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LARGE AMPLITUDE NEUTRINO OSCILLATIONS WITH MAJORANA·MASS EIGENSTATES?

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1. QUALITATIVE CONSIDERATIONS

We wish to give here some physical arguments in favour of the idea that large amplitude and probably large length neutrino oscillations might be expected naturally in some schemes with Majorana mass neutrinos. Thus the observation of large amplitude oscillations might indicate the presence of Majorana mass neutrinos. Due to the high sensitivity of neutrino oscillation experiments /1,2/, this is of special significance if neutrino masses are smaller than a few eV, since in such a case the classical way to search for Majorana mass neutrinos - the neutrinoless double β -decay /3/,is impracticable(the corresponding decay rates being too small to be observed). From this point of view cosmic /4,5/ and solar /6/ neutrino investigations are of the utmost importance. We present at the beginning some qualitative and intuitive remarks. Let us first discuss oscillations with Dirac mass neutrinos /7/ whereby there are possible only "flavour oscillations" (the total lepton number must be conserved whereas electron, muon, tauon... lepton number are not conserved separately)*. For the sake of simplicity and of explanation of our point of view a definite oscillation between two neutrino states is considered below. Guessing the value of the mixing angle θ from general considerations is impossible. It might be close to the value $\pi/4$ (maximum mixing), but that would look, at least to us, like an accident in the case of Dirac mass neutrinos. This statement comes about if one recognizes that e , μ , τ are particles of widely different masses (and so would be presumably the neutrinos of definite masses). In other words the lepton mixing angle reminds us of the Cabibbo mixing angle and is not expected to be large for Dirac mass neutrinos.

On the other hand the situation might be different in the case of Majorana mass neutrinos. If neutrinos have a Majorana mass one should not be very surprised facing special situations, whereby one definite neutrino oscillation has an amplitude close

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[*] PC-invariance in the lepton sector is assumed. There are used the notations ν_1 , ν_2 , ν_3,... for particles of definite masses and ν_e , ν_μ , ν_τ ,... for the "phenomenological" particles undergoing the usual weak interaction. Among the neutrinos which have no definite masses,those which do not undergo the usual weak interaction will be called sterile(for example the right handed ν_e and the left handed $\bar{\nu}_e$: $\nu_{eR\,(ster)}$, $\bar{\nu}_{eL\,(ster)}$, etc.).

to the maximum one. We moved from an analogy with the $K^\circ \to \bar{K}^\circ$ [8] and $n \rightleftarrows \bar{n}$ [9] oscillations (if any), where oscillation amplitudes are large. Under PC-invariance in the case of such oscillations the particles with definite masses are neutral particles described by PC-eigenstates having opposite PC-values. As a matter of fact this means maximum mixing. The analogy for Majorana mass neutrinos is in no way full for the following reasons:

a) The mass difference $(m_1 - m_2)$ may be comparable with, and even larger than, the mass of the less heavy neutrino (in $K^\circ \rightleftarrows \bar{K}^\circ$ and $n \rightleftarrows \bar{n}$ oscillations $\Delta m \ll m_1, m_2$, the notations being obvious). The requirement that m_1 and m_2 must be positive, leads to the possibility that in certain cases there result two PC-eigenvalues of equal signs, in other cases of opposite signs [10]. The importance of this point for the neutrinoless double β -decay has been emphasized by L.Wolfenstein [11]. Incidentally, it should be noted that for Majorana fermions PC-eigenvalues, are pure imaginary [12] and only relative PC-eigenvalues have physical meaning.

b) The presence of at least a substantial neutrino helicity leads to effects typical only of neutrinos, effects which do not take place in the case of neutrons, the interaction of which is parity conserving. This can be seen right away by putting the question: how about the neutrino oscillation amplitude in the case of two Majorana neutrinos with different masses (four states) whereby the mass term is entirely right-left symmetrical? The answer is: the oscillation amplitude is maximum.

2. SOME APPLICATIONS

We would like to emphasize here that there are known only a few charged leptons: e , μ , τ. Therefore there is a relative small number of conceivable oscillations between neutrino states [13]: $\nu_e \rightleftarrows \nu_\mu$, $\nu_e \longrightarrow \nu_\tau$, $\nu_e \rightleftarrows \bar{\nu}_{eL(ster)}$, $\nu_e \rightleftarrows \bar{\nu}_{\mu L(ster)}$, $\nu_e \rightleftarrows \bar{\nu}_{\tau L(ster)}$, $\nu_\mu \rightleftarrows \nu_\tau$, $\nu_\mu \rightleftarrows \bar{\nu}_{\mu L(ster)}$, $\nu_\mu \rightleftarrows \bar{\nu}_{eL(ster)}$, $\nu_\mu \rightleftarrows \bar{\nu}_{\tau L(ster)}$, $\nu_\tau \rightleftarrows \bar{\nu}_{\tau L(ster)}$, $\nu_\tau \rightleftarrows \bar{\nu}_{eL(ster)}$, $\nu_\tau \rightleftarrows \bar{\nu}_{\mu L(ster)}$. The assumption is made that for every experimental facility there is only one "relevant" oscillation. So are defined oscillations of substantially large amplitudes and of effective oscillation length adequate (that is smaller or about equal to the distance source-detector).

Below 1) $\nu_e \rightleftarrows \bar{\nu}_{eL(ster)}$, 2) $\nu_\mu \rightleftarrows \bar{\nu}_{\mu L(ster)}$, 3) $\nu_e \rightleftarrows \nu_\mu$ oscillations* will be considered in schemes with Majorana mass eigenstates. The main application of the oscillation modes 1), 3) is in solar neutrino experiments: as we have stated large oscillation amplitudes are expected, and, as far as the effective oscillation length is concerned, either the "Sun-Earth facility" is adequate or the oscillations at issue will never be observed. The modes $\nu_\mu \rightleftarrows \nu_e$, $\nu_\mu \rightleftarrows \nu_\tau$, $\nu_\mu \rightleftarrows \bar{\nu}_{\mu L(ster)}$ are of importance also in the analysis of cosmic ray neutrino experiments, whereby the "atmospheric" ν_μ intensity is measured underground and compared with the intensity expected in the absence of oscillations (see especially the Chudakov group experiment) [5]. A similar statement can be made on ν_μ experiments of the Dumand type [14] and also on ν_μ experiments which will be the byproduct of the investigations in which proton decay is searched for in multi-kiloton detectors [15].

In solar neutrino and atmospheric ν_μ experiments a clear signature for oscillations may be discovered only if the oscillation amplitude is large. Hence the importance of Majorana mass schemes, which might just accomodate such a large amplitude. The solar experiments and atmospheric ν_μ experiments, in addition, are quite sensitive from the point of view of the well known parameter [2] $M^2 = |m_1^2 - m_2^2|$ (that is the experiment is adequate if $M^2 \geq 10^{-12}\,eV^2$ in the solar case and if $M^2 \geq 10^{-4}\,eV^2$ in the case of atmospheric cosmic ν_μ).

How about accelerator and reactor neutrino experiments? The main interest of such experiments is due to the possibility of getting information on small amplitude oscillations if the oscillation length is adequately small. Therefore the present note, dealing with oscillation lengths much larger than the distance source - neutrino detector in such experiments, is of no relevance for them.

We have been discussing essentially schemes with two Majorana massive neutrinos. Let us examine the situation with oscillation amplitudes in two concrete examples.

3. EXAMPLE 1, TWO MAJORANA MASS NEUTRINOS, PURE MAJORANA MASS TERM

Let us consider the case of two Majorana mass neutrinos, the masses of which are not both identically equal to zero. We shall assume a purely Majorana mass term. This implies that the neutrino field components which are present in the mass term are also present in the ordinary weak current. There are no sterile neutrinos. The striking two component weak current structure is fully preserved and the oscillations which arise are of the type, say, $\nu_e \longrightarrow \nu_\mu$ [10]. If we wish to preserve the notion of lepton charge (useful even in the case considered, where it is violated), we must recognize that there is only one lepton

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*Of course, instead of $\nu_e \rightleftarrows \nu_\mu$ oscillations, $\nu_e \rightleftarrows \nu_\tau$ oscillations might be more important. Oscillations $\nu_\mu \rightleftarrows \nu_\tau$ also might be relevant. Oscillations into sterile states of different flavour ($\nu_e \rightleftarrows \bar{\nu}_{\mu L(ster)}$, etc.) are not considered here, as they are probably of less importance.

charge*, which has opposite signs for e^- and μ^-. But this is the scheme proposed by Zeldovich[16] and by Konopinsky and Mahmoud[17], (renewed in connection with lepton charge non conservation), according to which e^- and μ^+ are particles with identical lepton charge. Maximum mixing appears to be very natural in this scheme. As a matter of fact the charged lepton current is

$$j_\alpha = 2[(\bar{\nu}_L \gamma_\alpha e_L) + (\bar{\mu}_R \gamma_\alpha \nu_R)] . \qquad (1)$$

All the four components of the neutrino field (only one!) are present in the current on the same footing.

Within the framework of the scheme considered here, it is natural to assume that ν_L and ν_R are present symmetrically also in the neutrino mass term. Such a term of the Lagrangian has the form

$$-\mathcal{L} = (\bar{\nu}^c \nu) \begin{pmatrix} m & \delta m \\ \delta m & m \end{pmatrix} \begin{pmatrix} \nu^c \\ \nu \end{pmatrix}, \qquad (2)$$

where m and δm are real parameters, and $\nu^c = C\bar{\nu}^T$ is the charge conjugate spinor. After diagonalization we have

$$-\mathcal{L} = \sum_{i=1,2} m_i \bar{\nu}_i \nu_i . \qquad (3)$$

Here $\nu_{1,2} = \frac{\nu \pm \nu^c}{\sqrt{2}}$ and $m_{1,2} = m \pm \delta m$. The analogy with the case of neutral kaons is evident. As mentioned above, the analogy is limited by the possibility that δm may be not small in comparison with m. For example, if $m_2 < 0$, the neutrino field of mass $-m_2$ is $\nu_2' = \gamma_5 \nu_2$, as it was remarked by V.Gribov[10]. In such a case neutrinos of definite masses have equal PC parities, and not opposite PC parities, as in the case when m_1 and m_2 are positive[11].

The physical reason why the Z-K-M scheme yields naturally a maximum mixing amplitude can be seen also as follows. In such a scheme there is one flavour shared by the electron and the muon and the oscillations $\nu_e \rightleftarrows \nu_\mu$ are, in fact, particle $\rightleftarrows$ antiparticle oscillations, similar to neutral kaon oscillations. Incidentally, one would expect the oscillations considered here to have a quite large oscillation length (small δm).

4. EXAMPLE 2, TWO MAJORANA MASS NEUTRINOS, COEXISTENCE OF MAJORANA AND DIRAC MASS TERMS

This example seems to be at a first glance quite unrealistic, but it serves the purpose of illustrating the situation with oscillations to and from a sterile particle, which are implied by the coexistence of Majorana and Dirac mass terms[13]. We may think of dealing with only one type of neutrinos, let us say ν_e, and putting ourselves the question as to whether oscillations $\nu_e \rightleftarrows \bar{\nu}_{eL(ster)}$ may have a maximum amplitude. The oscillation picture is fully described by 3 parameters having the dimensions of a mass, $m_{act;\,act}$, $m_{ster;\,ster}$, $m_{act;ster}$, which are present in the lepton number violating mass term:

$$-\mathcal{L} = \bar{\nu}^c_{eR} \nu_{eL} m_{act;\,act} + \bar{\nu}_{eR} \bar{\nu}^c_{eL} m_{ster;\,ster} + + (\bar{\nu}_{eR}\nu_{eL} + \bar{\nu}^c_{eR} \nu^c_{eL}) m_{act;\,ster} + \text{h.c.} \qquad (4)$$

If the equality

$$m_{act;\,act} = m_{ster;\,ster} \qquad (5)$$

holds, the Lagrangian (4) has the form (2) (with the obvious replacement $m \rightarrow m_{act;ster}$ and $\delta m \rightarrow m_{act;\,act}$). The Majorana neutrino fields with masses $m_{1,2} = m_{act;ster} \pm m_{act;\,act}$ are $\nu_{1,2} = (\nu_e \pm \nu_e^c)/\sqrt{2}$. The oscillation $\nu_e \rightleftarrows \bar{\nu}_{eL(ster)}$ has maximum amplitude.

5. CONCLUSIONS

Of course the oscillations of the type $\nu_e \rightleftarrows \bar{\nu}_{eL(ster)}$, $\nu_\mu \rightleftarrows \bar{\nu}_{\mu L(ster)}$, which we have just been discussing for the case of Majorana mass eigenstates, are not possible in the case of Dirac mass eigenstates.

Oscillations of the type $\nu_e \rightleftarrows \nu_\mu$, which we have been discussing in the case of two Majorana mass eigenstates (four neutrino states), instead, can take place also in the case of two Dirac mass eigenstates[7] (eight neutrino states). It is instructive to compare the conditions for maximum oscillation amplitudes for Majorana and Dirac mass eigenstates. In the Majorana case the mass term can be written as[10]

$$-\mathcal{L} = \bar{\nu}^c_R \nu_L m^M_{ee} + \bar{\nu}_R \nu^c_L m^M_{\mu\mu} + + (\bar{\nu}^c_R \nu^c_L + \bar{\nu}_R \nu_L) m^M_{\mu e} + \text{h.c.} \qquad (6)$$

and the condition for maximum amplitude is

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* If there were two lepton charges, which are not conserved exactly, we would be faced with eight objects and not with four, as we should. Thus the Zeldovich and Konopinsky-Mahmoud scheme is not far fetched. It is required by our assumptions.

$$m^M_{ee} = m^M_{\mu\mu} \tag{7}$$

a symmetry which we have found plausible on physical grounds (the renewed ZKM scheme, implied in the case at issue, see §3). In the Dirac case the mass term can be written as

$$-\mathcal{L} = \bar{\nu}_{eR}\nu_{eL}\, m^D_{ee} + \bar{\nu}_{\mu R}\,\nu_{\mu L}\, m^D_{\mu\mu} + \bar{\nu}_{eR}\,\nu_{\mu L}\, m^D_{e\mu} + \bar{\nu}_{\mu R}\,\nu_{eL}\, m^D_{\mu e} + \text{h.c.} \tag{8}$$

and the conditions for maximum oscillation amplitude are*

$$\begin{aligned} m^D_{ee} &= m^D_{\mu\mu}\,, \\ m^D_{e\mu} &= m^D_{\mu e} \end{aligned} \tag{9}$$

The parameters m_{ee} and $m_{\mu\mu}$ physically, are the "bare" masses of the electron and the muon neutrinos and it is hard to expect them to be equal.

Thus the "true neutrality" of Majorana mass neutrinos (at a variance with the case of Dirac mass eigenstates), seems to make plausible the full analogy between neutrino oscillations and $K^\circ \rightleftarrows \bar{K}^\circ$, $n \rightleftarrows \bar{n}$ oscillations. This analogy in fact implies maximum oscillation amplitude.

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REFERENCES

1. Pontecorvo B. JETP, 1957, 33, p.549; 1958, 34, p.247; 1967, 53, p.1717.
2. For a review see: Bilenky S.M., Pontecorvo B. Phys.Rep., 1978, 41, p.226.
3. Doi M. et al. Prog.Theor.Phys., 1981, 66, p.1769; Rosen S.P. Proc. of the Int.Conf. "Neutrino-81", Hawaii, 1981, vol.II, p.76.
4. Crouch M.F. et al. Phys.Rev., 1978, D18, p.2239.
5. Chudakov A.E. et al. Proc. of the XVII Conf. on Cosmic Rays, Paris, 1981.
6. Davis R., Evans J.C., Cleveland B.T. Proc. of the Int. Conf. "Neutrino-78", Purdue, 1978, p.53.
7. Nakagawa M. et al. Prog.Theor.Phys., 1963, 30, p.727; Bilenky S.M., Pontecorvo B. Phys.Lett., 1976, 61B, p.248; Fritzsch H., Minkovsky P. Phys.Lett., 1976, 62B, p.72.
8. Gell-Mann M., Pais A. Phys.Rev., 1955, 97, p.1387.
9. Kuz'min V. Pis'ma JETP, 1978, 12, p.335.
10. Gribov V., Pontecorvo B. Phys.Lett., 1969, 28B, p.493.
11. Wolfenstein L. Phys.Lett., 1981, 107B, p.77.
12. Carruthers P. Spin and Isospin in Particle Physics. Gordon and Breach, New York, 1971, p.132.
13. Bilenky S.M., Pontecorvo B. Proc. of the XVII Int.Conf. on High Energy Physics, Tbilisi, 1976. JINR, D1,2-10400, Dubna, 1977, vol.2, p.B122; Bilenky S.M., Pontecorvo B. Lett.Nuovo Cim., 1976, 17, p.569.
14. See for example: Proc. of the 1978 DUMAND Summer Workshop. A.Roberts (Editor), 3 cols. DUMAND, Scripps Inst. of Oceanography, 1979.
15. See for example: Fiorini E. Proc. of the Int.Conf. "Neutrino-82", Balatonfured, 1982.
16. Zeldovich Ya.B. DAN SSSR, 1952, 86, p.505.
17. Konopinsky E.J., Mahmoud H. Phys.Rev., 1953, 92, p.1045.

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*It should be noticed that we have not been discussing here the conditions of a different character which would also lead to a maximum amplitude of the oscillations considered: $m^M_{e\mu} \gg m^M_{ee}$, $m^M_{\mu\mu}$; $m^D_{e\mu} \simeq m^D_{\mu e} \gg m^D_{ee}$, $m^D_{\mu\mu}$ for $\nu_e \rightleftarrows \nu_\mu$ oscillations and $m_{ster;\,act} \gg m_{act;\,act}$, $m_{ster;\,ster}$ for $\nu_e \rightleftarrows \bar{\nu}_{eL(ster)}$ oscillations.

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Биленький С.М., Понтекорво Б.М. E2-82-722
Нейтрино с майорановскими массами
и большие амплитуды осцилляций?

Приводятся аргументы в пользу того, что в случае нейтрино с майорановскими массами лидирующие осцилляции между двумя нейтринными состояниями характеризуются большой амплитудой и, возможно, большой длиной осцилляции. В связи с этой гипотезой подчеркивается особая роль опытов с солнечными и космическими нейтрино.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1982

Bilenky S.M., Pontecorvo B.M. E2-82-722
Large Amplitude Neutrino Oscillations with Majorana Mass Eigenstates?

Arguments are given in favour of the hypothesis that, for Majorana mass neutrinos, a leading oscillation between two neutrino states could well be characterized by a large amplitude (and probably a large oscillation length): hence the great importance of solar neutrino and cosmic ray neutrino observations.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1982