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дубна

317/83

17/I-83

E2-82-725

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INTERFERENCE CORRELATIONS OF IDENTICAL PARTICLES IN MODELS WITH CLOSELY SPACED SOURCES

Submitted to "ЯФ"

1982

1. As is well-known, the two-particle distribution of identical particles emitted with nearly equal 4-momenta p_1 and p_2 by a system of independent one-particle sources is described by the formula

$$W(p_1, p_2) = W_0(1 + \cos(q\ell)), \tag{1}$$

where $q = p_1 - p_2$, $\ell = (t, \vec{\ell})$, $\vec{\ell}$ is the distance between any two of the sources, t is the time interval between the moments of particle emission, $q\ell = q_0 t - \vec{q}\vec{\ell}$. It is assumed that the one-particle distributions are given by smooth enough functions of 4-momentum p and that the probability of three or more particles having nearly equal momenta can be neglected. Both these conditions are fulfilled better for larger space-time dimensions of particle emission region and larger momenta p_1 and p_2.

In the analysis of experimental data, eq. (1) is usually changed by a more complicated expression

$$W(p_1, p_2) = W_0(1 + \lambda \langle\cos(q\ell)\rangle), \tag{2}$$

with the factor $\lambda \neq 1$. It has been shown in our papers [1,2] that the parameter λ can appear due to a number of simple reasons, e.g., due to the presence of two or several space-time characteristics of the process (see also ref. [3]). On the other hand, interesting papers have been published in which the parameter λ is connected with the hypothesis of pion emission in the so-called coherent states or in a mixture of coherent and noncoherent states [4-10]. The main point is that the correlations due to symmetrization of wave functions are absent for the pions in the same coherent state. Thus the role of coherent states could be, in principle, estimated by the measurement of parameter λ.

In our opinion, there may be some doubts in practicability of such a program since the difference of parameter λ from unity can be, as mentioned above, connected with the more trivial reasons. Besides, coherent states, to a definite probability, contain any large number of particles. This circumstance is somewhat unsatisfactory since the finite pion mass with the limited total energy of the system prevents from the literal use of the apparatus of coherent states.

At the same time it is well-known that the absence of interference correlations is characteristic not only for the particles

in coherent states but also for any fixed number of particles in the same quantum state. Thus, from our point of view, another, more adequate, approach to the "λ -problem" is possible. The corresponding remarks can be found already in our papers [1,2]. Below we want to discuss this approach more thoroughly. It seems to us that it is more reliable, very simple mathematically and, moreover, it allows one to clarify the limiting cases when the corresponding correlations turn into the predictions obtained in the framework of coherent state theory. As the possibility of production of several identical pions in the same quantum state is concerned, although it seems not to be a typical mechanism of multiparticle production, at least, it cannot be excluded. The near threshold s-state pion production is an illustrative example [11].

2. If a heavy source is at rest at the point $\vec{r}_1$ and emits a pion at the moment t_1 and another source at the point $\vec{r}_2$ emits a pion at the moment t_2, the corresponding amplitude for the production of two pions with 4-momenta p_1 and p_2 is given by

$$A(p_1, p_2) = u(p_1)\, u(p_2)\, (e^{i(p_1 r_1 + p_2 r_2)} + e^{i(p_1 r_2 + p_2 r_1)}), \tag{3}$$

where $r_1 = (\vec{r}_1, t_1)$ and $r_2 = (\vec{r}_2, t_2)$ and the one-particle distribution $u(p)$ is assumed to be the same for both sources. The probability is given by

$$W(p_1, p_2) = 2|u(p_1)|^2 |u(p_2)|^2 [1 + \langle\cos(p_1 - p_2)(r_1 - r_2)\rangle], \tag{4}$$

where the averaging is done over r_1 and r_2. The interference term in eq. (4) vanishes, i.e., the two-particle distribution is given by the product of one-particle ones $W(p_1, p_2) = 2|u(p_1)|^2 |u(p_2)|^2$ provided that the characteristic interval between the points r_1 and r_2 is large enough. In the opposite situation, when the points r_1 and r_2 are very close to each other, we have $\langle\cos(p_1 - p_2)(r_1 - r_2)\rangle = 1$, and the probability $W(p_1, p_2) \approx 4|u(p_1)|^2 |u(p_2)|^2$ is again determined by the one-particle distributions only, but it is now two times larger than in the previous case. It should be noted that the formula (3) is valid only in the case when the distance between the sources is large as compared to their space-time dimensions and to the wave-length of emitted particles. In particular, this formula is not valid for the sources situated at the same point and emitting particles at the same moment. In such a case the probability $W(p_1, p_2)$ coincides with the one for pion emission by two very remote sources.

Having in mind the above comments and wishing to stress as much as possible the principal peculiarities of the problem, below the one-particle amplitudes $u(p)$ are considered to be independent of the 4-momentum p and the sources to be point-like in the sencé that their dimensions are small as compared to distances of the sources situated at different points. Besides, we consider some of the sources situated strictly at the same space-time point, the others at different points, as well as the situation when several such groups of sources (and also unit sources) are connected with different points. Finally, we consider the probability when all sources are situated at different points (i.e., the distances between them are large as compared to the particle wave lengths), but some part of them forms one or several groups with dimensions Δ smaller than the dimension R characterizing the total system, i.e.,

$$\lambda \ll \Delta \ll R, \qquad (5)$$

and the same inequalities are supposed to be valid for the corresponding time parameters.

We consider correlations in pairs of particles in inclusive approach when the momenta of any two particles are fixed and averaging over the others is performed. Thus each pair of sources can be considered independently of all other pairs. Besides, as mentioned above, we consider the particle momenta to be large enough so that the interference maximum occupies only a very small part of phase space, and the rare configurations with three or more particles having nearly equal momenta can be neglected. This allows one just simply to add the probabilities corresponding to various pairs of sources, taking into account that the probability is independent of the difference q of pion 4-momenta for two coinciding sources; for two "near-by" sources it is two times larger and also independent of q so far as the momentum difference is small as compared to $1/\Delta$, and for pairs of "distant" sources the q-dependence appears and is described by formula (1).

The following is reduced to simple combinatorics, i.e., to calculation of the number of pairs of various types. In particular, below, we consider, from this point of view, several concrete situations analyzed in the literature in terms of coherent or partly coherent states.

3. We start with a system of $n+m$ one-particle sources, n of them situated at one and the same space point and emitting particles simultaneously, the others at different space-time points. In other words, pions from the first group are in the same quantum state, while the others in different states. We denote by W_0 the probability in the "plateau" region when the momenta of all pions are essentially different. The question is how this probability should be modified in the case when the momenta of any

two pions become close to each other without changing the momenta of remaining $n+m-2$ particles. The answer to this question depends on the origin of a pair (the total number of possible pairs is equal to $(m+n)(m+n-1)/2$). If the momenta of two pions from the first group approach close to one another (the number of such pairs is equal to $n(n-1)/2$), the probability remains equal to W_0, and for pions from the second group (there are $m(m-1)/2$ such pairs) the probability changes due to appearing the interference term, i.e., $W = W_0(1+\langle \cos q\ell\rangle)$. Exactly the same change takes place when we make close the momenta of pions from different groups (the number of such pairs is equal to mn). As a result, the total probability is expressed as

$$W = \frac{2W_0}{(n+m)(n+m-1)}\left\{\frac{n(n-1)}{2} + \left[\frac{m(m-1)}{2} + nm\right](1+\langle\cos(q\ell)\rangle)\right\} =$$

$$= W_0\left\{1 + \frac{m(m-1)+2nm}{(m+n)(m+n-1)}\langle\cos(q\ell)\rangle\right\}.$$

Consequently,

$$W = W_0\,(1+\lambda\langle\cos(q\ell)\rangle), \qquad \lambda = \frac{m(m-1)+2mn}{(m+n)\,(m+n-1)} \leq 1. \qquad (6)$$

As could be expected, $\lambda = 0$ at $m = 0$, $\lambda = 1$ at $n = 0$ and $n = 1$, and $\lambda < 1$ if $n \geq 2$.

Assuming that $n, m \gg 1$ and introducing the patameter $\gamma = n/m$, we get $\lambda = (m^2 + 2m^2\gamma)/(m^2(1+\gamma)^2) = (1+2\gamma)/(1+\gamma)^2$, i.e.,

$$W = W_0\left(1 + \frac{1+2\gamma}{(1+\gamma)^2}\langle\cos(q\ell)\rangle\right). \qquad (7)$$

In the particular case, when all $n+m$ sources are situated at one and the same space point, we arrive at the formula which is completely equivalent to the results obtained in refs. [4,5,7] in the framework of the theory of coherent and partly coherent states. Note, however, that this formula is valid only in the case of large enough n and m.

Formula (7) has been obtained under the assumption of the same distributions of space-time intervals between the sources from the second group and the sources belonging to different groups. Consequently, only a single average quantity $\langle\cos q\ell\rangle$ enters into formulae (6) and (7). If these distributions are different, instead of formula (7) we have the following slightly more complicated expression

$$W = W_0\left(1 + \frac{1+2\gamma}{(1+\gamma)^2}\langle\cos(q\ell)\rangle_{\alpha\beta} + \frac{1}{(1+\gamma)^2}\langle\cos(q\ell)\rangle_{\beta\beta}\right). \qquad (8)$$

Here the indices α , β denote the averaging over the sources from different groups ($\alpha\beta$) and from the second group only ($\beta\beta$). The results obtained coincide with the formula (4.66) from ref.[8]. Expression (8) also leads to formulae (13), (14) and (18) from ref.[9] if we make the averaging over the concrete space distributions of sources used in ref.[9].

We consider now the situation when n pions are emitted simultaneously from the same point and m pions from the other common point also simultaneously, but, generally speaking, at the different moment. If we make close the momenta of pions from one of the two groups, the interference is absent, while the interference term contributes to the probability for pions coming from different groups. The number of pairs of the first type is equal to $\frac{1}{2}n(n-1)+\frac{1}{2}m(m-1)$ and of the second type to nm. Thus the probability can be written as

$$W=\frac{2W_0}{(n+m)(n+m-1)}\{\frac{n(n-1)}{2}+\frac{m(m-1)}{2}+nm\,(1+<\cos(q\ell)>)\},$$

or

$$W=W_0\,(1+\lambda<\cos q\ell>),\quad \lambda=\frac{2nm}{(n+m)(n+m-1)}\leq 1. \tag{9}$$

The factor $\lambda=0$ at $m=0$ or $n=0$; $\lambda=1$ at $n=m=1$. In the other cases $\lambda<1$. Assuming n and m large enough and introducing the parameter $\gamma=n/m$, we get

$$W=W_0(1+\frac{2\gamma}{(1+\gamma)^2}<\cos(q\ell)>). \tag{10}$$

From the considered example it follows that the factor $\lambda<1$ can appear not only due to a joint contribution of multi- and one-particle sources. The interpretation based on formula (10) concerning the multiparticle sources only is also quite possible.

We continue our discussion with the case of N independent multi-particle sources situated at N different points, each of them simultaneously emitting n identical pions in the same (for a given group) quantum state. The number of pion pairs from the same source is equal to $n(n-1)N/2$, while for pairs formed by pions from different groups it is equal to $N(N-1)n^2/2$. This immediately leads to the formula

$$W=W_0(1+\lambda<\cos q\ell>),\quad \lambda=\frac{n(N-1)}{nN-1}. \tag{11}$$

For $n\gg 1$ we get $\lambda=1-1/N$ which is in agreement with formulae (10) and (11) from ref.[10]. Allowing for fluctuations of the number of sources N when we come from one event to another, in a similar way we get the formula

$$W=W_0(1+(1-\frac{<N>}{<N^2>})<\cos(q\ell)>), \tag{12}$$

obtained earlier in ref.[8]. Note that the formulae (11) and (12) would change provided that the sources emit an unequal number of pions.

In fact, if we add to the above system one more point-like source emitting m identical pions in the same quantum state, then the interference effect is absent for $\frac{1}{2}Nn(n-1)+\frac{1}{2}m(m-1)$ pion pairs, while $\frac{1}{2}N(N-1)n^2$ pairs, as before, make a contribution to the probability proportional to $1+<\cos(q\ell)>_{\alpha\alpha}$ and nmN pairs to $1+<\cos(q\ell)>_{\alpha\beta}$. This combinatorics leads to the formula

$$W=W_0\,(1+\lambda<\cos(q\ell)>_{\alpha\alpha}+\mu<\cos(q\ell)>_{\alpha\beta}),$$

$$\lambda=\frac{N(N-1)n^2}{n(n-1)N+m(m-1)+N(N-1)n^2+2nmN},\quad \mu=\frac{2nmN}{n(n-1)N+m(m-1)+N(N-1)n^2+2nmN}. \tag{13}$$

Assuming $n, m\gg 1$ and introducing the parameter $\gamma=Nn/m$, formula (13) yields

$$W=W_0\,[1+(1-\frac{1}{N})\frac{<\cos(q\ell)>_{\alpha\alpha}}{(1+\gamma)^2}+\frac{2\gamma}{(1+\gamma)^2}<\cos(q\ell)>_{\alpha\beta}]. \tag{14}$$

Finally, if $N\gg 1$, we come again, except for the changed notation, to the previous result (8) despite the absence of one-particle noncoherent sources (in this context see also the discussion of formula (10)).

For comparison with the results of ref.[10] we consider here the case when m one-particle sources situated at different space-time points, are added to N previous n-particle sources (i.e., the m-particle source from the above example is divided into m independent one-particle sources). Then, making close the momenta of two pions the probability is not changed for $\frac{1}{2}Nn(n-1)$ pairs, the contribution of $\frac{1}{2}N(N-1)n^2$ pairs is proportional to $1+<\cos(q\ell)>_{\alpha\alpha}$, of $\frac{1}{2}m(m-1)$ pairs to $1+<\cos q\ell>_{\beta\beta}$ and of mnN pairs to $1+<\cos(q\ell)>_{\alpha\beta}$. This leads to the total probability

$$W=W_0\,[1+\lambda<\cos(q\ell)>_{\alpha\alpha}+\mu<\cos(q\ell)>_{\alpha\beta}+\nu<\cos(q\ell)>_{\beta\beta}], \tag{15}$$

where $\nu=\frac{m(m-1)}{Nn(n-1)+n^2N(N-1)+m(m-1)+2mnN}$, and λ , μ are the same as in eq. (13). Assuming again, $m, n \gg 1$ and introducing $\gamma = n/m$, we get

$$W = W_0 \left[1 + \frac{\langle\cos(q\ell)\rangle_{\beta\beta}}{(1+\gamma N)^2} + \frac{2\gamma N}{(1+\gamma N)^2}\langle\cos(q\ell)\rangle_{\alpha\beta} + \frac{N(N-1)\gamma^2}{(1+\gamma N)^2}\langle\cos(q\ell)\rangle_{\alpha\alpha}\right], \quad (16)$$

which is equivalent to the formula (20) from ref./10/. If the n-particle and one-particle sources have the same space-time distributions, formula (16) yields

$$W = W_0 \left[1 + \left(1 - \frac{N\gamma^2}{(1+N\gamma)^2}\right)\langle\cos(q\ell)\rangle\right] \quad (17)$$

in agreement with the corresponding formula in ref./10/.

4. Now we discuss the changes arising when a group of sources situated at different points inside small space-time region Δ satisfying the condition (5) is considered instead of the sources connected with the one and the same point. Of course, the number of pairs of sources of various types is not changed, but the weight corresponding to pairs of sources from the same compact group is doubled. E.g., in the situation corresponding to formula (6) in the previous consideration, each of $n(n-1)/2$ pairs of sources from the compact group contributes with the weight equal to 2, $m(m-1)/2$ pairs formed by "distant" sources contribute with the average weight $1 + \langle\cos(q\ell)\rangle$ each, and exactly the same weight corresponds to "mixed" pairs. Thus the probability is given by

$$W(p_1,p_2) = \frac{2W_0}{(n+m)(n+m-1)}\left[\frac{n(n-1)}{2}\,2 + \left(\frac{m(m-1)}{2} + nm\right)(1 + \langle\cos(q\ell)\rangle)\right],$$

or by

$$W(p_1,p_2) = W_0(1 + \lambda\langle\cos(q\ell)\rangle), \quad \lambda = \frac{m(m-1) + 2nm}{(n+m)(n+m-1) + n(n-1)}. \quad (18)$$

Assuming $n, m \gg 1$ and introducing $\gamma = n/m$, we get

$$W(p_1,p_2) = W_0\left(1 + \frac{1+2\gamma}{(1+\gamma)^2+\gamma^2}\langle\cos(q\ell)\rangle\right). \quad (19)$$

In an analogous way the other situations considered above can be easily reformulated as well (see also Appendix).

The change of coinciding sources by the compact group of near-by sources always leads to an essential change of the final result. In particular, formulae (18) and (19) do not coincide with (6) and (7), respectively. In order to reveal better the origin of this difference, we analyze in some detail the simplest case of three sources situated at space-time points r_1, r_2, r_3 and emitting pions with 4-momenta p_1, p_2 and p_3. The amplitude of this process is

$$\begin{aligned} A(p_1,p_2,p_3) = u(p_1)u(p_2)u(p_3)[&e^{i(p_1r_1+p_2r_2+p_3r_3)} + \\ &+ e^{i(p_1r_1+p_3r_2+p_2r_3)} + e^{i(p_2r_1+p_1r_2+p_3r_3)} + e^{i(p_2r_1+p_3r_2+p_1r_3)} + \\ &+ e^{i(p_3r_1+p_1r_2+p_2r_3)} + e^{i(p_3r_1+p_2r_2+p_1r_3)}]. \end{aligned} \quad (20)$$

The probability for emission of three identical pions containing six diagonal terms and 30 nondiagonal terms (half of them complex conjugated) is of the form

$$\begin{aligned} W(p_1,p_2,p_3) = B_0\{&6 + 2\langle\cos(p_1-p_2)(r_1-r_2)\rangle + \\ &+ 2\langle\cos(p_1-p_2)(r_1-r_3)\rangle + 2\langle\cos(p_1-p_2)(r_2-r_3)\rangle + \\ &+ 2\langle\cos(p_1-p_3)(r_1-r_2)\rangle + 2\langle\cos(p_1-p_3)(r_1-r_3)\rangle + \\ &+ 2\langle\cos(p_1-p_3)(r_2-r_3)\rangle + \\ &+ 2\langle\cos(p_2-p_3)(r_1-r_2)\rangle + 2\langle\cos(p_2-p_3)(r_1-r_3)\rangle + \\ &+ 2\langle\cos(p_2-p_3)(r_2-r_3)\rangle + \\ &+ 2\langle\cos[p_1(r_1-r_3) + p_2(r_2-r_1) + p_3(r_3-r_2)]\rangle + \\ &+ 2\langle\cos[p_1(r_1-r_2) + p_2(r_2-r_3) + p_3(r_3-r_1)]\rangle + \\ &+ 2\langle\cos[p_1(r_1-r_3) + p_2(r_3-r_2) + p_3(r_2-r_1)]\rangle + \\ &+ 2\langle\cos[p_1(r_1-r_2) + p_2(r_3-r_1) + p_3(r_2-r_3)]\rangle + \\ &+ 2\langle\cos[p_1(r_2-r_3) + p_2(r_1-r_2) + p_3(r_3-r_1)]\rangle + \\ &+ 2\langle\cos[p_1(r_3-r_2) + p_2(r_1-r_3) + p_3(r_2-r_1)]\rangle\}, \end{aligned} \quad (21)$$

where $B_0 \simeq |u(p)|^6$. Averaging is made over the space-time points r_1, r_2 and r_3. First we assume that $r_1 = r_2 = \rho$. This allows us to rewrite the formula (21) in a simpler form

$$W(p_1,p_2,p_3) = B_0[12 + 8\langle\cos(p_1-p_2)(\rho-r_3)\rangle + 8\langle\cos(p_1-p_3)(\rho-r_3)\rangle + 8\langle\cos(p_2-p_3)(\rho-r_3)\rangle]. \tag{22}$$

We now make the momenta p_1 and p_2 close to one another and also integrate formula (22) over the momentum p_3 as required by the considered inclusive approach. Then in the dominating part of phase space the following inequalities are fulfilled*

$$|\vec{p}_1-\vec{p}_3|\,R \gg 1, \quad |\vec{p}_2-\vec{p}_3|\,R \gg 1$$

leading to vanishing the last two terms in (22). Thus the corresponding two-particle probability

$$W(p_1,p_2) = \tilde{B}_0[12 + 8\langle\cos(p_1-p_2)(\rho-r_3)\rangle] =$$
$$= 12\tilde{B}_0[1 + \frac{2}{3}\langle\cos(p_1-p_2)(\rho-r_3)\rangle], \quad \tilde{B}_0 = \int B_0\, d^3\vec{p}_3$$

coincides with formula (6) for $n=2$ and $m=1$.

We now change the situation assuming the points r_1 and r_2 not coinciding but situated in a small region Δ satisfying the inequalities (5). Since the quantities $p_1\Delta$, $p_2\Delta$ and $p_3\Delta$ are assumed to be large enough, the six last terms in formula (21) vanish and the probability is written as

$$\begin{aligned} W(p_1,p_2,p_3) = B_0[6 &+ 2\langle\cos(p_1-p_2)(r_1-r_2)\rangle + 2\langle\cos(p_1-p_2)(r_1-r_3)\rangle + \\ &+ 2\langle\cos(p_1-p_2)(r_2-r_3)\rangle + \\ &+ 2\langle\cos(p_1-p_3)(r_1-r_2)\rangle + 2\langle\cos(p_1-p_3)(r_1-r_3)\rangle + 2\langle\cos(p_1-p_3)(r_2-r_3)\rangle + \\ &+ 2\langle\cos(p_2-p_3)(r_1-r_2)\rangle + 2\langle\cos(p_2-p_3)(r_1-r_3)\rangle + 2\langle\cos(p_2-p_3)(r_2-r_3)\rangle]. \end{aligned} \tag{23}$$

Approaching the momenta p_1 and p_2 close to one another and assuming the differences p_1-p_3 and p_2-p_3 large abough so that the inequalities

$$|\vec{p}_1-\vec{p}_2|\,R \lesssim 1, \quad |\vec{p}_1-\vec{p}_2|\Delta \ll 1, \quad |\vec{p}_1-\vec{p}_3|\Delta \gg 1, \quad |\vec{p}_2-\vec{p}_3|\Delta \gg 1 \tag{24}$$

are fulfilled, the second term in (23) yields 2, while the last six terms vanish. Integration over p_3 in formula (23) thus leads

* Here and below we assume that an analogous inequality for time components corresponds to each inequality for space ones.

to the following two-particle probability

$$W(p_1,p_2) = \tilde{B}_0[8 + 2\langle\cos(p_1-p_2)(r_1-r_3)\rangle + 2\langle\cos(p_1-p_2)(r_2-r_3)\rangle].$$

Since $\Delta \ll R$, we can put $r_1 \simeq r_2 \simeq \rho$ and rewrite this formula in the form

$$W(p_1,p_2) = 8\tilde{B}_0[1 + \frac{1}{2}\langle\cos(p_1-p_2)(\rho-r_3)\rangle]$$

coinciding with the formula (18) for $n=2$ and $m=1$.

If we increase the difference p_1-p_2, the second term in (23) becomes smaller and vanishes at $|\vec{p}_1-\vec{p}_2|\Delta \gg 1$. Consequently, in the considered situation there appears a peculiar dependence on the momentum difference $\vec{p}_1-\vec{p}_2$. Namely, at $|\vec{p}_1-\vec{p}_2|\,R \lesssim 1$ we have a narrow interference maximum with height $12\tilde{B}_0$, then at $\frac{1}{\Delta} \gg |\vec{p}_1-\vec{p}_2| \gg \frac{1}{R}$ it turnes into a "false plateau" with height $8\tilde{B}_0$, and after that a further decrease of the probability leads, at $|\vec{p}_1-\vec{p}_2|\Delta \gg 1$, to a "true plateau" with height $6\tilde{B}_0$ which is two times smaller than the height of the interference maximum.

The discussed example is absolutely typical, similar phenomena appear also in the case when not only two but several near-by one-particle sources or several groups of near-by sources are considered if only the characteristic parameters λ, Δ and R satisfy the inequalities (5). As a result, we come to the situation analyzed in our previous paper[1] where the interference correlations in the systems possessing two characteristic and essentially different space parameters have been considered.

Therefore the appearance of parameter $\lambda < 1$ in formulae of the type (2) certainly doesn't show evidence for simultaneous emission of pions by sources situated at one point. Similar phenomena take place also in the case when pions are emitted by two (or several) groups of sources - "near-by" and "distant". On the other hand, both alternative differ in the value of parameter λ and also in the presence or absence of "false plateau". Thus, in principle, experiment can separate one from the other. However, there is a number of complications (calculation of the concrete form of one-particle distributions, the problem of choosing background distributions, etc.) preventing, at the present time, from the implementation of such a program.

APPENDIX.

We consider $n+m$ one-particle sources situated at the space-time points $r_1, r_2, \ldots, r_{n+m}$. The amplitude for emission of identical pions with the momenta $p_1, p_2, \ldots, p_{n+m}$ is of the form

$$A(p_1,p_2,\dots,p_{n+m}) = A_0\Sigma\left(\prod_{\ell=1}^{n+m} e^{ip_k \ell^{r}\ell}\right), \qquad (A.1)$$

where the summation should be done over all permutations of momenta p_k. First, we assume that n sources are situated at the same space-time point and the other m sources, at different points. Since any permutation of n momenta connected with coinciding points $r_1 = r_2 = .. = r_n = \rho$ does not change at all the correspoinding n! terms in the sum in formula (A.1), the common factor n! can be taken out of the sum so that the summation should be done only over $N = (n+m)!/n!$ terms with different phases. Squaring this amplitude yields N diagonal terms equal to $|A_0|^2(n!)^2$ and N(N-1) nondiaginal interference terms (half of them complex conjugated).

We assume now that any two momenta (we denote them by p_1 and p_2) are so close that $|\vec{p}_1 - \vec{p}_2|R \lesssim 1$, while for all other pairs $|\vec{p}_i - \vec{p}_j|R \gg 1$. Then, after averaging over the space-time distribution of the sources, the probability contains only diagonal terms and terms corresponding to the interference of those addends in (A.1) which differ from each other in the interchange of momenta p_1 and p_2 only, and both these momenta should be connected with the sources situated at different points. The number of interfering pairs in (A.1) corresponding to the momenta p_1 and p_2 connected with different one-particle sources is equal to $\frac{1}{2}m(m-1)(m+n-2)!/n!$, each contributing to the probability of the process $2|A_0|^2(n!)^2 \langle \cos(p_1-p_2)(r_k - r_\ell)\rangle$. In our previous notation their total contribution is $[2|A_0|^2(n!)^2 m(m-1)\times (m+n-2)!/2(n!)] \langle\cos(q\ell)\rangle_{\beta\beta}$. If one of the considered pions with momenta p_1 and p_2 is emitted by the multi-particle source and the other by some of the one-particle sources, the number of such interfering pairs in (A.1) is equal to $nm(n+m-2)!/n!$, and their total contribution to the probability of the process composes $[2|A_0|^2(n!)^2 nm(n+m-2)/n!]\langle\cos(q\ell)\rangle_{\alpha\beta}$. Summing all the terms and integrating over the momenta $p_3, p_4, \dots, p_{n+m}$, we obtain for the inclusive two-particle probability the expression

$$W(p_1,p_2) = \tilde{B}_0(n!)(n+m-2)!\,\{(n+m)(n+m-1) + m(m-1)\langle\cos(q\ell)\rangle_{\beta\beta} + 2nm\langle\cos(q\ell)\rangle_{\alpha\beta}\}$$

which can be rewritten in the form

$$W(p_1,p_2) = \tilde{B}_0(n!)(n+m)!\,\{1 + \frac{m(m-1)}{(n+m)(n+m-1)}\langle\cos(q\ell)\rangle_{\beta\beta} + \frac{2nm}{(n+m)(n+m-1)}\langle\cos(q\ell)\rangle_{\alpha\beta}\},\quad \tilde{B}_0 = \int|A_0|^2 d^3\vec{p}_3\, d^3\vec{p}_4 \dots d^3\vec{p}_{n+m}. \qquad (A.2)$$

For $\langle\cos(q\ell)\rangle_{\alpha\beta} = \langle\cos(q\ell)\rangle_{\beta\beta}$ the formula (A.2) coincides with (6). Assuming $n,m \gg 1$ and introducing $\gamma = n/m$, the formula (A.2) takes the form

$$W(p_1,p_2) = (n!)(n+m)!\,\tilde{B}_0\{1 + \frac{2\gamma}{(1+\gamma)^2}\langle\cos(q\ell)\rangle_{\alpha\beta} + \frac{1}{(1+\gamma)^2}\langle\cos(q\ell)\rangle_{\beta\beta}\}, \qquad (A.3)$$

i.e., it coincides with the formula (8).

We now change n coinciding sources by a compact group of near-by one-particle sources situated in a small region Δ assuming that the parameters λ, Δ and R satisfy the inequalities (5). Then, in distinction from the previous case, there are no coinciding terms in the sum (A.1). Thus its squaring yields $(n+m)!$ addends, equal to $|A_0|^2$ plus the sum of bilinear products of each term in (A.1) by the complex conjugated value of any of the other terms. After averaging over the points r_1, $r_2, \dots, r_{n+m}$, all these products vanish except those in which the momenta p_1 and p_2 "change sources", while each of $n+m-2$ remaining momenta in both cofactors is connected with the same source. We assume again as in the basic text that $|\vec{p}_1 - \vec{p}_2|\Delta \ll 1$ and $|\vec{p}_i - \vec{p}_j|\Delta \gg 1$ for $i,j \neq 1,2$. Thus after integration over the momenta $p_3, p_4, \dots, p_{n+m}$ and averaging over the space-time distribution of the sources, the contribution to the probability from each pair of the sources from the compact group is equal to $2B_0$, and their total contribution composes $2\tilde{B}_0\frac{1}{2}n(n-1)(n+m-2)!$. If the two pions with momenta p_1 and p_2 are emitted by a pair of "distant" sources, the interference term appears equal, on average, to $2\tilde{B}_0\langle\cos(q\ell)\rangle_{\beta\beta}$. Since the number of such pairs equals $\frac{1}{2}m(m-1)(n+m-2)!$, their total contribution composes $2\tilde{B}_0\frac{1}{2}m(m-1)(n+m-2)!\,\langle\cos(q\ell)\rangle_{\beta\beta}$. Finally, the number of "mixed" pairs equals $nm(n+m-2)!$, and their total contribution to the probability of the process composes $2\tilde{B}_0 nm(n+m-2)!\langle\cos(q\ell)\rangle_{\alpha\beta}$. Summing all these addends, we get the expression

$$W(p_1,p_2) = \tilde{B}_0(n+m-2)!\,\{(n+m)(n+m-1) + n(n-1) + 2nm\langle\cos(q\ell)\rangle_{\alpha\beta} + m(m-1)\langle\cos(q\ell)\rangle_{\beta\beta}\}$$

which can be rewritten in the form

$$W(p_1,p_2) = W_0\{1 + \frac{2nm\langle\cos(q\ell)\rangle_{\alpha\beta}}{(n+m)(n+m-1)+n(n-1)} + \frac{m(m-1)\langle\cos(q\ell)\rangle_{\beta\beta}}{(n+m)(n+m-1)+n(n-1)}\},$$

$$W_0 = (n+m-2)!\,[(n+m)(n+m-1)+n(n-1)]\tilde{B}_0,\quad \tilde{B}_0 = \int|A_0|^2 d^3\vec{p}_3\, d^3\vec{p}_4 \dots d^3\vec{p}_{n+m} \qquad (A.4)$$

coinciding with the formula (18) in the particular case when $\langle\cos q\ell\rangle_{\alpha\beta} = \langle\cos q\ell\rangle_{\beta\beta}$.

If we take off the limitation $|\vec{p}_1 - \vec{p}_2|\Delta \ll 1$, then the contribution of each pair from the compact group is $2\tilde{B}_0 \langle\cos(q\ell)\rangle_{\alpha\alpha}$ instead of $2\tilde{B}_0$ leading to the probability

$$W(p_1, p_2) = \tilde{B}_0 (n+m-2)!\,\{(n+m)(n+m-1) + n(n-1)\langle\cos(q\ell)\rangle_{\alpha\alpha} +$$

$$+ m(m-1)\langle\cos(q\ell)\rangle_{\beta\beta} + 2nm\langle\cos(q\ell)\rangle_{\alpha\beta}\},$$

i.e.,

$$W(p_1, p_2) \sim 1 + \frac{n(n-1)\langle\cos(q\ell)\rangle_{\alpha\alpha}}{(n+m)(n+m-1)} + \frac{m(m-1)\langle\cos(q\ell)\rangle_{\beta\beta}}{(n+m)(n+m-1)} + \frac{2nm\langle\cos(q\ell)\rangle_{\alpha\beta}}{(n+m)(n+m-1)}. \qquad (A.5)$$

Thus for $|\vec{p}_1 - \vec{p}_2|R \lesssim 1$ we have a narrow interference maximum, a "false plateau" at $\frac{1}{R} \ll |\vec{p}_1 - \vec{p}_2| \ll \frac{1}{\Delta}$ and a "true plateau" at $|\vec{p}_1 - \vec{p}_2|\Delta \gg 1$. The height of the latter composes half the height of the interference maximum.

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Received by P
on Oct

Ледницки Р., Любошиц В.Л., Подгорецкий М.И. E2-82-725
Интерференционные корреляции тождественных частиц в моделях с близко расположенными источниками

Показано, что формулы интерференционных корреляций пар тождественных пионов с близкими импульсами, полученные в ряде работ с привлечением аппарата когерентных состояний, являются предельными случаями более общих соотношений, которые к тому же следуют из существенно более простого представления о независимых одночастичных источниках.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1982

Lednický R., Lyuboshitz V.L., Podgoretsky M.I. E2-82-725
Interference Correlations of Identical Particles in Models with Closely Spaced Sources

It is shown that the formulae for interference correlations in pairs of identical particles with nearly equal momenta, obtained in a number of papers in the framework of coherent state theory, turn out to be limiting cases of more general relations. Moreover, the latter follow from essentially simpler approach based on the model of independent one-particle sources.

The investigation has been performed at the Laboratory of High Energies, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1982