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QCD ANALYSIS OF DEEP INELASTIC LEPTON SCATTERING DATA

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INTRODUCTION

Study of deep inelastic lepton-nucleon scattering is a suitable way to verify the theory of strong interactions - quantum chromodynamics (QCD). QCD predicts the experimentally found dependence of structure functions of these processes upon the square 4-momentum transfer q^2: $F_i = F_i(x, Q^2)$ (x is the Bjorken variable, $Q^2 = -q^2$). If the momenta transfer are high enough ($Q^2 \gg M^2$, M is the nucleon mass), the perturbation theory (PT) is applicable within QCD. Within the PT framework Q^2-dependence is presented both for $F_i(x, Q^2)$ and for their moments $\langle F_i(Q^2)\rangle_n = \int_0^1 dx\, x^{n-1} F_i(x, Q^2)$. Q^2- evolution has quite a simple explicit form for the moments $\langle F_i(Q^2)\rangle_n$, but it is necessary to know $F_i(x, Q^2)$ for all $x \in [0,1]$ to determine the values of these moments. Experiments yield values of structure functions in a limited range of x. On extrapolating experimental values of $F_i(x, Q^2)$ to points $x = 0, 1$, one can introduce considerable ambiguity in behaviour of the moments. Therefore it is more convenient to operate directly with the structure functions, when comparing QCD predictions and experimental data. Q^2-evolution of $F_i(x, Q^2)$ is given by the Lipatov-Altarelli-Parisi (LAP) integrodifferential equations /1/, which can be solved by various approximated methods /2-4/. In this paper the method developed in /5/ has been applied. The basic principles of the method and the way of comparison with the experimental data are given in section 1. Results of the QCD analysis of the data from BCDMS, EMC and CDHS groups are given in section 2. The final section presents basic conclusions.

1. THE METHOD OF QCD ANALYSIS

Structure functions $F_i(x, Q^2)$ have a parton interpretation in the QCD, i.e., they can be expressed through Q^2-dependent quark (antiquark) distributions $q_i(x, Q^2)(\bar{q}_i(x, Q^2))$ ($q_i = u, d, s, c, \ldots$), where one usually separates valence (v) and sea (s) parts, and gluon distributions $G(x, Q^2)$. Structure functions are connected with these distributions in the following forms:
for the scattering of μ-mesons on the proton

$$F_2^{\mu p}(x, Q^2) = \frac{5}{18} x\Sigma(x, Q^2) + \frac{1}{6} xV(x, Q^2), \qquad (1.1a)$$

on the isoscalar target

$$F_2^{\mu N}(x, Q^2) = \frac{5}{18} x\Sigma(x, Q^2) \qquad (1.1b)$$

and for the $\nu(\bar{\nu})$ scattering

$$F_2^{\nu(\bar{\nu})}(x, Q^2) = x\Sigma(x, Q^2), \qquad (1.2)$$

where

$$V(x, Q^2) = u_v(x, Q^2) - d_v(x, Q^2) \qquad (1.3)$$

is a non-singlet (NS) combination and

$$\Sigma(x, Q^2) = u_v(x, Q^2) + d_v(x, Q^2) + \sum_{i=1}^{f} (q_{is}(x, Q^2) + \bar{q}_{is}(x, Q^2)) \qquad (1.4)$$

is a singlet combination of quark distributions. Here $u_v(x, Q^2) = u_v^p(x, Q^2) = d_v^n(x, Q^2)$ and $d_v(x, Q^2) = d_v^p(x, Q^2) = u_v^n(x, Q^2)$, and expressions (1.1) correspond to the flavour-independent distributions of sea quarks.

The LAP evolution equations (EE) describe the evolution of combinations $V(x, Q^2)$, $\Sigma(x, Q^2)$ and $G(x, Q^2)$, if these distributions are known at some initial value $Q^2 = Q_0^2$. The Q^2-dependence of the quark and gluon distributions is presented in the form of the dependence upon the variable

$$s = \ln \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}, \qquad (1.5)$$

where $\alpha_s(Q^2)$ is a running coupling constant which includes a QCD parameter Λ. It allows to find Λ by comparing the QCD-predicted evolution upon s with the experimentally observed Q^2-dependence.

Initial conditions (IC) for the evolution of structure functions (1.1), (1.2) (i.e., distributions at $Q^2 = Q_0^2$) must contain both quark and gluon distributions, since EE for the singlet $\Sigma(x, Q^2)$ and gluons $G(x, Q^2)$ form a system of bound equations. To prescribe the IC and solve the EE we applied the technique we developed earlier /5/. Distributions $V(x, Q^2)$, $\Sigma(x, Q^2)$, $G(x, Q^2)$ are selected in the form obtained on the basis of the systematic reconstruction of the structure functions by their asymptotic form in the Regge region $x \to 0$ /6/:

$$V(x, Q^2) = x^{-1/2} \frac{(1-x)^r}{B(1/2, r+1)} \cdot \frac{\Phi(a, r+1; -\beta_q(1-x))}{\Phi(a, r+3/2; -\beta_q)}, \qquad (1.6a)$$

$$\Sigma(x,Q^2) = \frac{r-a}{x}(1-x)^{r+1/2}\frac{\Phi(a, r+3/2; -\beta_q(1-x))}{\Phi(a, r+3/2; -\beta_q)} + 3V(x,Q^2), \quad (1.6b)$$

$$G(x,Q^2) = \frac{a}{x}(1-x)^{r+1/2}\frac{\Phi(a, r+3/2; -\beta_G(1-x))}{\Phi(a, r+3/2; -\beta_G)} e^{-\beta_G x}, \quad (1.6c)$$

where $\Phi(a, b; z)$ is a degenerated hypergeometric function. Quark and gluon distributions are characterized by the parameters with physical sense [6]. (Below the set of these parameters will be denoted as $\{a\}$). The Q^2-dependence of the functions is concentrated in $\{a\}=\{a(s)\}$, s-dependence of which is selected in the form which ensures reproduction of the QCD evolution. The simplest linear dependence

$$\{a(s)\} = \{a^{(0)} + a^{(1)} s\} \quad (1.7)$$

is quite enough for the region of Q^2 values attainable at modern accelerators. Two steps are usually necessary to find parameters $\{a^{(\ell)}\}$. At first, through the known values of a structure functions at some $Q^2=Q_0^2$ $(s=0)$ $\{a^{(0)}\}$ is determined by minimizing the expression

$$\chi^2(\{a^{(0)}\}) = \sum_k \left(\frac{F_i^{exp}(x_k,Q_k^2) - F_i(x_k,\{a^{(0)}\})}{\Delta F^{exp}(x_k,Q_k^2)}\right)^2 \Bigg|_{Q_k^2=Q_0^2}, \quad (1.8)$$

where F_i are connected with (1.6) by correlations (1.1), (1.2). The minimization prescribes IC $V(x,Q_0^2)$, $\Sigma(x,Q_0^2)$ and reduces greatly randomness in selecting the gluon distribution $G(x,Q_0^2)$ which is not included in (1.1) and (1.2) (only parameter $\beta_G^{(0)}$ is not fixed). This is an advantage of (1.6) as compared with commonly used empiric parametrizations of the type

$$\sum_i A_i x^{\alpha_i}(1-x)^{\beta_i}, \quad (1.9)$$

where each quark and gluon distribution has its own set of parameters (A_i, α_i, β_i) what results in much randomness in determination of $\Sigma(x,Q^2)$ and $G(x,Q^2)$. Besides, the use of parametrizations (1.9) with s-dependent parameters as solutions for EE [2] yields lower accuracy of the QCD evolution reproduction as compared with parametrization (1.6), especially for gluons. That means that Q^2-evolution with the use of (1.9) has a considerably distorted form of x-dependence of quark and gluon distribution functions. In other words, the use of (1.9) to prescribe IC leads, at $Q^2=Q_0^2$, to the expressions which cannot be presented in the form of (1.9). It makes more preferable to prescribe IC in the form of (1.6), because of randomness in selecting Q_0^2.

Non-fixed evolution parameters $\{a^{(1)}\}$ are determined through the conditions of the best QCD evolution reproduction, what is achieved by the minimization:

$$\phi^2(\{a^{(1)}\}) = \sum_D \sum_n \int_0^{s_{max}} ds \left(\frac{\langle D^{QCD}(s)\rangle_n - \langle D(\{a(s)\})\rangle_n}{\langle D^{QCD}(s)\rangle_n}\right)^2, \quad (1.10)$$

where D takes values of V, Σ and G, and $\langle D^{QCD}(s)\rangle_n$ are calculated by the formulae of the QCD evolution of the moments, the initial values of the evolution being equal to the moments from (1.6) at $Q^2=Q_0^2$: $\langle D^{QCD}(0)\rangle_n = \langle D(\{a^{(0)}\})\rangle_n$. Both in the leading order (LO) and in the next-to-leading order (NO) in α_s evolution formulae calculated in the $\overline{MS}$ scheme [7] were used, and in NO we employed the evolution which retained the parton connection (1.1), (1.2) between the distribution and structure functions [8]. The value of s_{max} determines the s-range where we searched for the solution. In this paper we took $s_{max}=1$ what imposed the condition of the correct QCD behaviour in a quite wide Q^2-region (e.g., at $Q_0^2=10$ GeV^2 and $\Lambda=100$ MeV in LO $s_{max}=1$ corresponds to $Q^2_{max}=1.4\cdot10^6$ GeV^2). Besides, the parameter $r^{(1)}$ is fixed in QCD by the condition of the correct behaviour at $x\to1$:

$$r^{(1)} = \frac{16}{33-2f}, \quad (1.11)$$

what ensured the correct QCD behaviour of the moments at $n\to\infty$. This allows us to use in (1.10) only the sum from $n=2$* to a certain $n=n_{max}$ (we used $n_{max}=20$). The obtained parameters $\{a^{(1)}\}$ provide high accuracy of the QCD evolution reproduction both for the moments and for the distribution functions (1.6) (see Section 2).

Now that $\beta_G^{(0)}$ is known, one could find Λ through comparison of the s-evolution obtained by minimization of (1.10) with the experimental Q^2-dependence of structure functions, by minimization of

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* For expressions (1.6) at $\beta_q \neq \beta_G$ the longitudinal momentum remains approximately $\langle\Sigma(Q^2)\rangle_2 + \langle G(Q^2)\rangle_2 = 1 - K(Q^2)$. This weak violation $(K(Q^2) \leq 0.1)$ can be eliminated by introducing the gluon Bose-condensate: $\tilde{G}(x,Q^2) = G(x,Q^2) + K(Q^2)\delta(x)/x$, where $\tilde{G}(x,Q^2)$ is a new gluon distribution and $G(x,Q^2)$ is given by (1.6). It is obvious that this modification does not change the form of distributions (1.6) at $x>0$, and excludes the 2nd gluon moment from the sum in (1.10).

$$\chi^2(\Lambda) = \sum_k \left(\frac{F_i^{exp}(x_k, Q_k^2) - F_i(x_k, \{a(s(Q_k^2,\Lambda))\})}{\Delta F_i^{exp}(x_k, Q_k^2)} \right)^2 \Bigg|_{Q_k^2 \neq Q_0^2} . \quad (1.12)$$

In order to find still unknown gluon parameter $\beta_G^{(0)}$ in (1.10) and (1.12) a joint analysis of EE and experimental data is necessary. Besides, when there is not much data at the same $Q^2 = Q_0^2$, it is better to determine also other $(a^{(0)}, \tilde{x}^{(0)}, \beta_q^{(0)})$ initial parameters and Λ in the totality of the experimental values of the structure function. This is achieved by minimizing the sum of (1.8), (1.12) and the additional condition (1.10)(for satisfying QCD evolution) with a certain weight w:

$$\psi^2(\{a^{(\ell)}\},\Lambda) = \chi(\{a^{(\ell)}\},\Lambda) + w\phi(\{a^{(\ell)}\}) = \quad (1.13)$$

$$= \sum_k \left(\frac{F_i^{exp}(x_k, Q_k^2) - F_i(x_k, \{a(s(Q_k^2,\Lambda))\})}{\Delta F_i^{exp}(x_k, Q_k^2)} \right)^2 + w\phi^2(\{a^{(\ell)}\}) .$$

The value of w should be large enough, so that ϕ^2 was small in the obtained minimum (1.13). But if w is too large, the minima $\phi^2(\{a^{(\ell)}\})$ connected with the selected parametrization form will affect excessivly the selection of the initial parameters $\{a^{(0)}\}$. Therefore we have selected $w = 6.25\, N_p/N_m$, where N_p is a number of experimental points in (1.13) and $N_m = \sum_D \sum_n 1$ is a total number of moments in (1.10).

One often employs a simpler non-singlet (NS) approximation where gluon distributions are neglected for large x (usually $x > 0.25$), what gives NS evolution for $\Sigma(x,Q^2)$ also. Since non-singlet and singlet components of the structure function now have the same evolution, only a term with $D = V$ is left in $\phi^2(\{a^{(\ell)}\})$, here $\langle V \rangle_n \Rightarrow \langle NS = \frac{F_i}{x} \rangle_n$.

The approximation of valence quarks is also used, i.e., besides gluon distributions sea quark distributions are neglected as well, thus only valence quark distribution is left in $F_i(x,Q^2)$. The main characteristic feature of this NS approximation is the use of normalized functions for parametrization of $F_i(x, Q^2)$, because valence quarks obey the following sum rules:

$$\int_0^1 dx\, u_v(x, Q^2) = 2; \quad \int_0^1 dx\, d_v(x, Q^2) = 1 . \quad (1.14)$$

Taking account of (1.14) in (1.1)-(1.4), we used the following parametrizations:

$$(F_2^{\mu p}(x, Q^2), F_2^{\mu N}(x, Q^2), F_2^{\nu(\bar\nu)}(x, Q^2)) = (1, \frac{5}{6}, 3) \times V(x, Q^2) , \quad (1.15)$$

where $V(x, Q^2)$ is given by (1.6a) and is a function normalized to 1.

2. THE RESULTS OF QCD ANALYSIS

We have analysed on the basis of the method described in Section 1 the experimental data on deep inelastic scattering of μ-mesons on carbon (BCDMS)/9/, hydrogen (EMC)/10/ and iron (EMC)/11/ corresponding to $R = \sigma_L/\sigma_T = 0$, as well as the data on $\nu(\bar\nu)$ scattering on iron (CDHS)/12/. The analysis has been aimed at determining the value of the parameter Λ which was best for description of the experimental data by the structure functions reproducing QCD evolution (i.e., by the minimum (1.13)).

Table 1

The results of the QCD analysis of the BCDMS and EMC data ($R = 0$, $x = 0.35$-0.65, $Q^2 = 27$-$200\ GeV^2$) in LO ($\Lambda_{\overline{MS}}$ in MeV)

Variant \ Data	BCDMS	EMC_{H_2}	EMC_{Fe}
LO(NS, valence)	$\chi^2/\nu = 140/65$ $\Lambda = 139$	$\chi^2/\nu = 58/34$ $\Lambda = 303$	$\chi^2/\nu = 95/58$ $\Lambda = 56$
LO (NS)	$\chi^2/\nu = 72/65$ $\Lambda = 154$	$\chi^2/\nu = 54/34$ $\Lambda = 286$	$\chi^2/\nu = 78/58$ $\Lambda = 49$
LO	$\chi^2/\nu = 72/64$ $\Lambda = 216^{+127}_{-95}$	$\chi^2/\nu = 55/33$ $\Lambda = 411^{+289}_{-211}$	$\chi^2/\nu = 76/57$ $\Lambda = 102^{+146}_{-76}$

Table 1 shows values of χ^2 (here and below only statistical errors are presented), degrees of freedom ν and values of the parameter Λ obtained in LO in non-singlet approximations (LO (NS, valence), LO (NS)) and with allowance for sea quarks and gluons (LO). To compare the data of BCDMS and EMC groups we used the data in the same range of variables: $0.35 \le x \le 0.65$ and $27\ GeV^2 \le Q^2 \le 200\ GeV^2$ what corresponds to the BCDMS variation interval. The values of Λ coincide within the error limits in LO. However, we can say that Λ tends to decrease as the atomic weight grows (from hydrogen to iron). The EMC data have been obtained in a wider range of x and Q^2 values. To compare our results with the results of papers[10,11] we have analysed the EMC data in the region $x \geq 0.25$ and $Q^2 = 4.5$–200 GeV^2 in LO (NS) approximation (for comparison we give in brackets the corresponding values from[10] and [11]:

$$\mathrm{EMC}_{H_2}:\ \chi^2/\nu = 96/67\ (97/66),\quad \Lambda = 70^{+58}_{-36}\ \mathrm{MeV}\ (110^{+58}_{-46}\ \mathrm{MeV}),$$

LO (NS)

$$\mathrm{EMC}_{Fe}:\ \chi^2/\nu = 156/103\,(211/102),\ \Lambda = 584^{+128}_{-114}\ \mathrm{MeV}\ (122^{+22}_{-20}\ \mathrm{MeV}).$$

The hydrogen results are in good agreement. As far as iron is concerned, we have obtained a noticeably larger Λ and a much better χ^2/ν (almost the same as for hydrogen). It is interesting that when fixing Λ = 122 MeV χ^2 worsens, but remains better than in paper [11]:

$$\mathrm{LO\ (NS)\ EMC}_{Fe}:\ \chi^2 = 181\ (211),\quad \Lambda = 122\ \mathrm{MeV\ (fixed)}.$$

We think Λ = 122 MeV is questionable in case of iron since values of the structure function $F_2(x,Q^2)$, according to the EMC data, differ noticeably for iron and for hydrogen (the larger is x, the more noticeable is the difference).

We have analysed the EMC data for various intervals of x and Q^2 (see Table 2). Table 2 contains the data of both LO and NO analysis. In both cases the sea and gluon distributions have been taken into account, because NS approximation is not applicable at small x. The obtained results will be discussed below. Noteworthy is a rather high accuracy of the QCD evolution reproduction obtained in our analysis. Fig.1 shows, as an example, the ratio of the approximated QCD evolution to an accurate one((1.6) with parameters (1.7)). We have got these curves when processing the BCDMS data. The figure makes it obvious that the correct QCD behaviour of a great number of moments predetermines the correct behaviour of the structure functions: deviation from the QCD evolution does not exceed few per cent in the region of the processed experimental data. Now we proceed to the analysis of the results.

NS Approximation

Results in Table 1 show that the use of only valence quarks satisfying the normalization conditions (1.14) does not allow, generally speaking, to adequately describe the experimental data even for $x \geq 0.35$. Sea quarks added to the NS approximation change slightly the value of Λ, but improve essentially the description (for the BCDMS data, for example, χ^2 decreased

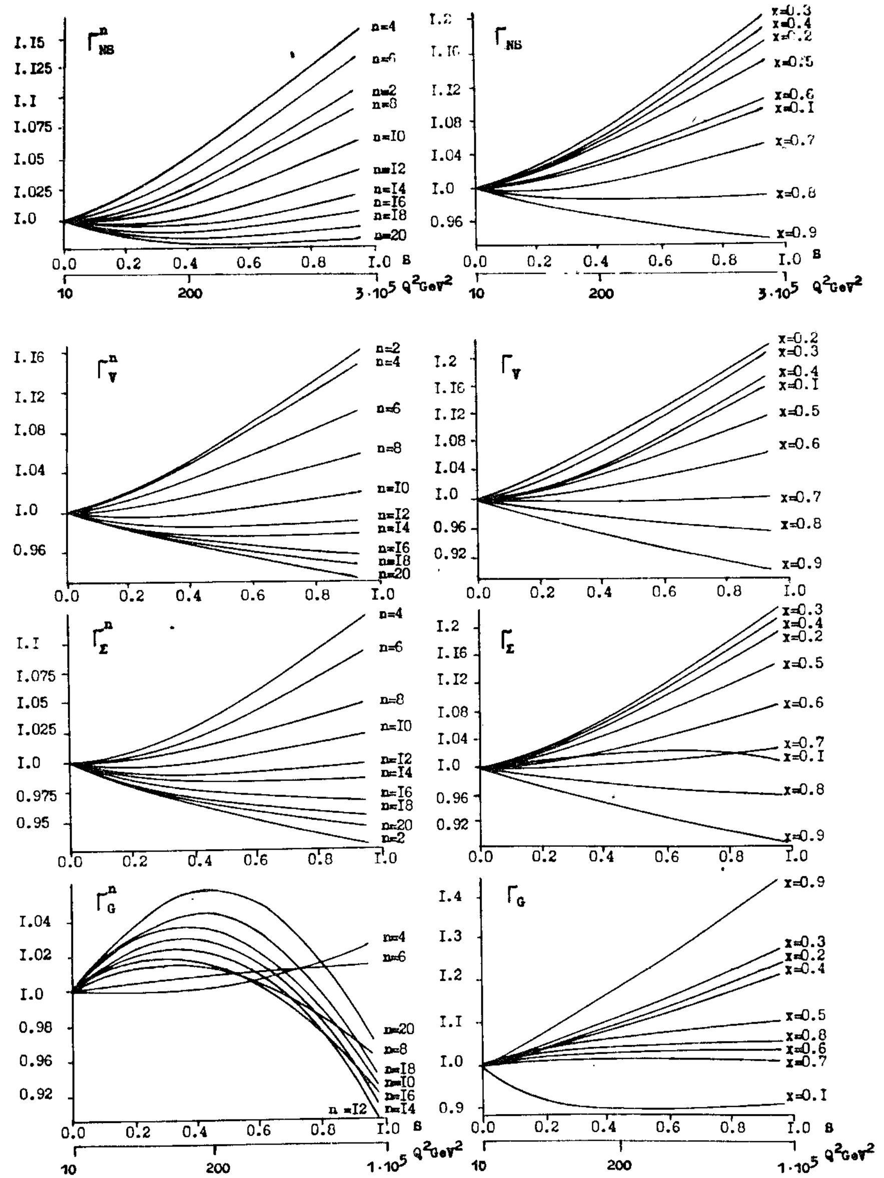

Fig.1. The ratio of distributions $r_D = D(x,\{a(s)\})/D^{QCD}(x,s)$ (calculations of $D^{QCD}(x,s)$ by the approximated method [3]) and moments $r^n_D = \langle D(\{a(s)\})\rangle_n / \langle D^{QCD}(s)\rangle_n$ when processing the BCDMS data in LO: D = NS is NS approximation, $D = V, \Sigma, G$ – with allowance for sea quarks and gluons.

Table 2

The results of the QCD analysis of the EMC data
($R = 0$, $\Lambda_{\overline{MS}}$ in MeV)

Variant \ Data		EMC_{H_2}	EMC_{Fe}
$x=0.03-0.65$, $Q^2=27-200$ GeV^2	LO	$\chi^2/\nu = 100/52$ $\Lambda = 477^{+250}_{-198}$	$\chi^2/\nu = 177/87$ $\Lambda = 26^{+67}_{-24}$
$x=0.35-0.65$, $Q^2=10-200$ GeV^2	LO	$\chi^2/\nu = 74/48$ $\Lambda = 140^{+109}_{-73}$	$\chi^2/\nu = 102/76$ $\Lambda = 676^{+138}_{-129}$
	NO	$\chi^2/\nu = 75/48$ $\Lambda = 136^{+95}_{-62}$	$\chi^2/\nu = 106/76$ $\Lambda = 527^{+83}_{-83}$
$x=0.03-0.65$, $Q^2=10-200$ GeV^2	LO	$\chi^2/\nu = 243/95$ $\Lambda = 161^{+90}_{-68}$	$\chi^2/\nu = 282/141$ $\Lambda = 631^{+85}_{-80}$
	NO	$\chi^2/\nu = 243/95$ $\Lambda = 147^{+66}_{-55}$	$\chi^2/\nu = 284/141$ $\Lambda = 516^{+56}_{-56}$
$x=0.03-0.65$, $Q^2=2.5-200$ GeV^2	LO	$\chi^2/\nu = 435/131$ $\Lambda = 398^{+142}_{-114}$	$\chi^2/\nu = 336/170$ $\Lambda = 559^{+84}_{-67}$

by the factor of 2), thus leading practically to the same values of χ^2 as in case with taking into account sea quarks and gluons. Our analysis leads to the following conclusion: the NS approximation can describe the experimental data in quite an appropriate way, but the value of Λ in this case will be somewhat lower regarding the analysis where sea and gluon distributions are taken into account.

NO

It is known that at the same Λ calculations in NO lead to a stronger Q^2-dependence of the structure functions. Hence, the observed Q^2-dependence will be achieved in NO at smaller Λ

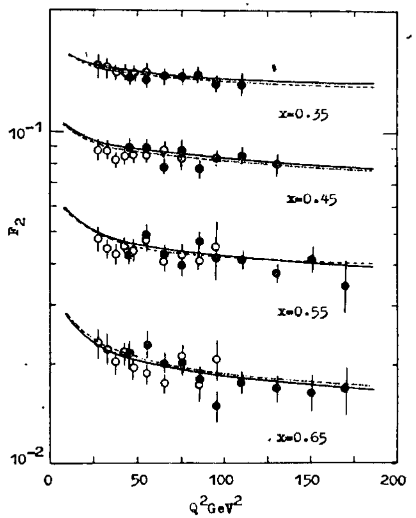

Fig.2. The structure function $F_2(R=0)$ of μ-meson scattering on carbon (BCDMS): ——— LO (Λ = 216 MeV), - - - - - NO (Λ = 190 MeV).

than in LO. Indeed, NO calculations (see Table 2) somewhat reduce Λ, while χ^2 remains practically unchanged. This is valid for the BCDMS data also (see Fig.2):

NO BCDMS: $\chi^2/\nu = 72/64$, $\Lambda = 190^{+98}_{-78}$ MeV.

(See Table 1 for the corresponding values of the BCDMS data in LO). Furthermore, when NO is taken into account, the values of Λ obtained from processing in various x ranges draw together (see Table 2):

$$EMC_{H_2}: \quad \Delta\Lambda^{NO} = 11 \text{ MeV} < \Delta\Lambda^{LO} = 21 \text{ MeV},$$

$$EMC_{Fe}: \quad \Delta\Lambda^{NO} = 11 \text{ MeV} < \Delta\Lambda^{LO} = 45 \text{ MeV},$$

where $\Delta\Lambda = |\Lambda\ (x = 0.03-0.65) - \Lambda\ (x = 0.35-0.65)|$ (Q^2 = 10-200 GeV). Thus, NO is more adequate for the description of the data from various x ranges.

Let us return to Tables 1,2. Up to now we have explained difference of results obtained in LO from processing in intervals different with respect to x and identical with respect to Q^2: allowance for NO eliminates the difference. Now we would like to give our reasons for difference in value of Λ for different targets and different Q^2 intervals chosen for the analysis (see Figs.3,4).

In the above given results we have not taken account of the following effects:

A. Contribution of Higher Twists (HT)

If HT taken into account, the terms appear in the structure function $F_i(x,Q^2)$, which are presented as a $1/Q^2$ power series, the coefficients of which cannot be calculated within the PT framework. Parametrizations used for this purpose reflect the

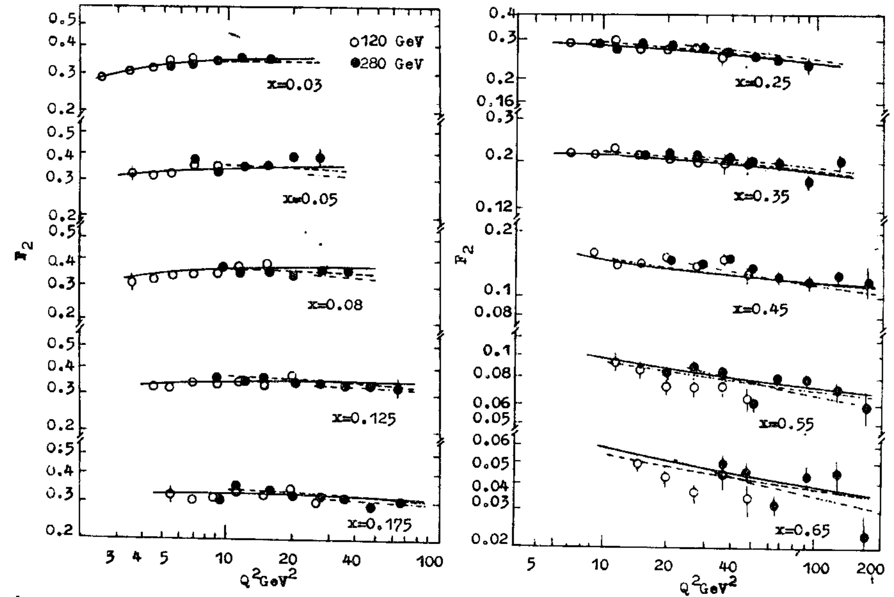

Fig.3. The structure function $F_2(R=0)$ of μ-meson scattering on hydrogen (EMC) in LO: ——— – $Q^2 = 2.5$-200 GeV^2 (Λ =398 MeV), - - - - - 10-200 GeV (Λ = 147 MeV), –·–·– – $Q^2 = 27$-200 GeV^2 (Λ = 477 MeV).

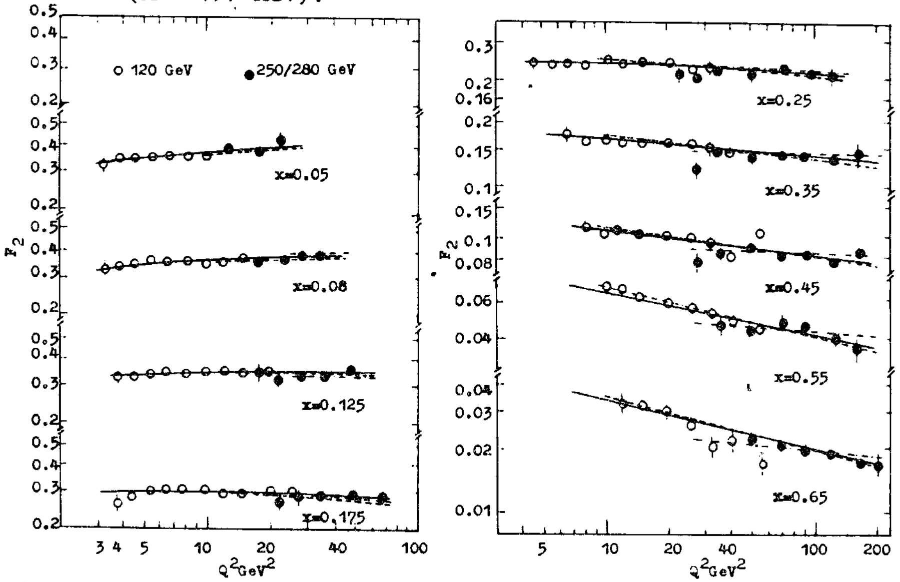

Fig.4. The structure function $F_2(R=0)$ of μ-meson scattering on iron (EMC) in LO: ——— – $Q^2 = 2.5$-200 GeV^2 (Λ=559 MeV), - - - - – $Q^2 = 10$-200 GeV^2 (Λ = 631 MeV), –·–·– – $Q^2 = 27$-200 GeV^2 (Λ = 26 MeV).

fact that HT effects decrease when $x \to 0$ [13]:

$$F_i(x, Q^2) = F_i^{QCD}(x, Q^2)\left(1 + \frac{a(x)}{Q^2} + \frac{b(x)}{Q^4} + \dots\right), \qquad (2.1)$$

where $F_i^{QCD}(x, Q^2)$ is a structure function with the QCD Q^2-evolution in accordance with EE, and $a(x)$, $b(x), \dots \sim 1/(1-x)$ or $x/(1-x)$, etc.

B. Nucleon Interaction in Nucleus

This interaction leads to a possible scattering not only on one nucleon, but also on 2-, 3-nucleon systems, etc. [14]:

$$F_i^A(x, Q^2) = \sum_{k=1}^{A} p_k F_i^{(k)}(x, Q^2), \qquad (2.2)$$

where p_k is a probability of interaction with k nucleons, $F_i^{(k)}(x, Q^2)$ is the structure function of this interaction. Nuclear effects grow as x increases and lead, in part, to the experimentally observed values of $F_i^A(x, Q^2)$ at $x > 1$, because the value of x in $F_i^{(k)}(x, Q^2)$, determined through the nucleon mass M, lies in the interval $0 \le x = Q^2/2M\nu \le k$. We would like to note, that even if HT are not taken into account, but (2.2) contains terms with $k > 1$, then the QCD evolution $F_i^A(x, Q^2)$ in the form of (2.2) differs from the evolution in the approximation of free nucleons (i.e., from our expressions (1.1), (1.2) corresponding to $p_k = \delta_{\ell k}$)) due to the evolution $F_i^{(k)}(x, Q^2)$ in various ranges with respect to $x \in [0, k]$, whereupon various k produce different Q^2-dependence at one and the same x. By substituting x_k for x/k and $F_i(x_k, Q^2)$ for $F_i^{(k)}(x, Q^2)$, it is suitable to consider the evolution $F_i^{(k)}$ as a usual evolution ($x \in [0, 1]$) for $F_i(x_k, Q^2)$ which, as is known, makes F_i grow at $x \le 0.2$ and fall at $x \ge 0.2$ with the increase of Q . Then at $x \approx 0.3$, for example, the fall of the main term $F_i^{(1)}(x, Q^2)$ in (2.2) is slowed down by other functions with $k > 1$, since the corresponding $x_k = 0.3/k < 0.2$. Here the following general statement is valid: allowance for the nucleon interaction (if there are no HT) weakens the Q^2-dependence of F_i^A (the larger A – the weaker dependence [14]). As a result, the value of Λ on the nucleus must be smaller than that on the nucleon, when processing the data in the approximation of free nucleons [14,15]. This can be seen in Tables 1,2 for $Q^2 \simeq 27$-200 GeV^2 where HT effects are quite weak: $\Lambda_H > \Lambda_C > \Lambda_{Fe}$. Additional points with smaller Q^2 require that power corrections should be taken into consideration, and the above conclusion is not applicable any more.

Apart from the considered effects disguising the QCD evolution of structure functions, one more thing affects greatly the analysis, that is

C. Choice of $R = \sigma_L/\sigma_T$

A wrong R may distort considerably the form of $F_2(x,Q^2)$, since for its extraction from the experimental cross section $d\sigma/dxdQ^2$ one should know also $F_1(x,Q^2)$ or

$$R = \frac{\sigma_L}{\sigma_T} = [-2xF_1 + (1+4x^2M^2/Q^2)F_2]/2xF_1 . \qquad (2.3)$$

It is difficult to study R in an experimental way. Usually $0 \le R \le 0.2$ is used, here the values of Λ, corresponding to $R = 0$ and $R = 0.2$, differ from each other by several times [10,11]. In the parton model

$$R^{P.M.} = 4x^2M^2/Q^2 \qquad (2.4)$$

appears when the Callan-Gross relation $(2xF_1 = F_2)$ is used. It is interesting to note that the results of the CDHS data processing with the help of the structure function of $\nu(\bar\nu)$ scattering on iron $F_2^{\nu(\bar\nu)}(x,Q^2)$ [12], extracted from $d\sigma/dxdQ^2$ using this relation, lead to Λ with weaker dependence upon the chosen Q^2-range (see Table 3).

Table 3

The results of the QCD analysis of the CDHS data $(2xF_1 = F_2)$ in LO ($\Lambda_{\overline{MS}}$ in MeV)

x=0.35-0.7	x=0-0.7	x=0-0.7	x=0-0.7
Q^2=27-200 GeV^2	Q^2=27-200 GeV^2	Q^2=10-200 GeV^2	Q^2=1-200 GeV^2
χ^2/ν =13/15	χ^2/ν =17/21	χ^2/ν =34/35	χ^2/ν =71/58
Λ =754	Λ =794	$\Lambda =528^{+389}_{-295}$	$\Lambda =773^{+72}_{-132}$

Now one more thing about the QCD evolution itself:

D. Allowance for the Threshold Effects

We have analysed the data of various experimental groups and determined the parameter Λ proceedings from the agreement between Q^2-dependence and EE which correspond to a fixed number of quark flavours $(f=4)$. However, these data belong to the region with heavy quark (c, b,...) production thresholds. If these thresholds are taken into account, the fixed f and Λ are replaced by their "effective" values which vary considerably near the threshold and slightly - in the intermediate region. The situation is approximately as follows: from the production threshold of a quark q_f to the production threshold of another quark q_{f+1} an f-quark evolution takes place, with the corresponding value of Λ_f. Therefore, in order to determine Λ_f, one should employ the data in a certain region of Q^2 (e.g., $f=4$ correspons to $Q^2 \sim 5$-100 GeV^2).

Now we shall try to perform the analysis with allowance for all the above-mentioned comments. Keeping them in mind, we have processed the EMC data from the $x = 0.03$-0.175 region, as the nuclear effects (B) and HT (A) are considerably weaker at small x. Our use of parton formulae (1.1), (1.2) for the analysis of the data with $R = 0$ is also justified in case of small x (due to (2.4)). Besides, practically all the data corresponding to this x range lie in the 4-quark evolution region. What value of Λ can we expect in this analysis? Up to now the results obtained from the processing of the EMC H_2 data (no nuclear effects) in the range Q^2=27-200 GeV^2 (HT effects are weakened) are most reliable. However, the corresponding value was affected by the data with Q^2>100 GeV^2, i.e., the 4-quark evolution described also the 5-quark evolution region, where Q^2-dependence of the structure functions is weaker ($f+1$-quark evolution is weaker than the f-quark evolution). Thus, the obtained $\Lambda_4 \sim 400$ MeV is too low, and the correctly chosen Q^2-range (if there are no nuclear and HT effects) must provide a larger value of Λ_4 what is proved by the results in Table 4. The obtained values of Λ_4 for hydrogen and iron are practically the same. On taking account of NO, one observes a greater reduction of Λ as compared with the cases considered earlier. This fact reflects a stronger influence of NO just in the region of small x (see also Fig.5).

CONCLUSION

In this paper we have carried out a QCD analysis of the structure functions $F_2(x,Q^2)$ of deep inelastic μ-meson (BCDMS, EMC) and $\nu(\bar\nu)$ (CDHS) scattering. We have used the parametrizations based on the phenomenological model [6] as the initial values of evolution equations for the considered structure functions. These parametrizations have some advantages as compared with usual empiric parametrizations (1.9). Method [5] has been applied for the solution of evolution equations.

Table 4

The results of the QCD analysis of the EMC data
($R = 0$, $\Lambda_{\overline{MS}}$ in MeV)

Variant \ Data		EMC_{H_2}	EMC_{Fe}
$x=0.03-0.175$, $Q^2=2.5-65$ GeV^2	LO	$\chi^2/\nu = 181/59$	$\chi^2/\nu = 144/63$
		$\Lambda = 876^{+180}_{-265}$	$\Lambda = 857^{+238}_{-219}$
	NO	$\chi^2/\nu = 176/59$	$\chi^2/\nu = 144/63$
		$\Lambda = 530^{+146}_{-228}$	$\Lambda = 536^{+123}_{-109}$

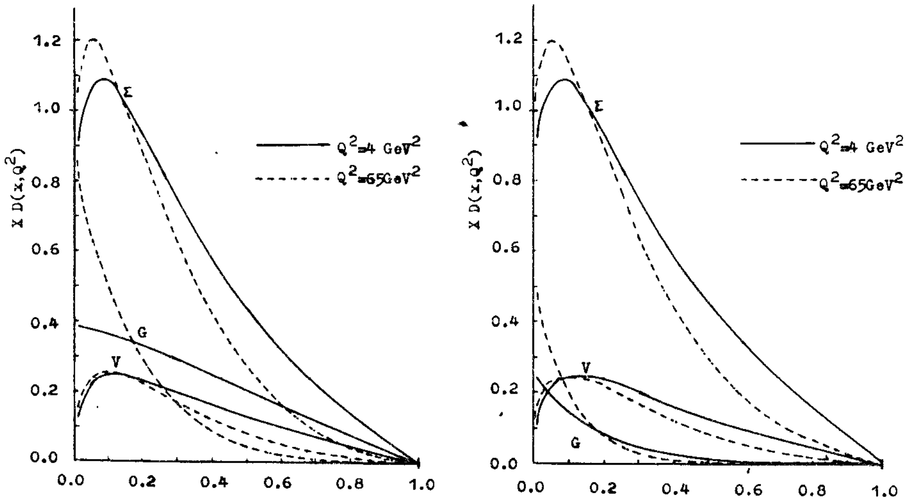

Fig.5. Distributions $xD(x, Q^2)(D = V, \Sigma, G)$ obtained when processing the EMC data on hydrogen ($R=0$, $x=0.03-0.175$, $Q^2 = 2.5-65$ GeV^2): a) LO; b) NO.

The following conclusions can be drawn from the analysis:

a) NS approximation for large $x > 0.25$ allows a good description of the experimental data, but yields too low values of Λ.

b) Allowance for NO leads to the reduction of Λ (the smaller is x, the stronger is reduction). For example, when analysing $F_2^{\mu p}(x, Q^2)$ for the interval $0.35 \le x \le 0.65$, the value of Λ decreases by 3 per cent, for the interval $0.03 \le x \le 0.175$ the reduction amounts to ~40 per cent.

c) The extraction of Λ from the experimental data, in our opinion, requires first of all allowance for threshold effects, apart from allowance for higher twists and nuclear effects and the use of a correct value of $R=\sigma_L/\sigma_T$. The value of Λ has a sense of the constant parameter in the region of much larger Q^2 than those attained in the experiment. We think that the value of Λ_4 corresponding to the commonly used 4-quark evolution must be noticeably higher than the popular figure ~100 MeV.

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Исаев П.С., Иванов Ю.П. E2-82-794
КХД анализ данных по глубоконеупругому рассеянию лептонов

Проведен КХД анализ данных по глубоконеупругому рассеянию лептонов /по данным групп BCDMS, EMC, CDHS/ в лидирующем и в следующем за ним порядках по константе связи α_s. Рассмотрено влияние кварков моря и глюонов. Обсуждается зависимость получаемого значения Λ от эффектов твистовых поправок, учета ядра мишени и порогов рождения тяжелых кварков.

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Isaev P.S., Ivanov Yu.P. E2-82-794
QCD Analysis of Deep Inelastic Lepton Scattering Data

The QCD analysis of deep inelastic scattering of leptons (according to the data of BCDMS, EMC, CDHS groups) has been performed in the leading and next-to-leading orders. The influence of sea quarks and gluons has been considered. The dependence of the obtained values of Λ upon the effects of twist corrections, allowance for the target nucleus and heavy quark production threshold has been discussed.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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