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THE PROPER TIME, SPATIAL DISTANCES, AND CLOCK SYNCHRONIZATION IN THE LOCALLY ANISOTROPIC SPACE-TIME

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As is known [1], the Mach principle is only partially reflected in the general theory of relativity. At each point of a space, the acceleration of a freely falling reference frame is given by the general distribution and motion of matter. Yet, if we try to accelerate a probing body relative to such a frame, its inertia turns out to be in no way associated with the distribution of matter beyond the body. In contrast to this, in the general relativistic theory of the locally anisotropic space-time and gravitation, the distribution and motion of matter manifest themselves in a freely falling reference frame. The anisotropy of a space-time in such a reference frame, as well as the inertia of a probe, depend on their localization and motion relative to the external matter. In this sense, the Mach principle is better reflected in the general relativistic theory of the locally anisotropic space-time and gravitation.

The special relativistic theory of the locally anisotropic space-time has been developed in refs. [2,3], the general theory being outlined in [4,5]. The underlying idea was that the space-time has not the Riemannian, but the more general Finslerian [6] geometry, the metric of the curved, locally anisotropic space of events being given by the formula [4]

$$ds = \left[\frac{(\nu_i\, dx^i)^2}{g_{ik}\, dx^i dx^k}\right]^{r/2} \sqrt{g_{ik}\, dx^i dx^k}\,, \qquad (1)$$

where $g_{ik}(x)$ is the field of the Riemannian metric tensor; $r = r(x)$, the scalar field that determines the magnitude of the local anisotropy; $\nu_i = \nu_i(x)$, the vector field of locally preferred directions in the given space-time, and $\nu_i \nu^i = g_{ik}\nu^i\nu^k = 0$. The flat anisotropic space with the metric [2]

$$ds = \left[\frac{(dx_0 - \vec{\nu}\, d\vec{x}\,)^2}{dx_0^2 - d\vec{x}^{\,2}}\right]^{r/2} \sqrt{dx_0^2 - d\vec{x}^{\,2}} \qquad (2)$$

is tangential to (1) at a given point. Here r and $\vec{\nu}$ are the values of the corresponding fields at a given space-time point. The Riemannian and pseudo-Euclidean space-time turn out to be the particular cases of spaces (1) and (2) at $r = 0$.

In Einstein's theory of gravitation, the field of the Riemann metric tensor is given by the distribution and motion of matter. In the general relativistic theory of the locally anisotropic space-time and gravitation it is natural to proceed from the assumption that the motion and distribution of matter determine not only $g_{ik}(x)$, but also the local space-time anisotropy, i.e., the fields $r(x)$ and $\nu_i(x)$. The second-order field equations, which relate $g_{ik}(x)$, $r(x)$, $\nu_i(x)$ and their derivatives to the motion and distribution of matter, have been found in /6/. Below, expressions for some of the observables will be obtained.

Formula (1) determines the metric of the locally anisotropic space of events referred to an arbitrary reference frame x^i. There arises the question of how one can determine the apparent spatial distances and time intervals from coordinate increments dx^i.

Let us first establish the relation between the proper time at a given reference point /7/ and the coordinate x^0. To do this, we consider, as usual, two infinitely close events that take place at the given point. Such events will have the same values of spatial coordinates. Assuming that in (1) $dx^1 = dx^2 = dx^3 = 0$ and bearing in mind that $ds = c d\tau$, we find

$$d\tau = \frac{1}{c}\left[\frac{\nu_0^2}{g_{00}}\right]^{r/2} \sqrt{g_{00}}\, dx^0 . \qquad (3)$$

In order to determine the spatial distance between two infinitely close reference points, we call attention to the fact that the equation of propagation of a light signal in the locally anisotropic space of events has the same form as it has in the Riemannian space-time of the general theory of relativity

$$g_{ik}\, dx^i dx^k = 0 . \qquad (4)$$

This enables us to apply a procedure similar to that used in ref. /8/ in the general theory of relativity to determine the spatial distance.

Consider two infinitely close reference points A and B with space coordinates x^α and $x^\alpha + dx^\alpha$, respectively. A light signal sent from point B at the moment of the coordinate time $x^0 + dx^0_{(1)}$ reaches point A at the moment x^0, and, having been reflected, will return to point B at the moment $x^0 + dx^0_{(2)}$. Proceeding from (4), one can calculate the coordinate time interval between the sending and arrival of the signal

$$(x^0 + dx^0_{(2)}) - (x^0 + dx^0_{(1)}) = \frac{2}{g_{00}}\sqrt{(g_{0\alpha}g_{0\beta} - g_{\alpha\beta}g_{00})\,dx^\alpha dx^\beta} . \qquad (5)$$

The corresponding interval of the proper time is found using (3)

$$\Delta\tau = \frac{2}{c}\left[\frac{\nu_0^2}{g_{00}}\right]^{r/2} \sqrt{\left(-g_{\alpha\beta} + \frac{g_{0\alpha}g_{0\beta}}{g_{00}}\right) dx^\alpha dx^\beta} . \qquad (6)$$

The spatial distance $d\ell$ between A and B is equal to $c\Delta\tau/2$ and we finally obtain

$$d\ell^2 = \tilde{\gamma}_{\alpha\beta}\, dx^\alpha dx^\beta , \qquad (7)$$

where

$$\tilde{\gamma}_{\alpha\beta} = \left[\frac{\nu_0^2}{g_{00}}\right]^{r} \gamma_{\alpha\beta} , \qquad (8)$$

$$\gamma_{\alpha\beta} = -g_{\alpha\beta} + \frac{g_{0\alpha}\, g_{0\beta}}{g_{00}} . \qquad (9)$$

It is interesting to note that the space geometry at the given reference frame turns out to be Riemannian, determined by the quadratic form (7) with the metric tensor (8), whereas the geometry of the space of events (1) is Finslerrian.

The contravariant three-dimensional metric tensor $\tilde{\gamma}^{\alpha\beta}$ corresponding to the metric (7) can readily be written if we take into account (8), (9) and the relation $-g^{\alpha\beta}(-g_{\beta\gamma} + g_{0\beta}g_{0\gamma}/g_{00}) = \delta^\alpha_\gamma$.

$$\tilde{\gamma}^{\alpha\beta} = -\left[\frac{\nu_0^2}{g_{00}}\right]^{-r} g^{\alpha\beta} . \qquad (10)$$

The element of the spatial volume is given by $dv = \sqrt{\tilde{\gamma}}\, dx^1 dx^2 dx^3$, where $\tilde{\gamma}$ is the determinant of the spatial metric tensor (8).

Let us now turn the question of synchronization of the coordinate clocks (i.e., clocks showing the coordinate time x^0) that are placed at two infinitely close reference points A and B. In other words, we wish to determine Δx^0 the difference between the times told by these neighbouring clocks corresponding to simultaneous events at A and B. Since the equations of propagation of a light signal in the locally anisotropic space and in the Riemannian space are of the same form, the determination of the simultaneity of events and the algorithm for calculating Δx^0 in the locally anisotropic space literally repeat the method for clock synchronization /8/ in the general theory of relativity. Namely, the time told by the clock at point B, which is between the moments of signal sending and its arrival at B, is simultaneius with the moment x^0 at A. Thus

$$\Delta x^0 = \frac{1}{2}(dx^0_{(2)} + dx^0_{(1)}) = -\frac{g_{0\alpha}\, dx^\alpha}{g_{00}} . \qquad (11)$$

Finally, we find an expression for the proper time interval between events taking place at the neighbouring points of space. If one event has occurred at A at the coordinate time moment and the other at x^0 at the moment $x^0 + dx^0$, then the coordinate time interval between these two events at B is equal to the difference between $x^0 + dx^0$ and the moment $x^0 - \frac{g_{0\alpha}}{g_{00}} dx^\alpha$ which is simultaneous at B to the moment x^0 at A. Multiplying this difference by $[\nu_0^2/g_{00}]^{r/2}\sqrt{g_{00}}/c$ we find, according to (3), the proper time interval

$$d\tau = \frac{1}{c}\left[\frac{\nu_0^2}{g_{00}}\right]^{r/2} \frac{g_{0i}\, dx^i}{\sqrt{g_{00}}} . \qquad (12)$$

The relation (12) extends formula (3) to the case when dx^1, dx^2, $dx^3 \neq 0$. Dividing dx^α by (12), we may write an expression for the three-dimensional particle velocity measured in the proper time determined from the clocks synchronized along the particle trajectory.

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Собственное время, пространственные расстояния и синхронизация часов в локально-анизотропном пространстве-времени

Получены соотношения, устанавливающие связь между финслеровой метрикой четырехмерного пространства-времени и метрикой трехмерного пространства, а также определены одновременность событий и способ синхронизации часов. Метод вывода выражений для наблюдаемых величин и синхронизации часов основан на обмене световыми сигналами и является следствием конкретной финслеровой структуры пространства-времени, которая приводит к такому же уравнению для распространения светового сигнала, что и общая теория относительности. В рамках общей релятивистской теории локально анизотропного пространства-времени и гравитации получены выражения для наблюдаемых величин: собственного времени, пространственного расстояния и промежутка собственного времени между событиями в соседних точках. Конкретная форма для метрики локально анизотропного пространства-времени, использованная в работе, приводит к разумным выражениям наблюдаемых величин теории через значения полей, определяющих локальную анизотропию.

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The Proper Time, Spatial Distances, and Clock Synchronization in the Locally Anisotropic Space-Time

The relations between the Finslerian metric of four-dimensional space-time and the metric of three-dimensional space is obtained and the simultaneity of events and the algorithm for synchronizing clocks are determined. The method for deriving the expressions of the observable quantities and synchronizing clocks is based on light-signal exchange and is a consequence of the specific Finslerian structure of space-time which gives the same equation of light-signal propagation as the general relativity. Within the framework of the general relativistic theory of the locally anisotropic space-time and gravitation, expressions have been obtained for the observable values: the proper time, the spatial distance, and the proper time interval between events at the neighbouring reference points. The specific form of the metric of locally anisotropic space-time used in the work has yielded reasonable expressions of the observable quantities through the values of the fields determining the local anisotropy.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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