204/83

объединенный
институт
ядерных
исследований
дубна

10/1-83

E2-82-714

S.N.Nikolaev, A.V.Radyushkin

LARGE POWER CORRECTIONS TO QCD CHARMONIUM SUM RULES

Submitted to "Письма в ЖЭТФ"

1982

1. INTRODUCTION

The QCD sum rule technique[1] is now very popular among the QCD practitioners. Although in calculational aspects this technique does not essentially differ from the perturbative QCD methods, it provides a possibility of analysing the long-distance (i.e., non-perturbative) properties of the hadrons, such as masses, leptonic widths, etc. This is achieved by inclusing into the theory non-zero vacuum expectation values (VEV's) of certain local operators like $<\bar{\psi}\psi>$, $<g^2 G^a_{\mu\nu} G^a_{\mu\nu}>$ which may be treated as fundamental constants characterizing the QCD vacuum structure.

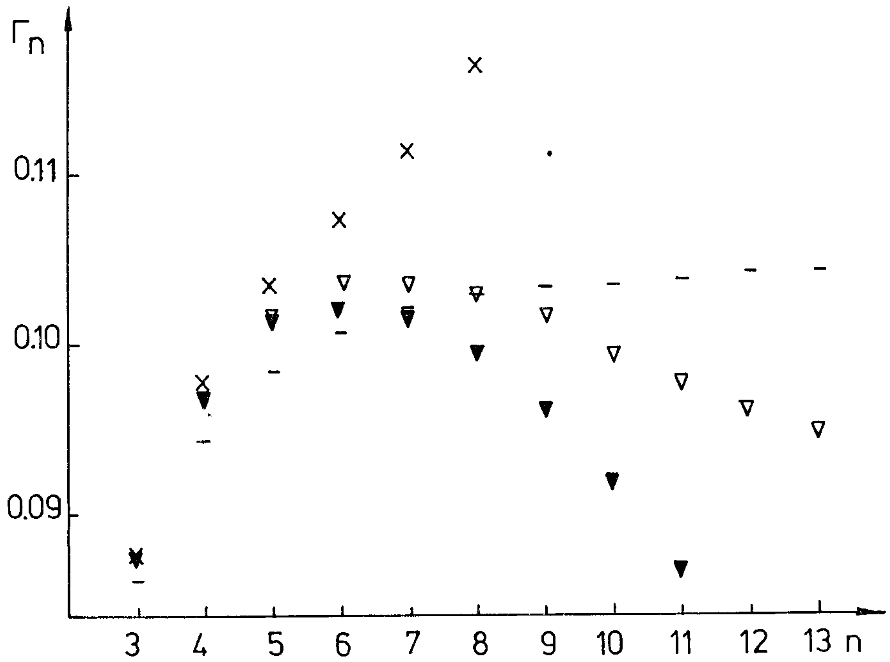
To test the soundness of its theoretical basis the method was applied to non-relativistic potential theory[2-4] and QCD in two dimensions[5,9], where the exact solution is known. Within the framework of these models it was established that while the whole picture is quite in accord with that implied in ref.[1], the power series over $1/m^2_q$ converges rather slowly, and as a consequence, the estimates of the confinement parameter ϕ (analogous to $<g^2G^2>/m^4_q$) obtained from fitting the exact result (i.e., "experimental data") by the lowest non-trivial approximation are very crude. Normally, ϕ is underestimated by a factor 2-3. It is important, therefore, to investigate whether the same is true in a more realistic (i.e., four-dimensional) QCD theory or maybe luckily the convergence of the $1/m^2_q$ series in this case is better than in two dimensions.

2. OPERATOR STRUCTURE OF $O(G^3)$ AND $O(G^4)$ CONTRIBUTIONS

To analyse the convergence of the $1/m^2_c$ series for QCD charmonium sum rules of ref.[1] we computed $O(m_c^{-6})$ and $O(m_c^{-8})$ corrections* to the relevant polarization operators $\Pi(Q^2,m^2_c)$ using the method described in refs.[8,9] and the computer program SCHOONSCHIP[7] written by M.Veltman. Our results for the ratio $r_n = M_n/M_{n-1}$ introduced in ref.[1] (where $M_n = (-d/dQ^2)^n \Pi^V(Q^2,m^2_c)/n!$ for $Q^2 = 0$, and $\Pi^V(Q^2,m^2_c)$ is the polarization operator related

---

* Since the $O(m_c^{-6}) = O(G^3)$ contribution is damped by an additional factor 1/15[3,10] one should calculate $O(m_c^{-6})$ and $O(m_c^{-8})$ terms together.



(–) experiment; (▼) theoretical curve including $O(\alpha_s)$ and $O(G^2)$ corrections, $\langle g^2G^2\rangle = (0.83\ \mathrm{GeV})^4$; (▽) the same with $O(G^3)$ added; (x) the same with $O(G^3)$ and $O(G^4)$ added.

to the vector, i.e., J/ψ -channel), are presented in the figure. We recall that the curve based on experimental data tends to $\mathfrak{M}^{-2}_{J/\psi} = 0.104$ as $n \to \infty$. Thus, if one is able to calculate theoretically the magnitude of r_n in the region where the experimental value is close to the asymptotic one (practically, for $n \geq 6$), one can estimate the relevant VEV's from known $\mathfrak{M}_{J/\psi}$ value; and vice versa – one can estimate $\mathfrak{M}_{J/\psi}$ (and/or the masses of the lowest charmonium states in other channels) provided that the VEV's values are known. Hence, for the lowest orders of the $\langle (G/m_c^2)^N\rangle$ expansion it is most interesting to study r_n for $n=6$:

$$r_6 = \frac{7}{36\, m_c^2}\Big\{ 1 - 0.40\,\alpha_s(2m_c) - 0.46\,\frac{\langle g^2 G^a_{\mu\nu} G^a_{\mu\nu}\rangle}{m_c^4} + $$

$$+ \frac{1}{m_c^6}\big[0.59 \langle g^3 f_{abc} G^a_{\mu\nu} G^b_{\nu\lambda} G^c_{\lambda\mu}\rangle - 2.08 \langle g^4 j^a_\mu j^a_\mu\rangle\big] +$$

$$+ \frac{1}{m_c^8}\big[3.63 \langle g^4 \mathrm{Sp}(\hat G_{\mu\nu}\hat G_{\mu\nu}\hat G_{\alpha\beta}\hat G_{\alpha\beta})\rangle - 0.49 \langle g^4 \mathrm{Sp}(\hat G_{\mu\nu}\hat G_{\alpha\beta}\hat G_{\mu\nu}\hat G_{\alpha\beta})\rangle +$$

$$+ 33.13 \langle g^4 \mathrm{Sp}(\hat G_{\mu\nu}\hat G_{\nu\alpha}\hat G_{\alpha\beta}\hat G_{\beta\mu})\rangle - 7.08 \langle g^4 \mathrm{Sp}(\hat G_{\mu\nu}\hat G_{\alpha\beta}\hat G_{\nu\alpha}\hat G_{\beta\mu})\rangle -$$

$$- 7.07 \langle g^5 f_{abc} G^a_{\mu\nu} j^b_\mu j^c_\nu\rangle - 2.92 \langle g^3 f_{abc} G^a_{\mu\nu} G^b_{\nu\lambda} G^c_{\lambda\mu;\,\alpha\alpha}\rangle + \qquad (1)$$

$$+ 3.03 \langle g^4 j^a_\mu j^a_{\mu;\,\alpha\alpha}\rangle - 0.36 \langle g^2 G^a_{\mu\nu} G^a_{\mu\nu}\rangle^2 \big]\Big\},$$

where $\hat G = G^a\lambda^a/2$, λ^a are the Gell-Mann matrices and the coloured vector current of light (u,d,s) quarks j^a_μ appeared due to equations of motion $G^a_{\mu\nu;\nu} = g j^a_\mu$. The notation $O^a_{;\mu}$ is used for the covariant derivative $D^{ab}_\mu O^b$. The origin of the $\langle g^2G^2\rangle^2$-term can be traced to the fact that r_n is the ratio of two quantities expanded into the $O(G^N)$ series.

The very structure of eq. (1) suggests that the expansion we have to deal with is more complicated than simply a series in powers of $\langle G^2\rangle/m_c^4$, and the magnitude of higher power corrections depends on the (poorly known) properties of the QCD vacuum structure, or in other words, on the estimates used to specify the values of the VEV's appeared in eq. (1).

3. ESTIMATES OF THE VEV's VALUES

The curves shown in the figure imply the following system of estimates for the relevant VEV's:

a) for $\langle g^2G^2\rangle$ we take the SVZ value $(0.83\ \mathrm{GeV})^4$ obtained in ref./1/ by fitting the experimental data by the $O(G^2)$ curve (see the figure);

b) for $\langle jj\rangle$ and $\langle fGjj\rangle$ we assume the dominance of the intermediate vacuum state /1/ (vacuum dominance hypothesis, VDH): $\langle j^a_\mu j^a_\mu\rangle = -\frac{4}{3}\langle \bar u u\rangle^2$, $\langle f_{abc} G^a_{\mu\nu} j^b_\mu j^c_\nu\rangle = -\frac{3}{4}\langle \bar u u\rangle\langle \bar u \tau_{\mu\nu}\hat G_{\mu\nu} u\rangle$. According to estimates made by various authors /1,11/ the VDH for 4-quark operators is accurate within at least 50%. For $\langle \bar u u\rangle$ we take the value $-(0.24\ \mathrm{GeV})^3$ extracted from the current algebra analysis /1/. Furthermore, according to available estimates $\langle g\bar u(\sigma\hat G)u\rangle/\langle \bar u u\rangle = (0.5-1)\ \mathrm{GeV}^2$ /12/. We take for this ratio the value $0.6\ \mathrm{GeV}^2$ that agrees with our recipe d) below;

c) $\langle g^3 fG^3\rangle$ is calculated within the dilute instanton gas approximation (DIGA). If one assumes that $\langle g^2G^2\rangle = (0.83\ \mathrm{GeV})^4$ then DIGA predicts that $\langle g^3 fG^3\rangle = (0.60\ \mathrm{GeV})^6$ /1/ in complete agreement with the estimate of this VEV in lattice gauge theory /13/.

d) for the VEV's of operators containing the combination $D^\alpha D_\alpha$ we assume that $\langle fGGG_{;\alpha\alpha}\rangle = M^2\langle fGGG\rangle$, $\langle jj_{;\alpha\alpha}\rangle = M^2\langle jj\rangle$ where $M^2 = \langle GG_{;\alpha\alpha}\rangle/\langle GG\rangle$ is the parameter characterizing the average off-shellness of the vacuum gluons. Using the relation $\langle GG_{;\alpha\alpha}\rangle = 2\langle gfG^3\rangle - 2\langle g^2 j^2\rangle$ and the recipes a)–c) we find the value $M = 0.52$ GeV that agrees with the physical interpretation of this parameter. In a similar way, using $\langle g\bar{u}(\sigma\hat{G})u\rangle = 2\langle\bar{u}\hat{D}_\alpha\hat{D}_\alpha u\rangle$ and assuming that the average off-shellnesses of the vacuum quarks and gluons are equal, we obtain the estimate $\langle g\bar{u}(\sigma\hat{G})u\rangle = 2M^2\langle\bar{u}u\rangle$ (cf. with b));

e) for VEV's of $\langle g^4 Sp(G^4)\rangle$ type we also incorporate the VDH, that gives the values $\frac{110}{1152}$, $\frac{20}{1152}$, $\frac{47}{1152}$, $\frac{29}{1152}$ for the ratio $\langle g^4 Sp(G^4)\rangle/\langle g^2G^2\rangle^2$ related to the 1st-4th terms of the $O(m_c^{-8})$-contribution in eq. (1), respectively. It is worth noting here that there exist some claims in the literature[11,14] that the VDH is not applicable to estimate the VEV's of operators constructed from the gluon fields. According to refs.[11,14] the VDH underestimates the ratio of $\langle G^4\rangle/\langle G^2\rangle^2$ by a factor 5-10;

f) all VEV's are normalized at $\mu^2 = -4m_c^2$, $m_c = 1.26$ GeV and it is taken into account that the combinations $\langle g^2G^2\rangle$, $\langle g^3 fG^3\rangle$ and $g\langle\bar{\psi}\psi\rangle$ may be treated as renorm-invariant quantities. We take $\Lambda = 0.1$ GeV, i.e., $\alpha_s(2m_c) = 0.2$ and $\alpha_s(\mu_0) = 0.7$, where μ_0 is the renormalization point, where $\langle\bar{u}u\rangle = -(240\ \mathrm{MeV})^3$ [1].

4. LARGE $O(G^4)$-CORRECTIONS

Using the VEV's values estimated according to the above recipes we obtain for r_6

$$r_6 = 0.1225\,\{1 - 0.080 - [0.086] + [0.007 + 0.003] + \\ + [0.012 - 3\times10^{-4} + 0.048 - 0.006 + 0.002 - 0.006 - 8\cdot10^{-4} - 0.013]\}\,. \tag{2}$$

From eqs. (1), (2) it is clear that the $O(G^4)$ contribution for $n = 6$ is large and that the largest part of it is due to the only VEV $\langle g^4 Sp(\hat{G}_{\mu\nu}\hat{G}_{\nu\lambda}\hat{G}_{\lambda\alpha}\hat{G}_{\alpha\mu})\rangle \equiv O_4^{(3)}$ associated with the largest coefficient in eq. (1). The latter exceeds all other coefficients by roughly an order of magnitude*. Notice also that non-negligible negative contributions in eq. (2) are due to $\langle GGG_{;\alpha\alpha}\rangle$, $\langle G^2\rangle^2$ and the fourth VEV of the $\langle G^4\rangle$ tape. In any reasonable model of the QCD vacuum these VEV's cannot be increased without increasing also $O_4^{(3)}$. Another observation is that if one increases all $\langle G^4\rangle$-VEV's by a factor of ten (i.e., if one adheres to the model[11]) then the $O(G^4)$-correction for $n = 6$ will be 5 times larger than the $O(G^2)$ one. However, even if we take the moderate estimate for $O_4^{(3)}$ given by the VDH, the inclusion of the $O(G^4)$ terms radically changes the theoretical curve for r_n (see the figure). In particular, the latter has no plateau and can fit the experimental data only for $n = 2,3,4$. The value of $\langle g^2G^2\rangle$ extracted from such a fitting procedure (for fixed mass $m_c = 1.26$ GeV) is about two times larger than the SVZ estimate of ref.[1] (cf. refs.[4-6]).

*It should be emphasized that in our system of estimates all VEV's contributing to the $O(m_c^{-8})$ -correction have the same order of magnitude.

5. CONCLUSIONS

Thus, in the real QCD the series over $1/m_c^2$ converges rather slowly. In particular, to get a stable (with respect to higher power corrections) plateau of the function r_n one should take into account $O(G^5)$- and $O(G^6)$-corrections, at least. In such a situation the procedure of extracting $\langle g^2G^2\rangle$ proposed in ref.[1] reduces to finding an effective parameter absorbing (for $n = 5$-7) the higher corrections up to $O(G^6/m_c^{12})$ or even $O(G^8/m_c^{16})$. Noting that the $(G^2/m_c^4)^k$ terms have alternating signs one should expect that $\langle g^2G^2\rangle_{eff}$ for bottomium states must be greater than for charmonium ones, just as it was observed by Voloshin[10]. Furthermore, the value of $\langle g^2G^2\rangle_{eff}$ in the pseudoscalar charmonium channel may differ from that in the vector one; though the successful prediction of the η_c mass[15] indicates rather that these values are close to each other. However, in the light of the observed slow convergence of the series the successes of the QCD charmonium sum rule approach look as highly surprising rather than well-founded; and the soundness of the theoretical basis for the existing applications of this approach deserves further thorough investigation.

We are indebted to A.V.Efremov and B.M.Barbashov for discussion of results and to V.A.Meshcheryakov for interest in this work and support.

REFERENCES

1. Shifman M.A., Vainshtein A.I., Zakharov V.I. Nucl.Phys., 1979, B147, pp.385,447.
2. Vainshtein A.I. et al. Yad.Fiz., 1980, 32, p.1622; Novikov V.A. et al. Nucl.Phys., 1981, B191, p.301.
3. Voloshin M.B. Nucl.Phys., 1979, B154, p.365.
4. Bell J.S., Bertlmann R.A. Nucl.Phys., 1981, B177, p.218; B187, p.285.
5. Bradley A., Langensiepen C.S., Shaw G. Phys.Lett., 1981, 102B, p.359.
6. Ditsas P., Shaw G. Preprint M/C-TH-82-09, Manchester Univ., 1982.
7. Strubbe H. Comp.Phys.Comm., 1974, 8, p.1.

8. Nikolaev S.N., Radyushkin A.V. Phys.Lett., 1982, 110B, p.476.
9. Nikolaev S.N., Radyushkin A.V. JINR, E2-82-521, Dubna, 1982.
10. Voloshin M.B. Preprint ITEP-21, Moscow, 1980.
11. Shuryak E.V. Nucl.Phys., 1982, B203, pp.93,116, 140.
12. Novikov V.A. et al. Proc.Int.Conf. "Neutrino-78". Lafayethe, 1978, p.C278.
13. Di Giacomo A. et al. Preprint IFUP-TH 13/82, Pisa, 1982.
14. Baier V.N., Pinelis Yu.F. Preprint IYaF 81-141, Novosibirsk, 1981.
15. Shifman M.A. et al. Phys.Lett., 1978, 77B, p.80.

Received by Publishing Department
on October 4 1982.

SUBJECT CATEGORIES OF THE JINR PUBLICATIONS

Index	Subject
1.	High energy experimental physics
2.	High energy theoretical physics
3.	Low energy experimental physics
4.	Low energy theoretical physics
5.	Mathematics
6.	Nuclear spectroscopy and radiochemistry
7.	Heavy ion physics
8.	Cryogenics
9.	Accelerators
10.	Automatization of data processing
11.	Computing mathematics and technique
12.	Chemistry
13.	Experimental techniques and methods
14.	Solid state physics. Liquids
15.	Experimental physics of nuclear reactions at low energies
16.	Health physics. Shieldings
17.	Theory of condenced matter
18.	Applied researches
19.	Biophysics

Николаев С.Н., Радюшкин А.В. E2-82-714

Большие степенные поправки к КХД правилам сумм для чармония

Показано, что учет $O(G^3)$- и $O(G^4)$ -поправок радикально меняет вид теоретической кривой для отношения r_n, используемого при анализе низших состояний чармония методом КХД правил сумм, что ставит под сомнение надежность теоретической базы существующих приложений этого метода.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1982

Nikolaev S.N., Radyushkin A.V. E2-82-714

Large Power Corrections to QCD Charmonium Sum Rules

It is shown that inclusion of the $O(G^3)$ and $O(G^4)$ corrections radically changes the theoretical curve for the ratio r_n used to analyse the lowest charmonium states within the QCD sum rule approach. This casts doubt on the soundness of the theoretical basis underlying the existing applications of this approach.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1982