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DIAGRAM SUMMATION METHOD IN SCHWINGER SOURCE THEORY

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1. In the source theory [1] it is shown that the total photon propagator can be represented in the Kallen-Lehmann form:

$$D(-k^2) = D_0(-k^2) + \int_0^{+\infty} \frac{dM^2}{k^2+M^2-i0}\,\rho(M^2), \qquad (1)$$

$$D_0(-k^2) = \frac{1}{k^2-i0}, \quad \rho(M^2) > 0.$$

Here our aim is to obtain an equation that connects $D(-k^2)$ with the polarizable operator.

We shall consider the detection photon source $J_1^\mu(x)$ localized in the remout future with respect to the production source $J_2^\mu(x)$. The first term in (1) describes the real ($k^2=0$) photon exchange. However, the source J_2^μ can emit also the virtual ($k^2\neq 0$) photon. Its field

$$A_2^\mu(x) = \int dx' D_0^P(x-x')J_2^\mu(x')$$

is localised in the source vicinity. Indeed, if one eliminates the value $k^2=0$ in propagator, that means the change $\frac{1}{k^2-i0} \to P\frac{1}{k^2}$ (P is the symbol of the Cauchy principal value for integrals, the propagators obtained in this way are marked by P), it is easy to make oneself sure in the exponential damping of the field with moving away from the function $J_2^\mu(x)$ carrier. Then the virtual photon $(-k^2=M^2)$ cannot reach the remote detector $J_1^\mu(x)$. But in an immediate vicinity of the source the photon can turn into a system of real particles (that is described by the polarizable operator) capable of propagating to the detector, near which the reverce transformation into the virtual photon and the absorption of the latter are possible. It is just the mechanism (it can be infinitely iterated also between the sources) that corresponds to the spectral integral in (1)(see [1]).

All these decays-recombinations can be summed in the following way. Let us consider the last transformation of the photon to the system of real particles before detector J_1. Then the corresponding term in the vacuum amplitude [1] has the form

$$-\int_0^{+\infty} dM^2 d\omega_k\, A_1^\mu(-k) I(M^2) A_{2\mu}(k), \qquad (2)$$

where $d\omega_k = \frac{d^3k}{(2\pi)^3}\frac{1}{2k_0}$, $-k^2 = M^2$, A_1 corresponds to the "bare" virtual photon, A_2 - to the "dressed" virtual photon, and $I(M^2)$ describes the system of real particles (see below (5)). So

$$A^{\mu}_1(-k) = D^P_0(-k^2)\, J^{\mu}_1(-k), \qquad A^{\mu}_2(k) = D^P(-k^2)\, J^{\mu}_2(k).$$

The use of propagators D^P_0 and D^P instead of D_0 and D is of great importance, because only in this way the causal sequence can be controlled. Besides, as we have already seen, virtual particles must be described just by these propagators /1/.

The expression (2) is reduced to

$$i \int dx\, dx'\, J^{\mu}(x) \times$$
$$\times \int dM^2 D^P_0(M^2) I(M^2) D^P(M^2) [\, i \int d\omega_k \exp[\, ik(x-x')]] J_{2\mu}(x'),$$

that corresponds to the virtual-photon contribution in the total propagator:

$$\int_0^{\infty} dM^2 D^P_0(M^2) I(M^2) D^P \frac{1}{k^2 + M^2 - i0}.$$

The equation sought follows immediately:

$$D(-k^2) = D_0(-k^2) + \int_0^{\infty} \frac{dM^2}{k^2 + M^2 - i0} D^P_0(M^2) I(M^2) D^P(M^2). \qquad (3)$$

It should be noted that this expression has the form of (1).

2. It is interesting to compare equation (3) with the ordinary quantum-field theory correlation

$$D(-k^2) = D_0(-k^2) + D_0(-k^2) P(-k^2) D(-k^2), \qquad (4)$$

with the polarizable operator

$$P(-k^2) = \frac{1}{D^2_0(-k^2)} \int \frac{dM^2}{k^2 + M^2 - i0} I(M^2) D^2_0(M^2). \qquad (5)$$

Expression (4) in the finite-loop approximation leads to the well-known pole of $D(-k^2)$ at the space-like $-k^2 < 0$. It is obvious that expression (4) cannot be reduced to the form of (3) and in the source theory can be obtained only with the explicit violation of the causality principle, as it has been made in /2/.

3. When comparing (3) and (1) it is easy to obtain an equation for the spectral function $\rho(t)$:

$$\rho(t) = D_0(t) I(t) [\, D_0(t) + P \int_0^{\infty} \frac{dt'}{t'-t} \rho(t')] .$$

The investigation shows /3/ that there exists the only solution for this equation:

$$\rho(t) = \frac{I(t)}{t^2 V^2(t)} - \frac{I(t) Z^P(t)}{t V(t)} P \int_0^{\infty} \frac{dt'}{t'-t} \frac{I(t')}{t'^2 V(t') Z^P(t')},$$

where

$$V^2(t) = 1 + [\pi t^{-1} I(t)]^2, \qquad Z(t) = \exp \Gamma(t),$$

$$\Gamma(t) = -\frac{t}{\pi} \int \frac{dt'}{t' - t - i0} \frac{1}{t'} \operatorname{arctg}[\pi t'^{-1} I(t')]$$

and $\rho(t) \geq 0$ as $t \geq 0$.

It is interesting that $D(t) = D_0(t) Z(t)$.

In one-loop approximation of spinor electrodynamics (which is typical of finite-loop ones here)

$$I(t) = \frac{\alpha}{3\pi} t (1 + \frac{2m^2}{t})(1 - \frac{4m^2}{t})^{1/2} \qquad \text{as } t \geq 4m^2,$$

$I(t) = 0$ as $t < 4m^2$. $\alpha = 1/137$, m is the electron mass. On investigating asymptotics $|t| \to \infty$, we obtain $\rho(t) = O(t^{-1+\kappa})$, where $\kappa = \pi^{-1} \operatorname{arctg}(\alpha/3) \approx 10^{-4}$, what leads to $D(t) = O(t^{-1+\kappa})$ and to the divergence of the "invariant charge" with $t \to -\infty$, as it is required /4/ in the source theory. In the same approximation one can make sure that Cauchy-Riemann conditions are fulfilled and ρ is an analytic function of the coupling constant α in the vicinity of $\alpha = 0$.

4. The problem of construction the approximate expressions for $D(t)$ in form (1) has been solved in papers /5, 6/ with the help of the summation rule in the spectral integrand. The finite value of "bare" charge (i.e., the absence of zero-charge asymptotics) and the non-analyticity of $\rho(\alpha)$ near $\alpha = 0$ were observed. It is seen that the method investigated in the present paper is quite different from the mentioned one. It does not require additional hypotheses and utilizes only the basic principles of the theory.

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Вышенский С.В. E2-82-681
Метод суммирования диаграмм в теории источников Швингера

Найдена причина появления нефизических сингулярностей в полных пропагаторах квантовой электродинамики. Развит последовательный метод получения уравнений для полных функций Грина, точные и приближенные решения которых свободны от нефизических сингулярностей и аналитичны в нуле по константе связи. Рассмотрен фотонный пропагатор в однопетлевом приближении для поляризационного оператора.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Vyshensky S.V. E2-82-681
Diagram Summation Method in Schwinger Source Theory

The origin of non-physical singularities in total quantum electrodynamics propagators is discovered. A consistent method of obtaining equations for the total Green functions is developed. Exact and approximate solutions of these equations are free of non-physical singularities and analytic in coupling constant at zero. The photon propagator in one-loop approximation for the polarizable operator is considered.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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