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FLUCTUATIONS IN METRICS AND NEW DYNAMICS OF PARTICLES

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The idea to use stochasticity of space-time [1] and random fluctuations in metrics [2] as an origin of the stochastic interpretation of quantum mechanics [3] plays an important role in constructing the nonlocal theory of quantized fields [4] and in the possibility of describing spectra of elementary particle masses [5].

Dynamics of particles within the hypothesis of fluctuations in metrics is considered in this note; the basic idea is the following: we suppose that freely moving particles perturb space-time around themselves and at the same time the space-time metrics is generated and becomes a Riemannian one $\tilde{g}_{\mu\nu}(b_E, x)$ depending on a random vector $b_E = (\tau, \vec{b})$ with a distribution $w(b_E^2/\ell^2)$ obeying the conditions:

$$w(b_E^2/\ell^2) \geq 0, \quad \int d^4b\, w(b_E^2/\ell^2) = 1 \qquad (b_E^2 = \tau^2 + \vec{b}^2).$$

Parameter ℓ characterises the value (intensity) of fluctuation in metrics.

As a result of such fluctuations in metrics, as is shown below, a nonlinear dynamics of particles arises and freely moving particles are represented by soliton-type solitary waves. In a reg[illegible] corresponding to the width of the solitary wave-amplitude [illegible]e averaged Riemannian metrics

$$g_{\mu\nu}(\ell, x) = \langle \tilde{g}_{\mu\nu}(b_E, x) \rangle = \int d^4b\, w(b_E^2/\ell^2)\, \tilde{g}_{\mu\nu}(b_E, x)$$

may be nearly singular (an idea reminiscent of an old concept by Einstein et al. [6] and recently by Efinger [7]). Outside this singular region one expects

$$g_{\mu\nu} \approx \eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$$

(or the boundary condition $g_{\mu\nu} \to \eta_{\mu\nu}$ at infinity) for $\ell \to 0$, where $\eta_{\mu\nu}$ is the Minkowski-metrics.

A simple form of $\tilde{g}_{\mu\nu}$ is chosen, that is $\tilde{g}_{\mu\nu} = \phi^2(b_E^2 x^2)\eta_{\mu\nu}$ or $\tilde{g}_{\mu\nu} = \phi^2(b_E^2, \lambda^2 + x^2)\eta_{\mu\nu}$ $(x^2 = x_0^2 - \vec{x}^2, \lambda = \hbar/mc)$, then the averaged space-time metrics acting on the behaviour of particles is obtained by the formula

$$g_{\mu\nu}(\ell^2/x^2) = \int d^4b\, w(b_E^2/\ell^2)\, \phi^2(b_E^2, x^2)\eta_{\mu\nu} = \phi^2(\ell^2/x^2)\eta_{\mu\nu}.$$

Now we pass to obtain a particle dynamics in this conformally flat space-time. The action for a free particle acquires the form

$$S = -mc\int_a^b ds, \quad ds = \sqrt{g_{\mu\nu}(\ell^2/x^2)\,dx^\nu\,dx^\mu} = \phi\,ds_0, \quad ds_0 = \sqrt{dx^\nu\,dx_\nu}\,. \qquad (1)$$

According to the action principle

$$\delta S = -mc\int_a^b \delta\,ds = -mc\int_a^b \frac{\delta\,ds^2}{2\,ds} = -mc\int_a^b \delta\,(g_{\mu\nu}\,dx^\nu\,dx^\mu)/2\,ds =$$

$$= -mc\int\Big\{\frac{1}{2}\frac{dx^\nu}{ds}\frac{dx^\mu}{ds}\frac{\partial g_{\nu\mu}}{\partial x^\lambda}\delta x^\lambda + g_{\mu\nu}\frac{dx^\nu}{ds}\frac{d\,\delta x^\mu}{ds}\Big\}\,ds = 0.$$

Integration by parts in the second term gives

$$\delta S = -mc\,g_{\nu\mu}\frac{dx^\nu}{ds}\delta x^\mu\Big|_a^b - mc\int_a^b\Big\{\frac{1}{2}\frac{dx^\nu}{ds}\frac{dx^\mu}{ds}\frac{\partial g_{\nu\mu}}{dx^\lambda}\delta x^\lambda - \frac{d}{ds}\Big(g_{\nu\mu}\frac{dx^\nu}{ds}\Big)\delta x^\mu\Big\}\,ds.$$

From this we have the following nonlinear equation

$$\frac{D_\phi u_\nu}{ds} = g_{\lambda\nu}\frac{du^\lambda}{ds} + \Gamma_{\nu,\mu\lambda}\,u^\mu u^\lambda = 0, \qquad (u^\mu = dx^\mu/ds)$$

or (assuming $g_{\mu\nu} = \phi^2\eta_{\mu\nu}$, $ds = \phi\,ds_0$)

$$\frac{du^\circ_\nu}{ds_0} - \frac{\partial\ln\phi}{\partial x^\nu} + u^\circ_\nu\Big(u_0^\lambda\frac{\partial\ln\phi}{\partial x^\lambda}\Big) = 0 \qquad (2)$$

$$(u^\circ_\nu = dx_\nu/ds_0).$$

Here

$$\Gamma_{\nu,\mu\lambda} = \frac{1}{2}(\partial g_{\mu\nu}/\partial x^\lambda + \partial g_{\lambda\nu}/\partial x^\mu - \partial g_{\mu\lambda}/\partial x^\nu)$$

is the Christoffel symbol.

We solve equation (2) in the two-dimensional space-time (x,t) and in the simple case when $\phi = \phi(x)$ depends only on the spatial variable x (now the variable x is regarded as a parameter).

Since $u^\circ_x = -\mathcal{V}_x/\sqrt{1-\beta^2}$, $ds_0 = c\,dt\sqrt{1-\beta^2}$, $\beta^2 = \mathcal{V}_x^2/c^2$, from (2) we get $(\mathcal{V}_x = \mathcal{V})$:

$$\frac{dv}{dt} = \epsilon\,(1-v^2), \qquad (v = \mathcal{V}/c, \quad \epsilon = -c\frac{\partial\ln\phi}{\partial x}). \qquad (3)$$

This is the Riccati-type equation (for example, see ref.[8]). The substitution $v = \dot w(t)/\epsilon\,w(t)$, $\dot w = dw/dt$ converts it into a second-order linear equation for $w(t)$: $\ddot w - \epsilon^2 w = 0$. The solution to the last equation is $w = c_1\mathrm{ch}(\epsilon t + c_2)$ and therefore $v = \mathrm{th}(\epsilon t + c_2)$. Here the integration constant c_2 is given by an initial condition, say $v(0) = \mathcal{V}_0/c$.

So, the solution of the Cauchy problem for the Riccati-type equation (3) is

$$v(t) = \mathrm{th}[\epsilon t + \mathrm{Arth}(\mathcal{V}_0/c)]\,,$$

or (4)

$$\dot v(t) = \epsilon\,\mathrm{sech}^2[\epsilon t + \mathrm{Arth}(\mathcal{V}_0/c)]\,.$$

We see that these solutions are solitary-type waves. Assuming the simple form for $\phi = 1 - \ell^2/(x_0^2 - \vec{x}^2 + \lambda^2) \Rightarrow 1 + \ell^2/(x^2 - \lambda^2)$ it is easy to verify that the result obtained represents the freely moving particle with the velocity $\mathcal{V}_0$ in both the cases: at infinity $x\to\infty$ and in the limit $\ell\to 0$. The singularity is reached in the region $x \sim \lambda$.

It should be noted that because of the fluctuations in metrics the description of the behaviour of the particle requires probabilistic methods. As is shown above, the particle velocity depends in fact not only on the time variable but also on the spatial variable x ($\mathcal{V}(t) \Rightarrow \mathcal{V}(x,t)$). In other words, for a complete description of the particle motion in a fluctuated metric space-time at the same time with the particle velocity one should introduce one more quantity $\rho(x,t)$ – the probability density of finding the particle at point x and at time t. So, we see that fluctuations in metrics as an origin of real "quantum" forces lead to the random behaviour of particles; their dynamics is described by nonlinear partial differential equations of a type of (2) and the equation of continuity for $\rho(x,t)$: $\partial\rho/\partial t + \mathrm{div}(\rho\vec{\mathcal{V}}) = 0$. In this case the particle velocity is given by the formula $\mathcal{V}(t) = \int\rho(x,t)\mathcal{V}(x,t)\,dx$ and du°_ν/ds_0 in the expression (2) should be read as $u_0^\lambda\partial_\lambda u^\circ_\nu$.

Now consider the motion of a charged particle in an electromagnetic field characterized by a potential A_μ within our scheme. After some calculations analogous to the usual (gravitational) case (for example, see ref.[9]) we have the following equation

$$mc\frac{D_\phi u_\nu}{ds} = \frac{e}{c}F_{\nu\rho}u^\rho, \qquad (5)$$

where

$$F_{\nu\rho} = A_{\rho;\nu} - A_{\nu;\rho} = \partial A_\rho/\partial x^\nu - \partial A_\nu/\partial x^\rho .$$

It turns out that in our model the equations for the electromagnetic field acquire the forms:

$$\partial F_{\nu\mu}/\partial x^\lambda + \partial F_{\lambda\nu}/\partial x^\mu + \partial F_{\mu\lambda}/\partial x^\nu = 0$$

and

$$F^{\nu\mu}_{\ ;\mu} = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}(\sqrt{-g}\,F^{\nu\mu}) = -\frac{4\pi}{c} j^\nu , \qquad (6)$$

where j^ν is the four-vector current satisfying the equation of continuity $j^\nu_{;\nu} = (1/\sqrt{-g})\partial_\nu(\sqrt{-g}\,j^\nu) = 0$.

Finally, we write the equation of motion for the charge in the external force f_μ in accordance with equation (5):

$$mc\frac{D_\phi u_\mu}{ds} = \frac{e}{c}F_{\mu\nu}u^\nu + f_\mu , \qquad (7)$$

where f_μ has components: $f_\mu = \{(\vec{f}\cdot\vec{\mathcal{V}})/c^2\sqrt{1-\beta^2}, -\vec{f}/c\sqrt{1-\beta^2}\}$ in particular, for bremsstrahlung $\vec{f} = \delta\ddot{\vec{\mathcal{V}}}$, $\delta = 2e^2/3c^3$. Then, the equation of motion in the case of $\mathcal{V} \ll c$, $\phi = \phi(\vec{x})$ has the form (in the absence of the electromagnetic field):

$$\dot{\vec{\mathcal{V}}} = \frac{\delta}{m}\ddot{\vec{\mathcal{V}}} - (1-\beta^2)[c^2\frac{\partial \ln\phi}{\partial \vec{x}}(1-\beta^2) + \vec{\mathcal{V}}(\vec{\mathcal{V}}\cdot\frac{\partial\ln\phi}{\partial\vec{x}})] \qquad (8)$$

or in the two-dimensional case

$$\dot{v} = \frac{\delta}{m}\ddot{v} + \epsilon(1-v^2), \qquad (v = \mathcal{V}/c). \qquad (9)$$

This is an autonomous equation if it is assumed that t is an independent variable and x is a parameter.

Then a standard trick is to express $u(v) = \dot{v}(t)$ as a function of v and to find an equation for u and its derivatives. We thus let (in the two-dimensional case)

$$\dot{v} = u(v), \quad \ddot{v} = u'(v)u(v), \quad u'(v) = du/dv$$

and for $u(v)$ we have the following equation:

$$u(v) - \frac{\delta}{m}u'(v)u(v) = \epsilon(1-v^2).$$

This equation is still too difficult to solve in a closed form.

This is easy to show if one assumes that $\epsilon \sim \ell^2 (x \ll 1)$ is a small parameter. Then a solution is searched in the form

$$u(v) = u_0(v) + \epsilon u_1(v) + \epsilon^2 u_2(v) + \dots$$

First two terms of $u(v)$ satisfy the following equations:

$$u_0(v) - \frac{\delta}{m}u_0'(v)u_0(v) = 0, \quad u_1(v) - \frac{\delta}{m}u_1'(v)u_0(v) - \frac{\delta}{m}u_0'(v)u_1(v) = 1-v^2 . \quad (10)$$

Solutions of the former are $u_0^{(1)} = 0$, $u_0^{(2)} = \frac{m}{\delta}v + c_1$. Corresponding to these $u_0^{(1)}$ and $u_0^{(2)}$ solutions to the last equation (10) are $u_1^{(1)}(v) = 1-v^2$ and

$$u_1^{(2)}(v) = v^2/2 - c_1\delta\cdot v/m + (c_1^2\delta^2/m^2 - 1)\ln(c_1 + mv/\delta) + c_2 .$$

So, for equation (9) we have the following approximate solutions:

$$v^{(1)} = \mathrm{th}[\epsilon(t + c_4)]$$

$$\int^{v^{(2)}} \frac{dx}{mx/\delta + c_1 + \epsilon u_1^{(2)}(x)} = t + c_3 ,$$

where $c_i (i=1,\dots,4)$ are integration constants which may be defined by the initial and physical conditions for the given problem.

We believe that solutions to equations (7)-(9) should be both causal (without advanced effects) and free from run-away difficulties. This problem will be discussed in future investigations.

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Намсрай Х. E2-82-850

Флуктуация метрики и новая динамика частиц

Предложен метод введения флуктуации метрики, в рамках которого изучаются динамика частиц и уравнения для электромагнитного поля. Получена нелинейная динамика частиц, описывающая солитон-подобные волны в конформно-плоском пространстве и времени. В нашем подходе причиной случайного поведения частиц служит флуктуация метрики.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Namsrai Kh. E2-82-850

Fluctuations in Metrics and New Dynamics of Particles

A method of introducing fluctuations in metrics is presented within which the particle dynamics and equations for the electromagnetic field are investigated. A nonlinear dynamics of particles is derived which admits solitary-type waves on a conformally flat Riemannian space-time. In our model fluctuations in metrics as the origin of real "quantum" forces lead to the random behaviour of particles.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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