную σ-модель. Определены соответствующие формы Картана и показано, как с их помощью строить геометрические характеристики теории. Обсуждаются некоторые новые возможности, вытекающие из предложенной формулировки.

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Intrinsic Geometry of the N=1 Supersymmetric Yang-Mills Theory

The N=1 supersymmetric Yang-Mills theory is formulated analogously to the minimal N=1 supergravity in the Ogievetsky-Sokatchev approach. The intrinsic superspace geometry of the N=1 Yang-Mills is shown to be the complex geometry of embedding of the real superspace $R^{4|4}=\{x^m, \theta^\mu, \bar\theta^{\dot\mu}=(\theta^\mu)^+\}$ into the extended complex one $C^{4+M|2}=\{x^m, \theta^\mu_L=\theta^\mu, \phi^i_L\}$, $(i=1,\dots,M)$, ϕ^i_L being local coordinates on the group G^c, the complexification of gauge group G, and $M=\dim G$. The N=1 Yang-Mills prepotential is identified with $\operatorname{Im}\phi^i_L$ restricted to the hypersurface $R^{4|4}$. It takes values in the coset G^c/G, so the N=1 Yang-Mills theory can be interpreted as a generalized nonlinear σ model. The corresponding Cartan forms are defined and they are applied to the construction of relevant geometric objects. We discuss also some new possibilities following from the suggested formulation of the theory.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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