сообщения
объединенного
института
ядерных
исследований
дубна

E2-82-747

Th.Braunschweig, J.Hořejší, D.Robaschik

NONLOCAL LIGHT-CONE EXPANSION AND ITS APPLICATIONS TO DEEP INELASTIC SCATTERING PROCESSES

1982

1. INTRODUCTION

From the theoretical point of view the so-called nonlocal light cone expansion (LCE) is preferred to the usual local LCE. Whereas the nonlocal LCE is a true operator identity in the Fock space [1], the local LCE is valid on a dense subset of the Fock space only [2].

Moreover the renormalization group equations for the coefficients of the nonlocal LCE are similar to the evolution equations derived by Altarelli and Parisi [3].

The application of local LCE to exclusive processes is very restricted because the full scattering amplitude has to be reconstructed from an infinite sum. Let us illustrate this for the case of forward scattering: Writing the LCE for matrix elements

$$\langle p|Tj(x)j(0)|p\rangle \underset{x^2\to 0}{=} \sum (xp)^{2n} F_n(x^2)A_n(p^2) \qquad (1.1)$$

we can reconstruct the scattering amplitude in the Bjorken region of the momentum space (up to the terms $O(p^2/q^2)$) as

$$T(\nu, Q^2) = \int dx\, e^{iqx} \langle p|T(j(x)j(0))|p\rangle \approx$$

$$\overset{Bj}{\approx} \sum_n \Big(p\frac{\partial}{\partial iq}\Big)^{2n} F_n(q^2)A_n(p^2) \approx \sum_n \Big(-i\frac{2qp}{q^2}\Big)^{2n}\Big(q^2\frac{\partial}{\partial q^2}\Big)^{2n} \tilde{F}_n(q^2)A_n(p^2) \approx \qquad (1.2)$$

$$\approx \sum_n (-1)^n \xi^{-2n}\Big(Q^2\frac{\partial}{\partial Q^2}\Big)^{2n} \tilde{F}_n(-Q^2)A_n(p^2)\ ,$$

where

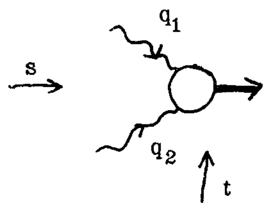
$$\tilde{F}_n(q^2) = \int dx\, e^{iqx} F_n(x^2), \quad \nu = 2qp\ , \quad \xi = -\frac{q^2}{2qp},\quad q^2 = -Q^2\ .$$

This series corresponds to the Taylor expansion of the scattering amplitude

$$T(\nu, Q^2) = \sum_n \nu^{2n}\frac{1}{(2n)!}\Big[\Big(\frac{\partial}{\partial\nu}\Big)^{2n} T(\nu, Q^2)\Big]_{\nu=0} =$$

$$= \sum_n \Big(\frac{\nu}{Q^2}\Big)^{2n}(-1)^n\frac{(-1)^n}{(2n)!}(Q^2)^{2n}\Big[\Big(\frac{\partial}{\partial\nu}\Big)^{2n} T(\nu, Q^2)\Big]_{\nu=0}\ . \qquad (1.3)$$

This series converges inside the analyticity domain of the scattering amplitude. This means for $Q^2 \geq 0$, $Q^2/|2qp| > 1$. For the usual processes as the virtual Compton scattering this region is outside the physical region. There exist, however, processes where this series converges inside the physical region. An example of such a process is $\gamma^*\gamma^* \to M$, where γ^* denotes virtual photon and M is a meson (see [4]):



$$Q = \frac{1}{2}(q_1 - q_2) \qquad w = \frac{Q^2}{PQ} = \frac{q_1^2 + q_2^2}{q_1^2 - q_2^2} > 1$$

$$P = q_1 + q_2 \qquad Q^2 \to \infty$$

$$q_1^2 < 0, \; q_2^2 < 0.$$

Here the physical channel is the s-channel whereas the LCE has been performed in the t-channel. The convergence radius is determined by the t-channel singularities which do not influence the coefficient functions for the s-channel processes. We will show here that the nonlocal LCE can be applied to all light-cone dominated processes since there are no convergence difficulties.

Furthermore, the anomalous dimensions of the relevant nonlocal operators are simply connected with the anomalous dimensions of standard local operators (essentially via Mellin transformation, see [3]). All anomalous dimensions of the nonlocal quark and gluon operators in QCD have been calculated in the present paper in the one-loop approximation. As it should be, they are directly related to the Altarelli-Parisi probability functions P_{ij}. The nonlocal renormalization group equations are formulated in terms of physical variables in momentum space. In principle they could also be used for the description of physical processes, thereby of course an unknown target function has to be taken into account. From the renormalization group equations one can immediately obtain the evolution equations in the leading order.

The paper is organized as follows: In the next section we derive an integral representation for the amplitude of a light-cone dominated process. In section 3 we elaborate on the renormalization group equations for the coefficients of the nonlocal LCE. In section 4 we present an alternative derivation of the Altarelli-Parisi equations (in the leading order) within the framework of our formalism. In section 5 the application to the QCD is described, together with the calculation of anomalous dimensions of the relevant quark and gluon nonlocal light-cone operators.

2. CONVERGENCE PROPERTIES OF THE NONLOCAL LIGHT-CONE EXPANSION

Here we will demonstrate that the nonlocal LCE can be applied to all light-cone dominated processes as there are no convergence difficulties in contrast to the local LCE.

For these general considerations we study scalar field theories and use also scalar currents. For simplicity we consider only the most simple LCE which contains only the operators with the lowest dimension (in the local case this corresponds to the operators with minimal twist). With this restriction the nonlocal LCE reads

$$RT j(x) j(0) S \approx \int_0^1 d\kappa_1 \int_0^1 d\kappa_2 \, F(x^2, \kappa_1, \kappa_2) (\bar{R}TO(\kappa_1, \kappa_2) S), \tag{2.1}$$

$$O(\kappa_1, \kappa_2) = \int dq_1 dq_2 \, e^{i\kappa_1 \tilde{x} q_1 + i\kappa_2 \tilde{x} q_2} :\phi(q_1)\phi(q_2): \tag{2.2}$$

Here we have used the following notation: ϕ - scalar field, $j(x)$ - scalar current, $R, \bar{R}$ - R-operations, S - unrenormalized S-matrix and $F(x^2, \kappa_1, \kappa_2)$ - coefficient function. In a somewhat oversimplified way we can state: The nonlocal LCE differs from the local expansion in two respects: Firstly, instead of an infinite sum we have an integral over a finite interval. Secondly, the light-cone operators $O(\kappa_1, \kappa_2)$ are not taken at $x = 0$ (local) but depend on two points $\kappa_1 \tilde{x}, \kappa_2 \tilde{x}$ lying on a light ray. The coefficient functions $F(x^2, \kappa_1, \kappa_2)$ are defined with the help of the x-proper functional of the product of the two scalar currents [1]

$$\bar{R}T(j(x)j(0)S)^{x\text{-prop}} = 1 + \sum_n \int \frac{1}{n!} dq_1 \ldots dq_n \, F_n^{x\text{-prop}}(x, q_i) :\phi(q_1) \ldots \phi(q_n): \tag{2.3}$$

then

$$F(x^2, \kappa_1, \kappa_2) = \frac{1}{2} \int d(xq_1) d(xq_2) F_2^{x\text{-prop}}(x^2, xq_1, xq_2, q_i q_j)\Big|_{q_i q_j = \mu_{ij}} e^{ixq_1\kappa_1 + ixq_2\kappa_2}, \tag{2.4}$$

μ_{ij} denotes the subtraction points. The coefficient function $F(x^2, \kappa_1, \kappa_2)$ has the support $0 \le \kappa_i \le 1$, which is a consequence of the analyticity properties of $F_2(x^2, xq_i, q_i q_j)$ w.r.t. variables xq_i.

For the matrix elements of the nonlocal light-cone operators $\langle p_1 | RT\phi(\kappa_1 \tilde{x})\phi(\kappa_2 \tilde{x}) S | p_2 \rangle$ similar statements are true. In analogy to the Dyson-Jost-Lehmann representation (which is now fixed at $x^2 = 0$) this is an entire function of the variables $\tilde{x} p_i \kappa_j$ so that

$$\langle p_1 | \bar{R}T : \phi(\kappa_1 \tilde{x})\phi(\kappa_2 \tilde{x}) : S | p_2 \rangle = \int_{|u_{ij}| \le a} du_{ij} \, e^{iu_{ij}\kappa_i \tilde{x} p_j} \chi(u_{ij}, p_k p_\ell). \tag{2.5}$$

The same conclusion can be drawn from the α-representations for the relevant graphs.

The main consequences of these facts are: Whereas the matrix elements $\langle p_1 | RT : \phi(\kappa_1 \tilde{x}) \phi(\kappa_2 \tilde{x}) : S | p_2 \rangle$ are entire functions of the variables $\kappa_j \tilde{x} p_i$ the coefficient functions $F(x^2, \kappa_i)$ are generalized functions w.r.t. κ_i. This implies that the matrix elements of current products in its approximate form (2.1) are well-defined quantities in x-space as it should be:

$$<p_1|RTj(x)j(0)S|p_2> \approx$$

$$\approx \int d\kappa_1 d\kappa_2 F(x^2,\kappa_1,\kappa_2) <p_1|\bar{R}T\phi(\kappa_1\tilde{x})\phi(\kappa_2\tilde{x})S|p_2> = \qquad (2.6)$$

$$= \int d\kappa_1 d\kappa_2 F(x^2,\kappa_i) \int_{|u_{ij}|\le a} du_{ij}\, e^{i\kappa_i u_{ij} q_j \tilde{x}} \chi(u_{ij}, p_k p_\ell).$$

The main question is what happens in momentum space? In other words, we have to find out whether such a representation makes also sense for the Fourier transform of the l.h.s. of (2.6). As we have seen, for the local expansion this is only true for a very restricted number of processes*. Of course, also in the nonlocal case we have approximated the complete scattering amplitude in the neighbourhood of the light cone. Therefore all conclusions are true for the Bjorken region of the momentum space, which is in one - to - one correspondence with the $x^2 \approx 0$ region of the x-space [5]. Having this in mind, we can apply Fourier transformation to eq. (2.6) (omitting for brevity the symbols $R, \bar{R}$ and S)

$$T(k,p_i) = \int dx\, e^{ikx} <p_1|T(j(x)j(0))|p_2> \approx$$

$$\approx \int dx\, e^{ikx} \int d\kappa_1 d\kappa_2 F(x^2,\kappa_i) <p_1|\phi(\kappa_1\tilde{x})\phi(\kappa_2\tilde{x})|p_2> \approx \qquad (2.7)$$

$$\approx \int dx\, e^{ikx} \int d\kappa_1 d\kappa_2 F(x^2,\kappa_i) \int du_{ij}\, e^{i\kappa_i u_{ij} p_j \tilde{x}} \chi(u_{ij}, p_r p_s) \approx$$

$$\approx \int d\kappa_i d\kappa_2 \int du_{ij}\, \tilde{F}((k+\kappa u p)^2, \kappa_i)\chi(u_{ij}, p_r p_s),$$

where $\tilde{F}$ denotes the Fourier transform

$$\tilde{F}(k^2,\kappa_i) = \int dx\, e^{ikx} F(x^2,\kappa_i). \qquad (2.8)$$

In the course of the calculation we have used $x - \tilde{x} = O(\sqrt{x^2})$ $(\tilde{x} = (|\vec{x}|, \vec{x})$ so that $\tilde{x} - x = (x^0 - \sqrt{x_0^2 - x^2}, 0)$, $|x^0 - \sqrt{x_0^2 - x^2}| \le \frac{1}{2}\sqrt{x^2})$ which is possible as the description is correct near the light cone only. This allows the substitution

$$e^{i\kappa u q \tilde{x}} = e^{i\kappa uqx}\, e^{i\kappa uq(\tilde{x}-x)} \approx e^{i\kappa uqx}.$$

The remaining integrals in eq. (2.7) exist. The proof is similar to that which shows the existence of convolutions of generalized functions. An essential role plays here the fact that all integrals have a finite range.

---

* Especially for deep inelastic scattering the Fourier transformed LCE does not converge in the physical region.

In this way we have obtained an integral representation of the scattering amplitude which exists in the generalized Bjorken region. This is in contrast to the results obtained with the help of the local LCE where it is in most cases not possible to get a representation which converges in momentum space. Our formula shows also the separation of calculable quantities as $F(x^2,\kappa_i)$ (hard subprocesses) and uncalculable quantities $\chi(u_{ij}, p_r p_s)$, which contain the target properties in compact manner.

3. RENORMALIZATION GROUP EQUATIONS

In this section we investigate the renormalization group equations (RGE) of the nonlocal LCE. Here we restrict ourselves to the case of deep inelastic inclusive scattering processes (forward scattering amplitude).

In the framework of nonlocal LCE there appear as calculable quantities the coefficient functions $F(x^2,\kappa_1,\kappa_2)$. They are defined according to (2.1), (2.4) with the help of the renormalized product of currents. In this way they are perturbatively calculable. Moreover they satisfy the RGE [3] (for simplicity we shall consider only massless theories)

$$\int d\kappa_1' d\kappa_2' [(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g})\delta(\kappa_1'-\kappa_1)\delta(\kappa_2'-\kappa_2) - \gamma(\kappa_1',\kappa_2',\kappa_1,\kappa_2)] F(x^2,\kappa_1',\kappa_2') = 0, \qquad (3.1)$$

where $\gamma(\kappa_i',\kappa_i)$ are the anomalous dimensions of the nonlocal operators. If we restrict ourselves to deep inelastic inclusive scattering processes (forward scattering), the equation (3.1) simplifies; if we define $\kappa_+ = \kappa_1 + \kappa_2$, $\kappa_- = \kappa_2 - \kappa_1$ and

$$F_f(x^2,\kappa_-) = \frac{1}{2}\int d\kappa_+ F(x^2,\kappa_i),\ \gamma(\kappa_-',\kappa_-) = \frac{1}{2}\int d\kappa_+ \gamma(\kappa_1',\kappa_2',\kappa_1,\kappa_2) \qquad (3.2)$$

then

$$\int d\kappa_-' [(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g})\delta(\kappa_-'-\kappa_-) - \gamma(\kappa_-',\kappa_-)] F_f(x^2,\kappa_-') = 0. \qquad (3.3)$$

Note that $\gamma(\kappa_-',\kappa_-) \neq 0$ only for κ_-, κ_-' satisfying the condition

$$0 < \frac{\kappa_-}{\kappa_-'} < 1. \qquad (3.4)$$

Eq. (3.3) reminds one of the Altarelli-Parisi equations [6] (see ref. [3]).

Knowing $F_f(x^2,\kappa_-)$ the complete amplitude $T(x,p)$ can be reconstructed in the form

$$T(x,p) = \int d\kappa_- F_f(x^2,\kappa_-) <p|R\bar{T}O(\kappa_1,\kappa_2)S|p>, \qquad (3.5)$$

where

$$\langle p|RTO(\kappa_1,\kappa_2)S|p\rangle = f(\tilde{x}\kappa\_p) = \int du\, e^{iu\kappa\_\tilde{x}p}\chi(u) \qquad (3.6)$$

is an entire function with the Fourier transform $\chi(u)$. The physically interesting amplitude $\tilde{T}(q,p)$ can be obtained by the Fourier transformation of (3.5)

$$\tilde{T}(q,p) = \int dx\, e^{iqx} T(x,p) \approx \int dx \iint d\kappa\_du\, e^{i(q+\kappa\_up)x} F_f(x^2,\kappa\_)\chi(u) =$$

$$= \iint d\kappa\_du \tilde{F}_f((q+\kappa\_up)^2,\kappa\_)\chi(u) = \int du G(q^2,\frac{\xi}{u})\chi(u); \qquad (3.7)$$

$$G(q^2,\frac{\xi}{u}) = \int d\kappa\_ \tilde{F}_f(q^2(1-\frac{\kappa\_u}{\xi}),\kappa\_).$$

In this way the target state is characterized by the function $\chi(u)$ which does not depend on other momenta*.

Now it is possible to derive also a RGE for the modified coefficient function G given by (3.7). Starting from

$$\mu\frac{d}{d\mu}\tilde{F}_f(k^2,\kappa\_) - \int d\kappa'\_\gamma(\kappa'\_,\kappa\_)\tilde{F}_f(k^2,\kappa'\_) = 0$$

we get

$$\mu\frac{d}{d\mu}G(q^2,\xi) = \mu\frac{d}{d\mu}\int d\kappa\_ \tilde{F}_f(q^2(1-\frac{\kappa\_}{\xi}),\kappa\_) =$$

$$= \iint d\kappa'\_d\kappa\_\,\gamma(\kappa'\_,\kappa\_)\tilde{F}_f(q^2(1-\frac{\kappa\_}{\xi}),\kappa'\_). \qquad (3.8)$$

In (3.8) we perform the substitution $\kappa\_ = \kappa'\_\xi/\eta$, i.e.,

$$\left|\frac{\partial(\kappa'\_,\kappa\_)}{\partial(\kappa'\_,\eta)}\right| = |\kappa'\_|\frac{\xi}{\eta^2}$$

and (3.8) becomes

$$\iint d\kappa'\_d\eta\, |\kappa'\_|\frac{\xi}{\eta^2}\gamma(\kappa'\_,\kappa'\_\frac{\xi}{\eta})\tilde{F}_f(q^2(1-\frac{\kappa'\_u}{\eta}),\kappa'\_) =$$

$$= \int\frac{d\eta}{\eta}\frac{\xi}{\eta}\tilde{\gamma}(\frac{\xi}{\eta})G(q^2,\eta). \qquad (3.9)$$

In arriving at (3.9) we have taken into account (cf. [3], (3.4))

$$\gamma(\kappa,\kappa') = \frac{1}{|\kappa|}\tilde{\gamma}(\frac{\kappa'}{\kappa})\theta(\frac{\kappa'}{\kappa})\theta(1-\frac{\kappa'}{\kappa}).$$

---

* The complete q-dependence is contained in $G(q^2,\xi)$. In what follows we take $-q^2/2pq \equiv \xi > 0$.

In this way we have obtained the following RGE* (writing γ instead of $\tilde{\gamma}$ for brevity)

$$(\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial g})G(q^2,\xi) - \int_\xi^\infty\frac{d\eta}{\eta}\frac{\xi}{\eta}\gamma(\frac{\xi}{\eta})G(q^2,\eta) = 0 \qquad (3.10)$$

for the expansion coefficients of the amplitude

$$\tilde{T}(q,p) = \int du\, G(q^2,\frac{\xi}{\eta})\chi(u). \qquad (3.11)$$

For the absorptive part of the forward scattering amplitude all the foregoing considerations remain valid, because one has only to perform the substitution

$$F_f(x^2,\kappa\_) \to C(x^2,\kappa\_) = \mathrm{Im}\,F_f(x^2,\kappa\_)$$

with the result (denoting schematically $H = \mathrm{Im}G$, $W = \mathrm{Im}T$ and using $\mathrm{Im}\,T = 0$ for $\xi > 1$)

$$(\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial g})H(q^2,\xi) - \int_\xi^1\frac{d\eta}{\eta}\frac{\xi}{\eta}\gamma(\frac{\xi}{\eta})H(q^2,\eta) = 0, \qquad (3.12)$$

$$W(q,p) = \int du\, H(q^2,\frac{\xi}{u})\chi(u). \qquad (3.13)$$

These renormalization group equations are also valid for higher order calculations (i.e., beyond the one-loop approximations). Here the full q^2-dependence is contained in the coefficient functions G or H, resp., whereas χ is an unknown (uncalculable) function describing the target properties.

4. EVOLUTION EQUATIONS

For practical calculations the evolution equations [6] are better suited than RGE (for practical aspects of the evolution equations, see, e.g., ref. [7]). From our calculations it is now very easy to obtain evolution equations in the one - loop approximation (leading order). If we take into account that in the leading order (and in massless theory) the RGE (3.12) is equivalent to the equation

$$-2q^2\frac{\partial}{\partial q^2}H(q^2,\xi) = \int_\xi^1\frac{d\eta}{\eta}\frac{\xi}{\eta}\gamma(\frac{\xi}{\eta})H(q^2,\eta), \qquad (4.1)$$

where in the anomalous dimension γ we substitute $g \to \bar{g}(q^2)$ and using the representation (3.13) we immediately obtain

$$-2q^2\frac{\partial}{\partial q^2}W(q^2,\xi) = \int_\xi^1\frac{d\eta}{\eta}\gamma(\frac{\xi}{\eta})\frac{\xi}{\eta}W(q^2,\eta). \qquad (4.2)$$

---

*Note that the lower limit ξ in the integral is due to (3.4).

Thus, we have obtained the Altarelli-Parisi equation (in a scalar theory) in the case when there is no mixing between the nonlocal operators of equal dimension (equal twist in the local LCE); this corresponds to the flavour non-singlet case in QCD. Our derivation is clearly equivalent to that of the original paper by Parisi /6/; there the differential equation for the moments of a structure function following from RGE for coefficient functions of the local LCE was converted into the integro-differential evolution equation by means of the inverse Mellin transformation. However, this Mellin transformation is built into our formalism from the very beginning, in view of the connection between the local and nonlocal LCE /3/.

Let us consider also the more complicated case of RGE containing a mixing matrix, in analogy with the flavour singlet case in QCD. In the rest of this section we shall present a schematic derivation of the system of evolution equations in the case when RGE contain a 2x2 mixing matrix of anomalous dimensions. Staying within the framework of scalar theories, such a case corresponds, e.g., to the system of two interacting scalar fields. We shall see in the next section that the relevant results carry over to the realistic case of QCD with minimal modifications.

Thus suppose that two independent nonlocal operators contribute to the forward scattering amplitude

$$\langle p|RTj(x)j(0)S|p\rangle \approx \sum_{i=1}^{2}\int d\kappa_- F_f^i(x^2,\kappa_-)\langle p|\overline{R}TO_i(\kappa_1,\kappa_2)S|p\rangle . \qquad (4.3)$$

The RGE for the coefficient functions read ($C_i \equiv \mathrm{Im}F_f^i$)

$$(\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial g})C_i(q^2,\kappa_-)-\sum_{j=1}^{2}\int d\kappa'_-\gamma_{ji}(\kappa'_-,\kappa_-)C_j(q^2,\kappa'_-)=0 \qquad (4.4)$$

and (cf. (3.12))

$$(\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial g})H_i(q^2,\xi)-\sum_{j=1}^{2}\int_\xi^1\frac{d\eta}{\eta}\frac{\xi}{\eta}\gamma_{ji}(\frac{\xi}{\eta})H_j(q^2,\eta)=0 . \qquad (4.5)$$

Now we look for an evolution equation for the complete absorptive part of the amplitude (4.3). Repeating the foregoing considerations and using (4.5) we obtain

$$-2q^2\frac{\partial}{\partial q^2}W_{ik}(q^2,\xi)=\int_\xi^1\frac{d\eta}{\eta}\frac{\xi}{\eta}\sum_{j=1}^{2}\gamma_{ji}(\frac{\xi}{\eta})W_{jk}(q^2,\eta), \qquad (4.6)$$

where $W_{ik}(q^2,\xi)$ are auxiliary quantities defined by

$$W_{ik}(q^2,\xi)=\int du\, H_i(q^2,\frac{\xi}{u})\chi_k(u) \qquad (4.7)$$

with

$$\langle p|\overline{R}TO_k S|p\rangle=\int du\, e^{iu\kappa p\tilde{x}}\chi_k(u) . \qquad (4.8)$$

To get the Altarelli-Parisi equations let us define first

$$W(q^2,\xi)=\sum_{i=1}^{2}W_{ii}(q^2,\xi). \qquad (4.9)$$

Then according to (4.6) (denoting $\bar\gamma_{ji}(x)=x\gamma_{ji}(x)$)

$$-q^2\frac{\partial}{\partial q^2}W(q^2,\xi)=\frac{1}{2}\sum_{i,j}(\bar\gamma_{ji}*W_{ji})(\xi), \qquad (4.10)$$

where the symbol $*$ denotes the Mellin convolution

$$(f*g)(x)=\int_x^1\frac{dy}{y}f(\frac{x}{y})g(y) .$$

Let us now define

$$D=W_{12}+\bar\gamma_{12}^{-1}*\bar\gamma_{21}*W_{21}+\bar\gamma_{12}^{-1}*(\bar\gamma_{22}-\bar\gamma_{11})*W_{22} ; \qquad (4.11)$$

in (4.11) $\bar\gamma_{12}^{-1}$ is given by*

$$\bar\gamma_{12}*\bar\gamma_{12}^{-1}=e, \qquad (4.12)$$

where $e=\delta(1-x)$; obviously

$$e*f=f \qquad (4.13)$$

for any f. Using (4.11)-(4.13) as well as the commutativity and associativity of the Mellin convolution, it is easy to rewrite (4.10) in the form

$$-q^2\frac{\partial}{\partial q^2}W(q^2,\xi)=\frac{1}{2}\int_\xi^1\frac{d\eta}{\eta}\frac{\xi}{\eta}[\gamma_{11}(\frac{\xi}{\eta})W(q^2,\eta)+\gamma_{12}(\frac{\xi}{\eta})D(q^2,\eta)]. \qquad (4.14)$$

Further, straightforward calculation gives

$$-q^2\frac{\partial}{\partial q^2}D(q^2,\xi)=\frac{1}{2}\sum_{j=1}^{2}[\bar\gamma_{j1}*W_{j2}+\bar\gamma_{21}*\bar\gamma_{12}^{-1}*\bar\gamma_{j2}*W_{j1}+\bar\gamma_{12}^{-1}*(\bar\gamma_{22}-\bar\gamma_{11})*\bar\gamma_{j2}*W_{j2}]=$$
$$=\frac{1}{2}\int_\xi^1\frac{d\eta}{\eta}\frac{\xi}{\eta}[\gamma_{21}(\frac{\xi}{\eta})W(q^2,\eta)+\gamma_{22}(\frac{\xi}{\eta})D(q^2,\eta)] . \qquad (4.15)$$

Eqs. (4.14) and (4.15) are the desired evolution equations. Let us stress again that they are valid in the leading order. Within the framework of the local LCE higher order corrections

*Note that in the practical example of QCD discussed in the next section the existence of γ^{-1} is guaranteed by the standard theorems (see, e.g., /8/ and cf. also ref. /7/).

can be incorporated in the standard way [9], employing the factorization of the solution of RGE and calculating inverse Mellin transforms of all relevant quantities. An analog of such a procedure within the framework of the nonlocal LCE has not been considered so far.

5. APPLICATION TO QCD

In this section we will show how the foregoing considerations can be generalized to the realistic case of QCD. We shall give the relevant nonlocal operators, their anomalous dimensions and also describe the necessary modifications of formulae derived for scalar theories.

To avoid kinematical complications, we study the LCE of the scalar product of electromagnetic currents $j_\mu(x)$. As it has been shown earlier, their expansion takes the form

$$RT(j^\mu(x) j_\mu(0)S) \approx \int d\kappa_1 d\kappa_2 \, \Sigma(x^2, \kappa_1, \kappa_2) \, \bar{R}(T\Omega_q S) + $$
$$+ \int d\kappa_1 d\kappa_2 \, \Pi(x^2, \kappa_1, \kappa_2) \, \bar{R}(T\Omega_G S) + \ldots$$

with the nonlocal operators

$$\Omega_q(\kappa_1, \kappa_2) = O_q(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) \, ,$$
$$O_q(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) = :\bar{\psi}(\kappa_2 \tilde{x})(\gamma^\mu \tilde{x}_\mu) P \exp\left(-ig \int_{\kappa_1}^{\kappa_2} A_\mu(\tau \tilde{x}) \tilde{x}^\mu d\tau\right) \psi(\kappa_1 \tilde{x}): \qquad (5.1)$$
$$\Omega_G(\kappa_1, \kappa_2) = O_G(\kappa_1 \tilde{x}, \kappa_2 \tilde{x})_{\mu\nu} \tilde{x}^\mu \tilde{x}^\nu \, , \qquad (5.2)$$
$$O_G(\kappa_1 \tilde{x}, \kappa_2 \tilde{x})_{\mu\nu} = :F^a_{\mu\rho}(\kappa_2 \tilde{x}) \left[P \exp\left(-ig \int_{\kappa_1}^{\kappa_2} A_\lambda(\tau \tilde{x}) \tilde{x}^\lambda d\tau\right)\right]_{ab} F^{b\rho}_\nu(\kappa_1 \tilde{x}):$$
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu \, ; \quad (A_\mu)_{bc} = -A^a_\mu i f_{bac}$$

and the coefficient functions Σ, Π.

In application to the forward scattering we have to consider

$$\langle p | RT j^\mu(x) j_\mu(0) S | p \rangle \approx \int d\kappa_- \, \Sigma_f(x^2, \kappa_-) \langle p | \bar{R} T \Omega_q S | p \rangle + $$
$$+ \int d\kappa_- \, \Pi_f(x^2, \kappa_-) \langle p | \bar{R} T \Omega_G S | p \rangle + \ldots$$

with

$$\Sigma_f(x^2, \kappa_-) = \frac{1}{2} \int d\kappa_+ \Sigma(x^2, \kappa_1, \kappa_2), \quad \Pi_f(x^2, \kappa_-) = \frac{1}{2} \int d\kappa_+ \Pi(x^2, \kappa_1, \kappa_2).$$

From general principles it follows that the forward scattering amplitude is a symmetric function of x_0. For coefficient functions this implies

$$\Sigma_f(x^2, \kappa_-) = - \Sigma_f(x^2, -\kappa_-), \quad \Pi_f(x^2, \kappa_-) = \Pi_f(x^2, -\kappa_-)$$

Therefore the integration range can be restricted to positive values of κ_- if we symmetrize or antisymmetrize the corresponding operators.

We shall now briefly describe the calculation of the anomalous dimensions of the new nonlocal operators (5.1) and (5.2). For the treatment of inclusive scattering processes we can restrict ourselves to the absorptive part of the forward scattering amplitude. For completeness we include also the flavour degrees of freedom

$$\mathrm{Im}\langle p | RT j_\mu(x) j^\mu(0) S | p \rangle \approx \int d\kappa_- C^8_q(x^2, \kappa_-) \langle p | \bar{R} T \Omega^8_q S | p \rangle +$$
$$+ \int d\kappa_- C^o_q(x^2, \kappa_-) \langle p | \bar{R} T \Omega^o_q S | p \rangle + \qquad (5.3)$$
$$+ \int d\kappa_- C^o_G(x^2, \kappa_-) \langle p | \bar{R} T \Omega^o_G S | p \rangle,$$

where

$$C^o_q(x^2, \kappa_-) = \mathrm{Im}\, \Sigma_f(x^2, \kappa_-)$$
$$C^o_G(x^2, \kappa_-) = \mathrm{Im}\, \Pi_f(x^2, \kappa_-)$$

and Ω^8_q is the flavour octet operator

$$\Omega^8_q(\kappa_1, \kappa_2) =$$
$$= :\bar{\psi}(\kappa_2 \tilde{x}) \gamma^\mu \tilde{x}_\mu \lambda^8 P \exp\left(-ig \int_{\kappa_1}^{\kappa_2} A_\mu(\tau \tilde{x}) \tilde{x}^\mu d\tau\right) \psi(\kappa_1 \tilde{x}): \, ; \qquad (5.4)$$

C^8_q is an appropriate coefficient function. The anomalous dimension of the operator Ω^8_q has been calculated earlier [10]. For the practical calculation we substitute the correct subtraction procedure contained in $\bar{R}$ in the course of the renormalization by the treatment of the divergent quantities only. This means we apply the dimensional regularization and look for the pole terms. Furthermore, our calculation is performed in Feynman gauge whereas the important representation (5.3) is proved for the axial gauge. For the leading operators this brings no complications. If nonleading terms are taken into account one has to be more careful.

In the actual calculations of the anomalous dimensions it is most convenient to start with the Feynman rules for the operator vertices. As an example we shall show very briefly how these can be obtained in the case of the flavour nonsinglet operator (5.4) (the relevant result has already been used in /10/). We have

$$A_\mu(x) = \int \tilde{A}_\mu(k)\, e^{ikx}\, dk, \qquad \psi(x) = \int \tilde{\psi}(p)\, e^{ipx}\, dp,$$

$$\int_{\kappa_1}^{\kappa_2} A^\mu(\tau\tilde{x})\tilde{x}_\mu\, d\tau = \int_{\kappa_1}^{\kappa_2} \tilde{x}^\mu d\tau \int dk\, \tilde{A}_\mu(k)\, e^{ik\tau\tilde{x}} =$$

$$= \int dk\, \tilde{A}_\mu(k)\tilde{x}^\mu\, \frac{e^{ik\tilde{x}\kappa_2} - e^{ik\tilde{x}\kappa_1}}{ik\tilde{x}},$$

$$\Omega_q^8(\kappa_1,\kappa_2) = \int dp_1 dp_2 : \bar{\tilde{\psi}}(p_2)\, O_q^8(p_1,p_2)\psi(p_1): +$$

$$+ \int dp_1 dp_2 dk : \bar{\tilde{\psi}}(p_2)\, O^8_{qa\mu}(p_1,p_2,k)\psi(p_1)\tilde{A}^a_\mu(k):$$

$$O_q^8(p_1,p_2) = \gamma\tilde{x}\, e^{ip_1\kappa_1 + ip_2\kappa_2}\lambda^8, \tag{5.5}$$

$$O^8_{qa\mu}(p_1,p_2,k) = (-igt_a)\lambda^8\,\tilde{x}_\mu\,\gamma\tilde{x}\, e^{ip_1\kappa_1+ip_2\kappa_2}\; \frac{e^{ik\tilde{x}\kappa_2} - e^{ik\tilde{x}\kappa_1}}{ik\tilde{x}}. \tag{5.6}$$

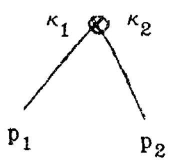
The expressions (5.5) and (5.6) represent the Feynman rules for the operator vertices of Ω_q^8 of order O(1) and O(g), resp.

In the flavour singlet case the calculations are more involved since one has to consider the mixing of Ω_q and Ω_G arising at the one-loop level. To deal with this problem it is convenient to consider instead of (5-1) the derivative w.r.t. κ_-: *

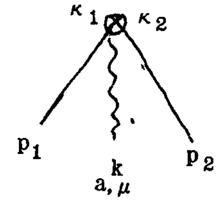
$$\frac{\partial}{\partial\kappa_-}\Omega_q(\kappa_1,\kappa_2). \tag{5.7}$$

The corresponding Feynman rules for the operators (5.7) and (5.2) are then (we use the notation $p\tilde{x} = \tilde{p}$, $\kappa_+ = \kappa_1 + \kappa_2$, $\kappa_- = \kappa_2 - \kappa_1$ and all external momenta are taken to be outgoing):

For $\frac{\partial}{\partial\kappa_-}\Omega_q(\kappa_1,\kappa_2)$



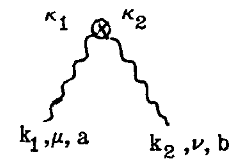
$$= \frac{1}{2}\tilde{\gamma}(\tilde{p}_2 - \tilde{p}_1)\, e^{i\tilde{p}_1\kappa_1 + i\tilde{p}_2\kappa_2}\Big|_{p_2 = -p_1 = p}$$



$$= \frac{1}{2}(-igt_a)\tilde{x}_\mu\tilde{\gamma}(\tilde{p}_2 - \tilde{p}_1 - \tilde{k})\cdot\left(\frac{e^{i\tilde{k}\kappa_-} - 1}{i\tilde{k}} - ie^{i\tilde{k}\kappa_-}\right) \times$$

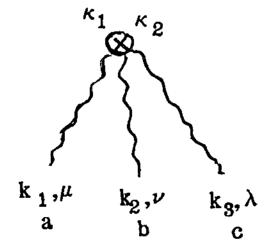
$$\times\, e^{\frac{i}{2}\kappa_+(\tilde{p}_1 + \tilde{p}_2 + \tilde{k}) + \frac{i}{2}\kappa_-(\tilde{p}_2 - \tilde{p}_1 - \tilde{k})}\Big|_{\substack{p_1+p_2+k=0,\\ p_2=-p_1=p}}$$

For Ω_G:



$$= -\delta^{ab}\left(e^{i\tilde{k}_1\kappa_1 + i\tilde{k}_2\kappa_2} + e^{i\tilde{k}_2\kappa_1 + i\tilde{k}_1\kappa_2}\right)\times$$

$$\times(\tilde{k}_1\tilde{k}_2 g_{\mu\nu} - \tilde{k}_1 k_{2\mu}\tilde{x}_\nu - \tilde{k}_2 k_{1\nu}\tilde{x}_\mu + k_1 k_2\tilde{x}_\mu\tilde{x}_\nu)\Big|_{-k_1=k_2=k}$$



$$= -igf_{abc}\Big\{ e^{i\tilde{k}_1\kappa_1 + i(\tilde{k}_2+\tilde{k}_3)\kappa_2}\times$$

$$\times\,[\tilde{x}_\nu(\tilde{k}_1 g_{\mu\lambda} - k_{1\lambda}\tilde{x}_\mu) - \tilde{x}_\lambda(\tilde{k}_1 g_{\mu\nu} - k_{1\nu}\tilde{x}_\mu)] -$$

$$-\frac{e^{i\tilde{k}_1\kappa_1 + i(\tilde{k}_2+\tilde{k}_3)\kappa_2} - e^{i(\tilde{k}_3+\tilde{k}_1)\kappa_1 + i\tilde{k}_2\kappa_2}}{\tilde{k}_3}\times$$

$$\times\,\tilde{x}_\lambda(\tilde{k}_1\tilde{k}_2 g_{\mu\nu} - \tilde{k}_1 k_{2\mu}\tilde{x}_\nu - \tilde{k}_2 k_{1\nu}\tilde{x}_\mu + k_1 k_2\tilde{x}_\mu\tilde{x}_\nu) +$$

$$+ (\kappa_1 \to \kappa_2) + \text{cyclic permatations of } \begin{pmatrix} k_1 & k_2 & k_3 \\ \mu & \nu & \lambda\end{pmatrix}\Big\}.$$

Taking into account the symmetry properties of the forward scattering amplitude it is easier to perform the calculation with the symmetrized operator

$$\frac{\partial}{\partial\kappa_-}\left[\Omega_q\left(-\frac{\kappa_-}{2}, \frac{\kappa_-}{2}\right) - \Omega_q\left(\frac{\kappa_-}{2}, -\frac{\kappa_-}{2}\right)\right].$$

In the course of the calculation we employ the dimensional regularization and isolate the pole terms, which determine the Z-factors. There appear integrals of the type

$$\int d^n q\; e^{i\kappa_-\tilde{x}q}\, \frac{1}{(q^2+M^2)^2}\, q\tilde{x}.$$

Because of the light-like character of $\tilde{x}$ such expressions are divergent. The divergences can be calculated with the help of the expansion of the exponential

$$e^{i\kappa_-\tilde{x}q} = 1 + i\kappa_-\tilde{x}q + \dots\,.$$

---

* Note that in the second term of the LCE (5.3) one may perform a partial integration (modifying correspondingly the coefficient function) which results in replacing (5.1) by (5.7); the "surface term" vanishes because of the symmetry properties of the coefficient functions mentioned earlier.

Again, because of the light-like character of $\tilde{x}$ only a few first terms of the expansion contribute. Expressions of the type

$$\int d^n q \; \frac{e^{i(\tilde{k}-\tilde{q})}-1}{\tilde{k}-\tilde{q}} \; \frac{1}{(q^2+M^2)^2}$$

can be treated with the help of the identity

$$\frac{e^{x+y}-1}{x+y} = \left(1 + y\frac{\partial}{\partial x} + y^2\frac{1}{2!}\frac{\partial^2}{\partial x^2} + \dots\right)\frac{e^x-1}{x}$$

so that

$$\int d^n q \; \frac{e^{i(\tilde{k}-\tilde{q})}-1}{\tilde{k}-\tilde{q}} \; \frac{1}{(q^2+M^2)^2} = \int d^n q \frac{1}{(q^2+M^2)^2}\, i\left(1 + (-i\tilde{q}\frac{\partial}{\partial i\tilde{k}}) + \right.$$

$$\left. + \frac{1}{2}(-i\tilde{q})^2\frac{\partial^2}{\partial(i\tilde{k})^2} + \dots\right)\frac{e^{i\tilde{k}}-1}{i\tilde{k}} = i\pi^2\Gamma(2-\frac{n}{2})\frac{e^{i\tilde{k}}-1}{\tilde{k}} .$$

An important difference in comparison with the case of local operators is that the nonlocal operators depend on continuous parameters, so that

$$\mathrm{div(TOS)}(\kappa_-) = g^2\int d\kappa'_- h(\kappa_-,\kappa'_-)O(\kappa'_-)$$

and the Z-factors appear as generalized functions of two variables (with finite support)

$$Z_2^{-1}\int d\kappa'_-\, Z(\kappa_-,\kappa'_-)(\phi(p)\Omega(\kappa_i)\phi(p))^{\mathrm{1PI\ ren}}(\kappa'_-) = (\phi(p)\Omega(\kappa_i)\phi(p))^{\mathrm{1PI\ unren.}}(\kappa_-),$$

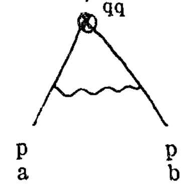
$$Z(\kappa_-,\kappa'_-) = \delta(\kappa_- - \kappa'_-) + g^2 z(\kappa_-,\kappa'_-) . \qquad (5.8)$$

In (5.8) we have generically denoted by ϕ an arbitrary external field (quark or gluon) and Z_2 denotes the corresponding Z-factor. Having calculated $Z(\kappa_-,\kappa'_-)$ according to (5.8), we may define the nonlocal anomalous dimension $\gamma(\kappa'_-,\kappa'_-)$ as follows (cf. also ref.[10]):

$$\int d\kappa'_-\, Z(\kappa_-,\kappa'_-)\,\gamma(\kappa'_-,\tau_-) = \mu\frac{\partial}{\partial\mu}Z(\kappa_-,\tau_-) . \qquad (5.9)$$

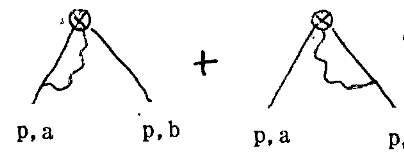
In practical calculations κ'_- arises as $\kappa'_- = \kappa_- z$, where z is a Feynman parameter (i.e., in resulting Feynman - parametric integrals we perform a suitable substitution $\kappa'_- = \kappa_- z$ in order to match the definition (5.9)); z lies in the interval (0,1) and this implies the constraint (3.4) mentioned earlier. Below we shall list the results of the evaluation of relevant Feynman diagrams (divergent parts are given, denoting $\frac{1}{2}\Gamma(2-\frac{n}{2}) \equiv \ln\frac{\Lambda}{\mu}$):

γ^q_{qq}:



$$I^{div} = \frac{g^2}{4\pi^2}C_N \ln\frac{\Lambda}{\mu}\int_0^{\kappa_-}\frac{d\kappa'_-}{\kappa_-} z(1-z)\tilde{\gamma}\tilde{p}(e^{i\tilde{p}\kappa'_-} + e^{-i\tilde{p}\kappa'_-})$$

$$z \equiv \kappa'_-/\kappa_-$$



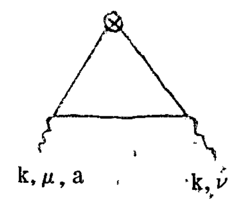
$$I^{div} = -2\frac{g^2}{4\pi^2}C_N \ln\frac{\Lambda}{\mu}\int_0^{\kappa_-}\frac{d\kappa'_-}{\kappa_-} \times \left(1+z-\frac{1}{(1-z)_+} - \delta(1-z)\right)\tilde{p}\tilde{\gamma}(e^{i\tilde{p}\kappa'_-} + e^{-ip\kappa'_-}) ;$$

$$h(\kappa_-,\kappa'_-) = \frac{g^2}{4\pi^2}C_N \ln\frac{\Lambda}{\mu}\int_0^{\kappa_-}\frac{d\kappa'_-}{\kappa_-}\left(\frac{z+z^3}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right)\tilde{\gamma}\tilde{p}(e^{i\tilde{p}\kappa'_-} + e^{-i\tilde{p}\kappa'_-})$$

$$\gamma^q_{qq}(\kappa_-,\kappa'_-) = \frac{1}{\kappa_-}\tilde{\gamma}_{qq}(z), \quad \tilde{\gamma}_{qq}(z) = -\frac{g^2}{4\pi^2}C_N z\left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right)$$

(we omit the obvious theta-functions due to the constraint $0 \le z \le 1$) γ^q_{GG}:

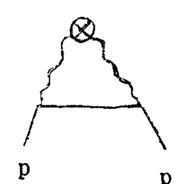
γ^q_{GG}:



$$I^{div} = g_{\mu\nu}\frac{g^2}{4\pi^2}2NT \ln\frac{\Lambda}{\mu}\int_0^{\kappa_-}\frac{d\kappa'_-}{\kappa_-} z(z^2+(1-z)^2)\tilde{k}(e^{i\tilde{k}\kappa'_-} + e^{-i\tilde{k}\kappa'_-}) + \text{other kinematical structures}$$

$$\gamma^q_{GG}(\kappa_-,\kappa'_-) = \frac{1}{\kappa_-}\tilde{\gamma}_{qG}(z); \quad \tilde{\gamma}_{qG}(z) = -\frac{g^2}{4\pi^2}2NT z(z^2+(1-z)^2) .$$

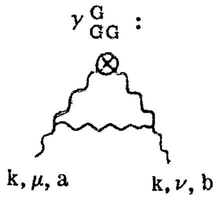
γ^G_{qq}:



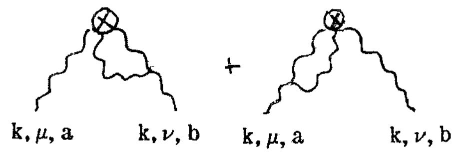
$$I^{div} = \frac{g^2}{4\pi^2}C_N \ln\frac{\Lambda}{\mu}\int_0^{\kappa_-}\frac{d\kappa'_-}{\kappa_-}(1+(1-z)^2)\tilde{\gamma}\tilde{p}(e^{i\tilde{p}\kappa'_-} + e^{-i\tilde{p}\kappa'_-}),$$

$$\gamma^G_{qq}(\kappa_-,\kappa'_-) = \frac{1}{\kappa_-}\tilde{\gamma}_{Gq}(z); \quad \tilde{\gamma}_{Gq}(z) = -\frac{g^2}{4\pi^2}C_N(1+(1-z)^2) .$$

γ^G_{GG} :



$$I^{div} = g_{\mu\nu}\frac{g^2}{4\pi^2}C_2(G)\ln\frac{\Lambda}{\mu}\int_0^{\kappa_-}\frac{d\kappa'_-}{\kappa_-}(2(1-z)^3-3(1-z)^2 + 2(1-z)+1)\,\tilde{k}^2\,(e^{i\tilde{k}\kappa'_-}+e^{-i\tilde{k}\kappa'_-}) + \dots \,.$$



$$I^{div} = -g_{\mu\nu}\frac{g^2}{4\pi^2}2C_2(G)\ln\frac{\Lambda}{\mu}\int_0^{\kappa_-}\frac{d\kappa'_-}{\kappa_-}\times \times\Big(z(1+z)-z\frac{1+z}{(1-z)_+}-\delta(1-z)\Big)\times \times\tilde{k}^2(e^{i\tilde{k}\kappa'_-}+e^{-i\tilde{k}\kappa'_-}) + \dots \,.$$

$$\gamma^G_{GG}(\kappa_-,\kappa'_-) = \frac{1}{\kappa_-}\tilde{\gamma}_{GG}(z);\quad \tilde{\gamma}_{GG}(z) = -\frac{g^2}{4\pi^2}2C_2(G)\Big(1-z+z^2(1-z) + \frac{z^2}{(1-z)_+} + \delta(1-z)\Big(\frac{11}{12}-\frac{1}{3}\frac{NT}{C_2(G)}\Big)\Big)\,*$$

In calculating the quantities $\tilde{\gamma}_{qG}$, $\tilde{\gamma}_{GG}$ we look for the terms proportional to $g_{\mu\nu}$. As it has been shown [11] this term characterizes the gauge invariant counterterm.

In collecting the final results we notice that in general ($i, j \equiv q, G$)

$$\tilde{\gamma}_{ij}(z) = zP^{AP}_{ij}(z)\,, \qquad (5.10)$$

where P^{AP}_{ij} are the well-known probability ("splitting") functions of Altarelli and Parisi [6]. The quantity $\tilde{\gamma}^8_{qq}(z)$ for the flavour non-singlet operator (5.4) has been already calculated in [10] with the result

$$\tilde{\gamma}^8_{qq}(z) = P^{AP}_{qq}(z)\,. \qquad (5.11)$$

Let us remark that the relations (5.10) and (5.11) can be easily understood if one realizes the connection between the nonlocal operators used here and the standard local quark and gluon operators (see [10]).

In the rest of this section we will show how the evolution equations in QCD follow from the RGE (3.3), (4.4). To this end

* In the expressions given above we employ the following notations for the group-theoretical factors: $C_2(G)\delta_{ab} = f_{acd}f_{bcd}$, $C_N\delta_{ij} = (t_a t_a)_{ij}$, $\mathrm{tr}(t_a t_b) = NT\delta_{ab} = \frac{1}{2}f$, where f is the number of flavours.

we must reconsider the integral representations (3.11), (3.13) for the forward amplitude and structure function resp., in the presence of the new nonlocal operators (5.1), (5.2) and (5.4). Formulae (3.11) and (3.13) with G and H satisfying (3.10) and (3.12) are strictly valid in scalar theory. To get the necessary modifications in QCD let us consider first the flavour non-singlet case. Taking into account that the operator Ω^8_q (5.4) involves one extra factor $\tilde{x}$ (multiplying the Dirac matrices) in comparison with the scalar theory, it is clear that the relation (3.6) is valid if we set $O(\kappa_1,\kappa_2) = \kappa_-\Omega^8_q(\kappa_1\tilde{x},\kappa_2\tilde{x})$ (as we want the matrix element to be a function of $\kappa_-\tilde{x}p$ only). Then the integral representation (3.7) is obviously modified to

$$T(q,p) \approx \int\frac{d\kappa_-}{\kappa_-}\int du\,\tilde{\Sigma}^8_f((q+\kappa_- up)^2,\kappa_-)\chi(u) \approx \int du\,G^8_q(q^2,\frac{\xi}{u})\,\chi(u)\,, \qquad (5.12)$$

where

$$G^8_q(q^2,\xi) = \int\frac{d\kappa_-}{\kappa_-}\tilde{\Sigma}^8_f(q^2(1-\frac{\kappa_-}{\xi}),\,\kappa_-)\,. \qquad (5.13)$$

The convergence of the integral on the r.h.s. of (5.12) may be proved in analogy with the scalar theory. The function $G^8_q(q^2,\xi)$ then satisfies the RGE (notice that the extra factor ξ/η present in (3.10) now disappears)

$$(\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial g})\,G^8_q(q^2,\xi) - \int_\xi^\infty\frac{d\eta}{\eta}\tilde{\gamma}^8_{qq}(\frac{\xi}{\eta})\,G^8_q(q^2,\eta) = 0 \qquad (5.13')$$

and similarly for the absorptive part. We thus finally obtain the evolution equation for the non-singlet structure function (in the leading order)

$$q^2\frac{\partial}{\partial q^2}W^8(q^2,\xi) = -\frac{1}{2}\int_\xi^1\frac{d\eta}{\eta}\tilde{\gamma}^8_{qq}(\frac{\xi}{\eta})\,W^8(q^2,\eta)$$

which, in view of (5.11) is just the Altarelli-Parisi equation.

Let us now consider the singlet case. Here two operators (5.2) and (5.7) contribute to the forward amplitude (see the footnote on page 12); these operators involve two extra factors of $\tilde{x}$ and therefore in analogy with the arguments given above we obtain the integral representation of the singlet structure function

$$W(q^2,\xi) = \sum_{i=1}^{2}\int du\,H_i(q^2,\frac{\xi}{u})\,\chi_i(u)$$

with $H_i(q^2,\xi)$ satisfying the RGE

$$(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g})H_i(q^2,\xi) - \int_{\xi}^{1}\frac{d\eta}{\eta}\bar{\gamma}_{ji}(\frac{\xi}{\eta})H_j(q^2,\eta) = 0, \qquad (5.14)$$

where

$$\bar{\gamma}_{ji}(z) = \frac{1}{z}\tilde{\gamma}_{ji}(z). \qquad (5.15)$$

Repeating all the steps which in the preceding section led to eqs. (4.14) and (4.15) we obviously obtain an analogous system of evolution equations which, owing to (5-15) and (5.10), coincides with the standard Altarelli-Parisi system in the leading order.

ACKNOWLEDGEMENTS

The authors are indebted to B.Geyer, N.B.Skatchkov and V.A.Matveev for useful discussions.

REFERENCES

1. Anikin S.A., Zavialov O.I. Ann. of Phys., 1978, 116, p.135. Zavialov O.I. Renormalized Feynman Diagrams, Nauka, M., 1980.
2. Bordag M., Robaschik D. Nucl.Phys., 1980, B169, p. 445.
3. Bordag M., Robaschik D. TMF, 1981, 49, p. 330.
4. Del Aguila F., Chase M.K. Nucl.Phys., 1981, B193, p. 517.
5. Vladimirov V.S., Zavialov B.I. JINR, E2-81-375, Dubna, 1981,
6. Altarelli G., Parisi G. Nucl.Phys., 1977, B126, p. 298.
7. Baulieu L., Kounnas C. Nucl.Phys., 1979, B155, p. 429.
8. Zemanian A.N. Generalized Integral Transformations, Interscience Publishers, New York, 1968.
9. Floratos E.G., Lacaze R., Kounnas C. Phys.Lett., 1980, B98, p. 285.
10. Bordag M. et al. JINR, E2-82-119, Dubna. 1981.
11. Floratos E.G., Ross D.A., Sachrajda C.T. Nucl.Phys., 1979, B152, p. 493.
Kluberg-Stern H., Zuber J.B. Phys.Rev.D, 1975, D12, p. 3159.

Received by Publishing Department
on October 21, 1982.

SUBJECT CATEGORIES
OF THE JINR PUBLICATIONS

Index	Subject
1.	High energy experimental physics
2.	High energy theoretical physics
3.	Low energy experimental physics
4.	Low energy theoretical physics
5.	Mathematics
6.	Nuclear spectroscopy and radiochemistry
7.	Heavy ion physics
8.	Cryogenics
9.	Accelerators
10.	Automatization of data processing
11.	Computing mathematics and technique
12.	Chemistry
13.	Experimental techniques and methods
14.	Solid state physics. Liquids
15.	Experimental physics of nuclear reactions at low energies
16.	Health physics. Shieldings
17.	Theory of condenced matter
18.	Applied researches
19.	Biophysics

Брауншвейг Т., Горжейши И., Робашик Д. E2-82-747

Нелокальное операторное разложение на световом конусе и его применение к процессам глубоконеупругого рассеяния

Процессы рассеяния, которые определяются поведением произведения токов на световом конусе, изучаются в рамках нелокального операторного разложения. Выведено интегральное представление Фурье-преобразования матричных элементов перенормированного произведения двух токов в обобщенной бьёркеновской области и доказана сходимость интеграла. Обсуждаются уравнения ренормгруппы для коэффициентных функций, принадлежащих рассеянию вперед. Показано, каким образом в ведущем порядке эволюционные уравнения для амплитуды рассеяния вперед или структурной функции непосредственно следуют из уравнений ренормгруппы. Аномальные размерности всех существенных нелокальных операторов в КХД вычислены в однопетлевом приближении и показано, что они имеют простое отношение к ядрам Альтарелли-Паризи.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1982

Braunschweig Th., Hořejší J., Robaschik D. E2-82-747

Nonlocal Light-Cone Expansion and Its Applications to Deep Inelastic Scattering Processes

Light-cone dominated scattering processes are studied within the framework of the nonlocal operator product expansion. An integral representation for the Fourier transform of matrix elements of the renormalized product of two currents in a generalized Bjorken region of the momentum space is derived and shown to be convergent everywhere. Renormalization group equations (RGE) for the coefficient functions pertinent to the forward scattering are discussed. It is demonstrated how the evolution equation for the forward amplitude and/or its absorptive part immediately follow (in the leading order) from such RGE. Anomalous dimensions of all relevant nonlocal operators in QCD are calculated in the one-loop approximation and shown to be simply related to the Altarelli-Parisi probability functions.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1982