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DIMENSIONAL REGULARIZATION OF SUPERGRAPHS

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The whole set of relations which defines (supersymmetric dimensional) regularization by dimensional reduction (RDR)[1] is known to be inconsistent[2]. In the component-field formalism one succeeded in finding the "redundant" relations (Fierz identities); giving them up makes the regularization consistent[3], still allowing one to compute diagrams. We carry out the same programme for a superfield formulation of the Wess-Zumino model, where in the component-field approach the RDR is equivalent[4] to conventional dimensional regularization. Up to a definite order the superfield RDR calculations using the contradictory set of relations give the same results, as our consistent formulation does. Hence, up to that order they are reliable.

In terms of chiral superfields $\Phi(x,\theta)$ the action for the Wess-Zumino model has the form:

$$
\begin{aligned}
S = \int dx \Big\{ & \iint d^2\theta\, d^2\bar\theta\, \bar\Phi(x,\bar\theta) \exp(-2i\theta\hat\partial\bar\theta)\, \Phi(x,\theta) \\
& + \frac{m}{2}\Big[\int d^2\theta\, \Phi^2(x,\theta) + \int d^2\bar\theta\, \bar\Phi^2(x,\bar\theta)\Big] \\
& + \frac{\lambda}{3!}\Big[\int d^2\theta\, \Phi^3(x,\theta) + \int d^2\bar\theta\, \bar\Phi^3(x,\bar\theta)\Big]\Big\} .
\end{aligned}
\tag{1}
$$

The notation for the two-component anticommuting spinors is

$$\theta\phi = \theta_A \epsilon_{AB}\phi_B, \quad \theta^2 = \theta_A\epsilon_{AB}\theta_B, \quad \bar\theta\bar\phi = \bar\theta_{\dot A}\bar\epsilon_{\dot A\dot B}\bar\phi_{\dot B}, \quad \hat p = \sigma_\mu p_\mu ,$$

$$\theta\sigma_\mu\bar\phi = \theta_A\epsilon_{AB}(\sigma_\mu)_{B\dot A}\bar\epsilon_{\dot A\dot B}\bar\phi_{\dot B}, \quad (\tilde\sigma_\mu)_{\dot B A} = \epsilon_{AB}(\sigma_\mu)_{B\dot A}\bar\epsilon_{\dot A\dot B} , \text{ etc.}$$

The basic relations have the following form:

$$\epsilon_{AB}\epsilon_{CD} = \delta_{AC}\delta_{BD} - \delta_{AD}\delta_{BC}, \tag{2}$$

$$\int d^2\theta\, \theta_{A_1}\cdots\theta_{A_n} = \frac{1}{2}\,\delta_{n2}\,\epsilon_{A_1A_2} , \tag{3}$$

$$\sigma_\mu\tilde\sigma_\nu + \sigma_\nu\tilde\sigma_\mu = 2g_{\mu\nu}\mathbf{1}, \quad \tilde\sigma_\mu\sigma_\nu + \tilde\sigma_\nu\sigma_\mu = 2g_{\mu\nu}\mathbf{1}, \tag{4}$$

$$\mathrm{tr}\,\mathbf{1} = \delta_{AA} = 2, \tag{5}$$

$$\epsilon_{AB} = -\epsilon_{BA}, \quad \epsilon_{AB}\epsilon_{BC} = -\delta_{AC} , \tag{6}$$

$$\theta_A\theta_B = \frac{1}{2}\epsilon_{AB}\theta^2 , \tag{7}$$

$$\theta_A \theta_B \theta_C = 0, \tag{8}$$

and the same for $\bar{\epsilon}_{\dot{A}\dot{B}}$ and $\bar{\theta}_{\dot{A}}$. Notice, that (8) follows from (5)-(7), which in turn can be derived from (2). Below we shall see that (2) has to be given up, and then (3)-(7) will be the axioms in place of (2)-(4).

After the reduction to a nonintegral dimension we get a projection operator $\hat{g}_{\mu\nu}$ with the property

$$g_{\mu\mu} = d = 4 - 2\varepsilon. \tag{9}$$

Since there are no vector fields in the model, we shall only need the d-dimensional σ-matrices (they appear together with momenta in the form $\hat{p}$) obeying (4), just as in the conventional dimensional regularization. Equations (4), (9) and the cyclicity of the traces lead to the following formulas for

$$T^{\pm}(\mu_1 \dots \mu_{2m}) = \mathrm{tr}(\sigma_{\mu_1} \tilde{\sigma}_{\mu_2} \dots \sigma_{\mu_{2m-1}} \tilde{\sigma}_{\mu_{2m}}) \pm \mathrm{tr}(\tilde{\sigma}_{\mu_1} \sigma_{\mu_2} \dots \tilde{\sigma}_{\mu_{2m-1}} \sigma_{\mu_{2m}}):$$

$$T^{+}(\mu_1 \dots \mu_{2m}) = \sum_{n=2}^{2m} (-)^n g_{\mu_1 \mu_n} T^{+}(\mu_2 \dots \not{\mu}_n \dots \mu_{2m}), \tag{10}$$

$$T^{-}(\mu_1 \dots \mu_{2m}) = 0. \tag{11}$$

Formula (11) is specific for d dimensions. It is analogous to the nullification of any $\mathrm{tr}(\gamma_5 \gamma_{\mu_1} \dots \gamma_{\mu_{2m}})$ with anticommuting γ_5 , $\{\gamma_5, \gamma_\mu\}_+ = 0$, in the nonintegral-dimensional space. As we shall see below, noncorrespondence of (11) to the four-dimensional limit is not seen in the super-invariance region.

The set of equations (2)-(11) is contradictory. Multiply (2) by $M_{AC} N_{BD}$, where $M = \sigma_{\mu_1} \dots \tilde{\sigma}_{\mu_{2m}}$, $N = \sigma_{\nu_1} \dots \tilde{\sigma}_{\nu_{2n}}$. Taking into account a consequence of the notation and (6),

$$\epsilon_{AB} \epsilon_{CD} N_{BD} = (\sigma_{\nu_{2n}} \dots \tilde{\sigma}_{\nu_1}) \equiv N^R_{CA},$$

we obtain a relation, which would be an identity for the σ-matrices of the four-dimensional space:

$$\Delta \equiv \mathrm{tr}[M(N + N^R)] - \mathrm{tr}(M)\,\mathrm{tr}(N) = 0.$$

But the calculation of the traces in the left-hand side with the use of (10), (11) and (5), when M and N each contain four σ-matrices, gives a nonzero answer:

$$\Delta = 4 \begin{vmatrix} g_{\mu_1\nu_1} & \cdots & g_{\mu_1\nu_4} \\ \vdots & & \vdots \\ g_{\mu_4\nu_1} & \cdots & g_{\mu_4\nu_4} \end{vmatrix} \equiv 4 \det(\mu_1 \dots \mu_4, \nu_1 \dots \nu_4) \neq 0.$$

To avoid this discrepancy, we have to exclude equation (2), which allows only two values for spinor indices. However, "simple" θ-variables entering (1) should be left two-component in the sense of (7), (8). It ensures the existence of an explicit δ-function for the θ-integration (3). That function is necessary for a correct construction of the propagator for the action (1). Therefore, we postulate (3)-(7) and (9), to make possible the use of (3)-(11).

In the quasi-two-dimensional space (see ref./3/),where (5) is true but (2) is not, a sum of two simple θ-variables is not a simple θ -variable: $(\theta+\phi)^3 \neq 0$, because (3) and (6) give

$$4 \int d^2\phi\, \phi_A \epsilon_{CE} \epsilon_{DF} \int d^2\dot{\theta}\, (\theta+\phi)_B (\theta+\phi)_E (\theta+\phi)_F =$$

$$= \epsilon_{AB} \epsilon_{CD} - \delta_{AC} \delta_{BD} + \delta_{AD} \delta_{BC} \neq 0.$$

It implies that shifts of the θ-integration variables are not generally permissible, and therefore, supersymmetry of the regularized action (1) can be broken.

The use of (3)-(11) only and the θ-shift ban in the supergraph techniques of ref./5/ lead to a consistent regularization scheme. It can be shown to be equivalent to component-field calculations/6/ with anticommuting γ_5. However, we cannot introduce differentiation with respect to simple θ-variables, since it would allow one to derive (2) from (7), the above contradiction following immediately. This fact does not permit us to construct a consistent version of the supergraph method with covariant derivatives/7/ to treat vector superfields.

Now we are to study the scope of the supersymmetric dimensional regularization. Compare the expressions for supergraphs before momentum integration in the consistent RDR version with those obtained in four dimensions and thus possessing supersymmetry. We only consider the $\Phi\bar{\Phi}$, $\Phi^3(\bar{\Phi}^3)$ and $\Phi^2\bar{\Phi}^2$ diagrams because other graphs are convergent due to power-counting rules with no reference to supersymmetry.

The only formula of those used which fails in four dimensions is (11). It is applied to compute the traces arising from closed cycles of lines in the diagrams. The corresponding four-dimensional formulas, for instance,

$$T^{-}(\mu_1 \dots \mu_4) = 4i\,\epsilon_{\mu_1 \dots \mu_4}, \tag{12}$$

involve the totally antisymmetric tensor. A nonzero contribution of that type after the integration over the internal momentum of the cycle can only be obtained if the latter has at least four independent external momenta. The minimal even cycle with such a property includes six lines and may contribute to the six-loop $\Phi\bar{\Phi}$, five-loop Φ^3 and four-loop $\Phi^2\bar{\Phi}^2$ graphs (or to the divergent parts of the five-loop $\Phi^2\bar{\Phi}^2$ ones).

Consequently, up to this limit the difference between (11) and (12) does not tell on the results, and our unambiguous expressions have the form which is supersymmetric in the four-dimensional space. However, to prove their supersymmetry properties, distinctive features of that space can turn out to be necessary. Namely, exactly four values for Lorentz indices and two for spinor ones (2). In terms of the $g_{\mu\nu}$-tensors the former implies that antisymmetrization over five indices is impossible in four dimensions:

$$\det(\mu_1 \cdots \mu_5, \nu_1 \cdots \nu_5) = 0. \qquad (13)$$

In turn, equation (2) allows additional simplifications in expressions with spinor-index quantities. Thus, the supersymmetry-breaking parts of the d-dimensional results may include evanescent momentum-combinations, to prove the nullification of which in four dimensions one would need formula (13), and θ-structures, which would be zeros if formula (2) were true.

A detailed analysis shows that the evanescent θ-structures cannot break supersymmetry properties of the propagators and triple vertices and also the finiteness of the quartic vertices. Therefore, nothing but the evanescent momentum-combinations can break supersymmetry. Such a combination has at least tenth power in momenta. On the other hand, the maximal power of the numerator of the L-loop diagram is $2L$ (for $\Phi\bar{\Phi}$ and Φ^3) or $2L+2$ (for $\Phi^2\bar{\Phi}^2$). Hence, supersymmetry can be broken in the consistent RDR version from five (in the $\Phi\bar{\Phi}$ and Φ^3 graphs) or four loops on (in the $\Phi^2\bar{\Phi}^2$ ones). Since the $\Phi^2\bar{\Phi}^2$ diagrams have only logarithmic divergencies, only the evanescent combinations which involve no external momenta can contribute to their divergent parts. Due to antisymmetry of (13), for such a combination at least five independent internal momenta are required. Thus, the four-loop quartic vertices remain finite.

We now consider computations with the use of the whole set (2)-(9) and θ-shifts or by the supergraph method of ref. /7/, i.e., in the inconsistent RDR versions. The following order of manipulations is natural: First the four-dimensional θ-(or D /7/) and σ-algebra (2)-(8), (10), (12) are carried out, and then the d-dimensional momentum integration is. Here the evanescent θ-structures and momentum-combinations become a source of ambiguities rather than of supersymmetry breaking. For the corresponding estimates one should use supergraph power-counting rules /5, 7/.

Calculations in contradictory versions are surely reliable while they have an external justification, i.e., while they agree with a consistent scheme which preserves supersymmetry up to a definite order. It is such a correspondence that we established above when studying super-invariance of our unambiguous

Table

The RDR in the Wess-Zumino model

Versions	Consistent				Inconsistent		
Graphs	$\Phi\bar{\Phi}$	Φ^3 $(\bar{\Phi}^3)$	$\Phi^2\bar{\Phi}^2$ divergent parts	$\Phi^2\bar{\Phi}^2$ finite parts	$\Phi\bar{\Phi}$	Φ^3 $(\bar{\Phi}^3)$	$\Phi^2\bar{\Phi}^2$
1. Unambiguity	∞	∞	∞	∞	6	6	5
2. Supersymmetry	5	5	5	4	∞	∞	∞
3. Four-dimensional limit	6	5	5	4	∞	∞	∞
4. External justification	is unnecessary				5	5	4

formulation. Therefore, in its invariance region one can also perform θ-shifts, considerably simplifying the computations, or employ techniques of ref. /7/. These arguments justify the use of the RDR in actual three- and four-loop calculations /8/.

The results of the above analysis are summarized in the Table. The consistent (superfield or component-field) RDR formulation and the inconsistent one (with θ-shifts or covariant derivatives) are considered. Their following properties are under study: 1. Unambiguity. 2. Supersymmetry. 3. Correspondence to the four-dimensional formulas such as (12). 4. Accordance with a consistent scheme. For each property the minimal number of loops is pointed out, when it can be broken for the first time. Our main conclusion is that for divergent parts of the diagrams the RDR can only prove incorrect from five loops on. Maybe, the search for an internal justification criterion in the contradictory RDR versions would extend their scope and give a satisfactory regularization for vector superfields.

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Определена область применимости суперсимметричной размерной регуляризации в модели Весса-Зумино. С этой целью сформулирована непротиворечивая версия регуляризации для суперполевых диаграмм.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Avdeev L.V., Kamenshchik A.Yu. E2-82-680
Dimensional Regularization of Supergraphs

The applicability region of the supersymmetric dimensional regularization is determined in the Wess-Zumino model. To achieve it, a consistent version of the regularization for supergraphs is formulated.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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