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Yu.P.Ivanov, S.V.Vyshensky

PHOTON PROPAGATOR ASYMPTOTICS AT $Q^2 \to \infty$ IN THE SOURCE THEORY

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1. On the basis of the Schwinger[1] source theory (ST) with the help of the causal diagram technique, the expression for the total photon propagator was established (ref.[2]) which is consistent with the Källen-Lehmann form (that is why the non-physical singularities are absent):

$$D(t) = D_0(t) + \int_0^{+\infty} \frac{dt'}{t' - t - i0} \partial(t') I(t') \partial(t'),$$

$$D_0(t) = \frac{1}{-t - i0}, \quad (1)$$

$$\partial(t) = -\frac{1}{t} \mathrm{Re}\exp\left[-\frac{t}{\pi} P \int_0^{+\infty} \frac{dt'}{t' - t} \frac{\phi(t')}{t'}\right],$$

P is the Cauchy principal value symbol,

$$\phi(t) = \mathrm{arctg}[\pi t^{-1} I(t)],$$

function I(t) is connected with the polarizable operator and in one-loop approximation is equal to

$$I(t) = \frac{\alpha}{3\pi} t \left(1 + \frac{t_0}{2t}\right)\left(1 - \frac{t_0}{t}\right)^{1/2}, \quad \alpha = 1/137, \quad (2)$$

$t_0 = 4m^2$, m is the electron mass.

The method from book[3] allows one to find the first term of the asymptotical expansion of form (I) in the vicinity of the point $t = -\infty$ (in the approximation (2)):

$$\frac{D(t)}{D_0(t)} \sim \mathrm{const} \times \left(\frac{-t}{t_0}\right)^{\kappa},$$

$$\kappa = \frac{2}{\pi} \mathrm{arctg} \frac{\alpha}{3} \approx \frac{2\alpha}{3\pi} \approx 1.5 \times 10^{-3}. \quad (3)$$

However the only thing we can say about the const is that it is positive. Its value may be determined with the help of the computer in the following way. Let us mention that expressions (1), (2) are analytic concerning coupling constant α. The structure of asymptotics (3) is obviously independent of the concrete physical value $\alpha = 1/137$. So, putting different values of α we shall obtain

$$\text{const} = \lim_{t=-\infty} \frac{D(t)}{D_0(t)}\left(\frac{t_0}{-t}\right)^{\kappa}.$$

Such calculations were performed for the wide interval of α variation. They brought to the correlation const = 1/2. A simple asymptotic formula is available:

$$\frac{D(t)}{D_0(t)} \approx \frac{1}{2}\left(\frac{-t}{t_0}\right)^{2\alpha/3\pi}, \qquad t \to -\infty. \qquad (4)$$

The region where this approximation works will be cleared up a little bit later.

2. The connection between the Gell-Mann-Low function $\psi(x)$, that describes the coupling constant $\alpha(t)$ evolution,

$$\ln\frac{t}{t_0} = \int_{\alpha}^{\bar{\alpha}(t)} \frac{dx}{\psi(x)}, \qquad (5)$$

with the spectral function ω from the total propagator representation:

$$\frac{D(t)}{D_0(t)} = \exp\left[-\frac{t}{\pi}\int \frac{dt'}{t'-t-i0}\,\frac{\omega(t')}{t'}\right].$$

was investigated in paper [4].

Function ω may be easily expressed through the functions ∂ and I from (1). ψ and ω are connected so:

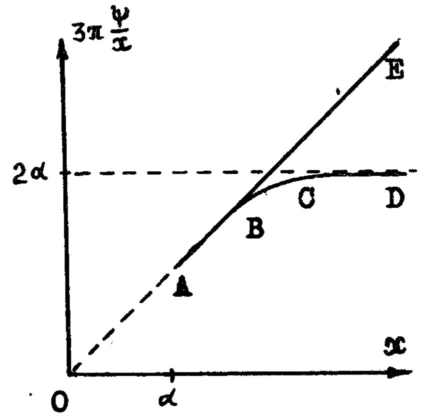
$$\psi(\bar{\alpha}) = \frac{\bar{\alpha}}{\pi}\,\omega[t(\bar{\alpha})].$$

This correlation is suitable at not very small t, where the renormgroup is usually applied.

The figure shows the dependence of $3\pi\frac{\psi(x)}{x}$ on x. The curve ABCD corresponds to our expression (1), (2). Standart electrodynamics based on the quantum field theory (QFT)[5] gives the straight AE according to

$$\psi(x) = \frac{x^2}{3\pi}. \qquad (6)$$

It is interesting to trace the motion of the point responsible for the increasing t value through the curve. In the AB section the point moves in the same manner through both the lines. In the BE section the motion gains the speed sharply, while in



Gell-Mann-Low function
ABCD - our curve, AE - standart approach.

the BC section the smooth regime is maintained. In the vicinity of B we are near the value of t where in QFT is a pole[6]:

$$\text{QFT:}\quad \bar{\alpha}(t) = \frac{\alpha}{1-\frac{\alpha}{3\pi}\ln\frac{t}{t_0}}. \qquad (7)$$

The point C is already "behind the pole". In the CD region our function rapidly achieves the asymptotics

$$\text{ST:}\quad \psi(x) = \kappa x, \qquad \kappa \approx 2\alpha/3\pi, \qquad (8)$$

that confirms with expressions (4), (5) and

$$\text{ST:}\quad \bar{\alpha}(t) = \frac{\alpha}{2}\left(\frac{t}{t_0}\right)^{\kappa}. \qquad (9)$$

It should be stressed that the formal substitution $t = t_0$ to this correlation violates the condition $\bar{\alpha}(t_0) = \alpha$. It should be recalled once more that expression (9) is suitable only for t from the CD section on the figure. Thus the value $\bar{\alpha}(t_0)$ has sence only while considering the low energy formula of type (7).

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Иванов Ю.П., Вышенский С.В. E2-82-824

Асимптотика фотонного пропагатора при $Q^2 \to \infty$ в теории источников

При использовании причинной диаграммной техники в теории источников Швингера исследовано поведение фотонного пропагатора при $|Q^2| \to \infty$. Возникающая в таком подходе связь функции Гелл-Манна-Лоу со спектральной функцией представления Челлена-Лемана гарантирует ренорминвариантность полученного выражения для бегущей константы связи. Результаты стандартного подхода в квантовой теории поля дают предасимптотику полученных выражений.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

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Ivanov Yu.P., Vyshensky S.V. E2-82-824

Photon Propagator Asymptotics at $Q^2 \to \infty$ in the Source Theory

On the basis of the Schwinger source theory with the help of the causal diagram technique the behaviour of the photon propagator at $|Q^2| \to \infty$ is investigated. The connection of the Gell-Mann-Low function with the Källen-Lehmann form spectral weight appeared here guarantees the renorm-invariance of the obtained expression for the running coupling constant. The standart quantum field approach results give the pre-asymptotics of the obtained expressions.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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