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SCALING OF SEMI-INCLUSIVE SPECTRA  
OF  $\pi^-$  MESONS FROM  $pp$  INTERACTIONS

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It is known that normalized semi-inclusive rapidity spectra of  $\pi^-$  mesons from pp interactions

$$\rho_n(y) \equiv \frac{1}{n\sigma_n} \frac{d\sigma_n}{dy} \quad (1)$$

( $y$  is c.m.s. rapidity) change with energy,  $\sqrt{s}$ , and  $\pi^-$  multiplicity,  $n$ . For example, the dispersion of the rapidity distribution

$$\langle y^2 \rangle_n \equiv \int y^2 \rho_n(y) dy \quad (2)$$

increases with energy and decreases with increasing multiplicity. Several distributions characterized by the same dispersion can be obtained choosing each following distribution for larger multiplicity and corresponding higher energy. The form of these distributions with the same dispersion must not be identical.

We show that experimental data obtained over a momentum range of primary protons 6.6-400 GeV/c<sup>1-7</sup> indicate that rapidity distributions having the same dispersion are identical. Certainly, any other characteristic of distribution (momentum, half-width, the value of  $\rho_n(y)$  for  $y=0$  and so on) can be used instead of dispersion. The above statement is illustrated in fig.1a, where  $y$  distributions are compared for  $1\pi^-$  events at 12 GeV/c,  $2\pi^-$  events at 24 GeV/c and  $5\pi^-$  events at 69 GeV/c. In fig.1b are presented data on  $1\pi^-$  events at 69 GeV/c,  $3\pi^-$  events at 205 GeV/c and 5 to  $8\pi^-$  events at 400 GeV/c. However, this way of comparing distributions is not very sensitive to a small difference in their form. A more sensitive way is to compare ratios of distribution moments

$$\langle y^k \rangle_n \equiv \left( \int y^k \rho_n(y) dy \right)^{1/k}, \quad (3)$$

where  $k$  is the order of moments (integer). This is pointed out, e.g., in paper<sup>8</sup> where the well-known "scaling in the mean" has been proposed for semi-inclusive spectra. In the case of direct comparison of spectra the scaling is probably satisfactory, whereas, comparing ratios of moments, one can see that this kind of scaling is only approximately valid<sup>8</sup>.

Figure 2 presents ratios of moments of 40 rapidity distributions for different multiplicities and 8 energies versus their dispersion. Errors of points are not presented due to large dif-

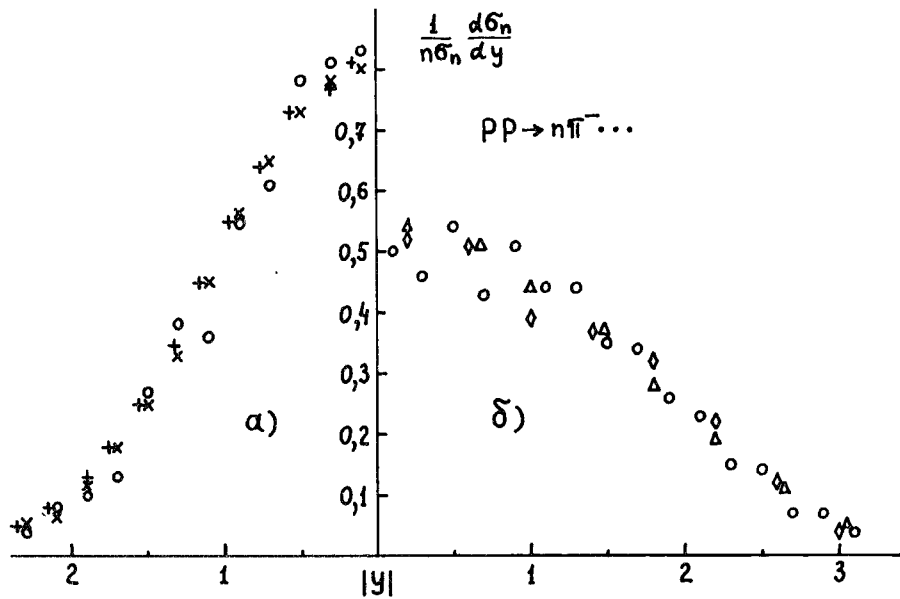


Fig.1. Normalized semi-inclusive rapidity spectra for  $\pi^-$ -mesons from pp interactions

$$\rho_n(y) \equiv \frac{1}{n \sigma_n} \frac{d\sigma_n}{dy}$$

- a) +12 GeV/c events with  $1\pi^-$ ,  $\times$  24 GeV/c -  $2\pi^-$ ,  $\circ$  69 GeV/c -  $\geq 5\pi^-$   
 b)  $\circ$  69 GeV/c -  $1\pi^-$ ,  $\diamond$  205 GeV/c -  $3\pi^-$ ,  $\Delta$  400 GeV/c -  $5-8\pi^-$ .

difficulties of obtaining data from figures of papers<sup>/1-7/</sup>. An estimation of the errors can be obtained from the scatter of experimental points around the overall monotonous behaviour of the data for each energy. It is seen that the groups of points corresponding to various energies are placed along the same curve. Thus, distributions with the same dispersion are characterized by the same values of other moments, i.e., they coincide completely. So, the two-parameter dependence of the rapidity spectrum on energy and multiplicity reduces to the single-parameter function of these variables

$$\rho_n(y, \sqrt{s}, n) = \phi(y, f(\sqrt{s}, n)). \quad (4)$$

Figure 3 shows average transverse momenta of  $\pi^-$ -mesons in events with fixed topology against the dispersion values of their rapidity distributions. The available data are much less abundant in this case. Nevertheless, one can see that average  $p_{\perp}$

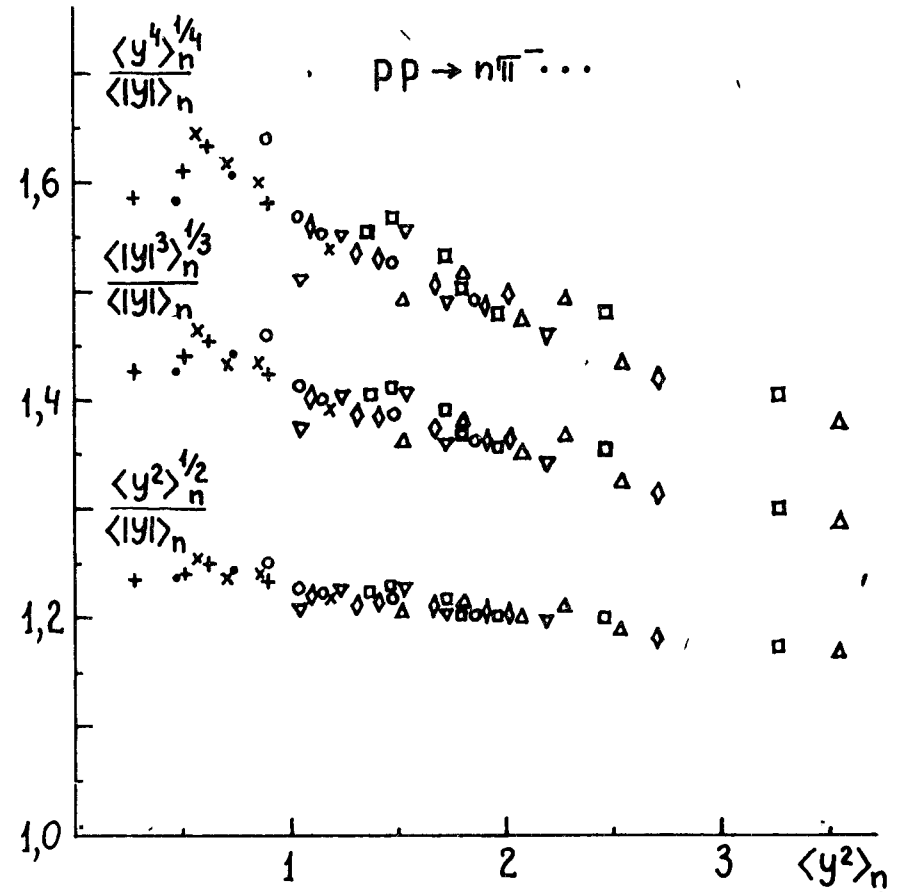


Fig.2. Ratios of higher moments for semi-inclusive rapidity distributions.  $\bullet$  6.6 GeV/c -  $1,2 \pi^-$ ; +12 GeV/c -  $1,2,3,4 \pi^-$ ;  $\times$  24 GeV/c -  $1,2,3,4 \pi^-$ ;  $\circ$  69 GeV/c -  $1,2,3,4, \geq 5 \pi^-$ ;  $\nabla$  102 GeV/c -  $1,2,3,4, > 5 \pi^-$ ;  $\diamond$  205 GeV/c -  $1,2,3,4,5,6,7 \pi^-$ ;  $\square$  300 GeV/c -  $1,2,3,4,5,6,7,8 \pi^-$ ;  $\Delta$  400 GeV/c -  $1,2,3,4,5-8, \geq 9 \pi^-$ .

values of  $\pi^-$ -mesons are the same for semi-inclusive events with identical  $y$  spectra. This allows us to assume that also the double-differential spectrum for a fixed energy and pion multiplicity is identical with the spectrum for another multiplicity and the energy properly chosen

$$\frac{1}{n \sigma_n} \frac{d\sigma_n}{dy dp_{\perp}}(y, p_{\perp}, \sqrt{s}, n) = \phi'(y, p_{\perp}, f(\sqrt{s}, n)). \quad (5)$$

Certainly, not only the equality of  $\langle p_{\perp} \rangle_n$  at equal  $\langle y^2 \rangle_n$  but also the identity of  $p_{\perp}$  distributions for each value of  $y$  are required for the validity of (5). If (5) is not true, the identity of rapidity spectra (4) and average transverse momenta can be apparently interpreted only as accidental. If (5) is valid, other distributions of  $\pi^-$ -mesons (over momentum, longitudinal momentum, energy and so on) are also single-parameter functions of  $f(\sqrt{s}, n)$ .

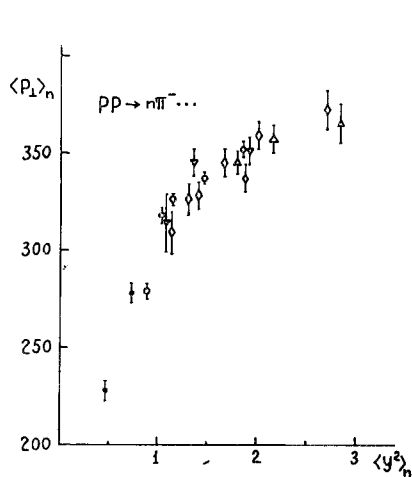


Fig. 3. Average  $p_{\perp}$  values of  $\pi^-$  mesons from events with fixed topology versus the dispersion of the rapidity spectra. Designations as in fig. 2.

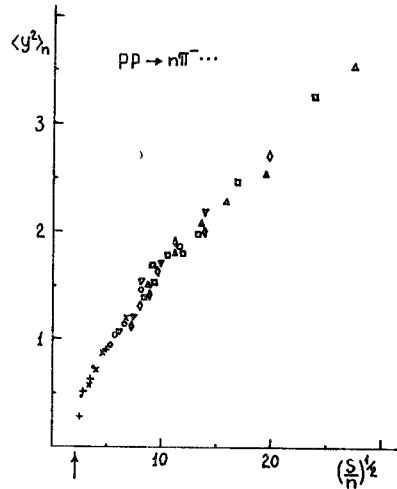


Fig. 4. Dependence of the dispersion of the semi-inclusive rapidity spectra on  $\sqrt{s}/\sqrt{n}$ .

The function  $f(\sqrt{s}, n)$  differs from  $\langle n \rangle/n$  predicted by KNO-II scaling of semi-inclusive spectra<sup>9/</sup>. Figure 4 shows the dependence of dispersion values of rapidity spectra on  $\sqrt{s}/\sqrt{n}$ . Experimental points satisfactorily lie on one curve, i.e.,

$$f(\sqrt{s}, n) = \frac{\sqrt{s}}{\sqrt{n}}. \quad (6)$$

The function  $f(\sqrt{s}, n)$  is intermediate between those corresponding to two extreme possible hypotheses: a) the average inelasticity coefficient for  $\pi^-$ -mesons in events with fixed multiplicity  $(\sum_n E_{\pi^-}/\sqrt{s})$  does not depend on the multiplicity, then

$f(\sqrt{s}, n) = \sqrt{s}/n$ ; b) this coefficient is proportional to the multiplicity, then  $f(\sqrt{s}, n) = \sqrt{s}$ .

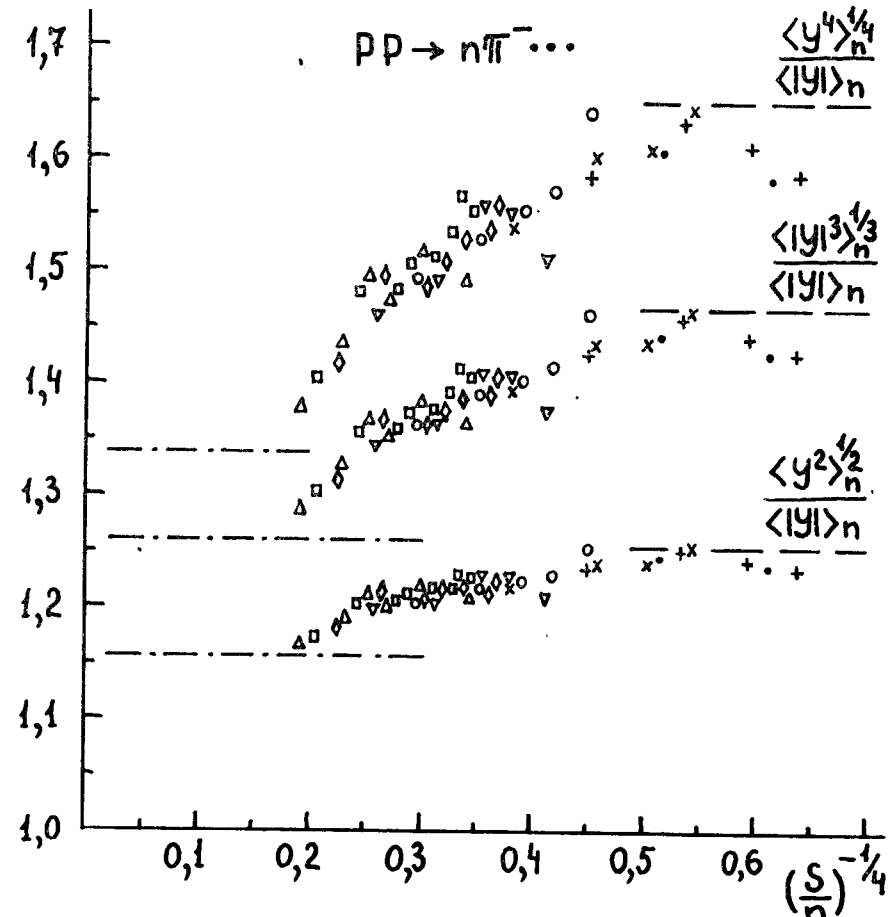


Fig. 5. Ratios of moments for the rapidity distributions versus  $(s/n)^{-1/4}$ . The data are denoted by the same symbols as in Fig. 2. Dashed lines correspond to the Gaussian form of distributions. The dot-dashed line corresponds to the rectangular form of distributions.

The single-parameter form of rapidity distribution allows one to obtain an evident picture of its dependence on energy and pion multiplicity. Figure 5 shows ratios of statistical moments for semi-inclusive rapidity spectra versus the function  $(s/n)^{-1/4}$ , the zero value of which corresponds to infinite energy. With increasing energy and decreasing multiplicity the ratios of statistical moments decrease, i.e., the form of the distributions changes towards flattening the top. The corresponding ratios of moments for Gaussian predicted by the hydrodynamic model<sup>10/</sup> are

given for comparison by dashed lines on the right of fig.5. Rectangular distributions expected from the multiperipheral<sup>/11/</sup> and parton<sup>/12/</sup> models are denoted by dot-dashed lines on the left. For higher energies the ratios of moments could achieve lower values denoted by dot-dashed lines (down to unity). This is possible if not Feynman scaling<sup>/13/</sup> but the hypothesis of limiting fragmentation<sup>/14/</sup> and other two-fireball models (e.g.<sup>/15/</sup>) are valid which lead to rapidity distributions with two maxima.

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Голохвастов А.И. E2-82-730  
Скейлинг полунклюзивных спектров  $\pi^-$ -мезонов  
в pp-взаимодействиях

Анализ экспериментальных данных при 6-400 ГэВ/с показывает, что двухпараметрическая зависимость спектров от энергии взаимодействия ( $\sqrt{s}$ ) и множественности отрицательных частиц (n) сводится к однопараметрической зависимости от отношения  $s/n$ .

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1982

Golokhvastov A.I. E2-82-730  
Scaling of Semi-Inclusive Spectra of  $\pi^-$  Mesons  
from pp Interactions

An analysis of experimental data for 6-400 GeV/c shows that a two-parameter dependence of spectra on energy ( $\sqrt{s}$ ) and the multiplicity of negative particles (n) reduces to a single-parameter dependence on ratio  $s/n$ .

The investigation has been performed at the Laboratory of High Energies, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1982