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PHASE TRANSITIONS IN THE EUCLIDEAN AND HAMILTONIAN APPROACHES IN THE LATTICE GAUGE THEORIES AT FINITE TEMPERATURE

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In recent years much progress has been achieved in the investigations of quark-gluon states of the matter and in the physics of the confinement-deconfinement phase transitions. This progress is to a great extent due to the formulation of the lattice gauge field theories[1].

The use of the lattice regularization in constructing the gauge field theories enables the use of the Monte-Carlo method for the numerical investigation of the quark-gluon interactions of different phases of the matter.

As is known, there exist different models of gauge field theories on lattice.

Physical quantities, determined in different renormalization schemes, can be compared in the case when the relation between the coupling constants entering different models is known.

Our paper is devoted to a direct nonperturbative comparison of bare coupling constants by the Monte-Carlo method in the Euclidean[1] and Hamiltonian[2,4] approaches at finite temperatures.

Recent papers[3-5] have been devoted to the calculations by the "background field" method in the limit of a weak coupling of the partition function in these two approaches. The calculations were performed in the one-loop approximation (in the limit $a \to 0$, where a is the lattice spacing) and enabled the relation between the interaction constants to be established. This method of calculation is limited by the region of small $g^2(a)$, whereas the most interesting phenomena occur in the region of $g^2(a) \sim 2$. Therefore, a direct calculation of the relation between the interaction constants is of interest. Though these calculations check the theory up on self-consistence, they also allow one to establish the relation between the interaction constants beyond the weak coupling region. In what follows we shall restrict ourselves to the case of SU(2)-symmetry with periodic boundary conditions.

In the gluon sector of the Euclidean field theory on lattice the action is

$$S_E(U) = \beta_E \cdot \sum_P \left(1 - \frac{1}{2}\,\mathrm{Sp}\,U_P\right)$$
$$U_P = U_{ij} \cdot U_{jk} \cdot U_{k\ell} \cdot U_{\ell i}\,. \qquad (1)$$

Here U_{ij} is the element of the SU(2) group, which corresponds

to the link (ij), and $\beta_E(a) = \frac{4}{g^2(a)}$, where $g(a)$ is the bare coupling constant depending on the spacing a between two neighbouring sites i and j. Summation in (1) is over all elementary squares (plaquettes). In the Euclidean approach the spacing a is independent of the link (i,j) direction. The partition function Z is determined by the following integral:

$$Z = \int [dU] \cdot e^{-S_E(U)},$$
$$[dU] = \prod_{(i,j)} dU_{ij}\,, \qquad (2)$$

and dU_{ij} is the Haar measure on the SU(2) group. The expectation values of any $O(U)$ are equal to

$$\langle 0 \rangle = Z^{-1} \cdot \int [dU] \cdot O(U) \cdot e^{-S_E(U)}. \qquad (3)$$

In the Hamiltonian approach we proceed from the Hamiltonian[2,4]

$$\hat H = \sqrt{\frac{g_t^2}{g_s^2}}\,\frac{g_H^2}{2a} \cdot \Big\{ \sum_{\text{links}} \vec E^{\,2} + \frac{2}{g_H^4} \cdot \sum_{P_s} \mathrm{Sp}\,(2 - \hat U_{P_s} - \hat U^{+}_{P_s}) \Big\}, \qquad (4)$$

where $g_H^2 = g_t \cdot g_s$ and the operators $\vec E(\vec x;\vec y)$ and $\hat U_{P_s} \equiv \prod_{(i,j)} \hat U_{ij}$ are determined by the following commutation relations:

$$[E^{\alpha}(\vec x;\vec x + a\cdot\vec e_k),\ E^{\beta}(\vec y;\vec y + a\cdot\vec e_{k'})] = i\epsilon^{\alpha\beta\gamma} \cdot E^{\gamma}(\vec x;\vec x + a\cdot\vec e_k)\cdot\delta_{\vec x;\vec y}\cdot\delta_{\vec e_k;\vec e_{k'}},$$
$$[E^{\alpha}(\vec x;\vec x + a\cdot\vec e_k),\ \hat U(\vec y;\vec y + a\cdot\vec e_{k'})] = \frac{1}{2}\sigma^{\alpha}\cdot\hat U(\vec y;\vec y + a\cdot\vec e_{k'})\cdot\delta_{\vec x;\vec y}\cdot\delta_{\vec e_k;\vec e_{k'}}, \qquad (5)$$

and the summation in (4) is over the spatial volume alone. The average value of any operator $\hat O$ is determined as follows:

$$\langle \hat O \rangle = Z^{-1} \cdot \mathrm{Sp}\,(\hat O\, e^{-\frac{1}{\theta}\hat H}),$$
$$Z = \mathrm{Sp}\, e^{-\frac{1}{\theta}\hat H}. \qquad (6)$$

Expression for the partition function (6) can be represented in the form analogous to (2)

$$Z = \int [dU] \cdot e^{-S_H(U)}, \qquad (7)$$

where

$$S_H(U) = \beta_H \{ \bar{\xi}^{-1} \cdot \sum_{P_s} (1 - \frac{1}{2} \mathrm{Sp}\, U_{P_s}) + \xi \cdot \sum_{P_t} (1 - \frac{1}{2} \mathrm{Sp}\, U_{P_t}) \} . \tag{8}$$

In expression (8) $\beta_H = \frac{4}{g_H^2}$; $\xi = \frac{a_s}{a_t}$; $\bar{\xi} = \sqrt{\frac{g_s^2}{g_t^2}} \cdot \xi \equiv \eta \cdot \xi$ and formula (7) defines the partition function as $\xi \to \infty$ ($a_t \to 0$; a_s - fixed). In this case the temperature $\theta = \frac{1}{a_t N_t}$ should remain fixed. In the limit $a_s \to 0$ the bare coupling constants $g_{E/H}(a)$ behave in a natural way (see, for instance [5])

$$g^2_{E/H}(a)_{a\to 0} = [\, b_0 \cdot \ln \frac{1}{a^2 \Lambda^2_{E/H}} + \frac{b_1}{b_0} \ln \ln \frac{1}{a^2 \Lambda^2_{E/H}} + O(g^2_{E/H}\,)]^{-1} . \tag{9}$$

For the SU(2) symmetry

$$b_0 = \frac{11}{3} \cdot \frac{1}{8\pi^2} ; \quad b_1 = \frac{34}{3} \cdot (\frac{1}{8\pi^2})^2 .$$

The thermal Wilson loop is used as an object of calculations

$$L = \frac{1}{2} \mathrm{Sp} \prod U_{ij} \equiv \frac{1}{2} \mathrm{Sp}\, U_{i_1 i_2}\, U_{i_2 i_3} \dots U_{i_{N_t - 1} i_{N_t}} , \tag{10}$$

where the sites $i_1, \dots i_{N_t}$ lie on one line along the time axis. The average value of the temperature Wilson loop is an order parameter for the global Z(2)-symmetry, which is convenient for the description of phase transitions [6,7].

The expectation value of L is the partition function Z_q of the Yang-Mills gas in the presence of the rest source

$$\langle L \rangle \equiv Z_q = e^{-\frac{1}{\theta} F_q} , \tag{11}$$

where F_q is the free energy in the presence of the rest source (quark) [8,9,10]. We have calculated the average value of L in the Euclidean approach on lattice 3×7^8 and in the Hamiltonian approach on lattice 15×7^3 with $\xi = 5$ and different η. Figure 1 shows the dependence of the Wilson loop in the Euclidean approach (solid line) and in the Hamiltonian approach of Kogut-Susskind with $\eta = 1$ (dashed line). The curve obtained in the Hamiltonian approach of Kogut-Susskind lies above the curve obtained in the Euclidean approach, that indicates the necessity to take into account the difference from unity of the second Hamiltonian variable η. In both the cases the dependence of $\langle L \rangle$ on β (β_E and β_H, respectively) is well described by the for-

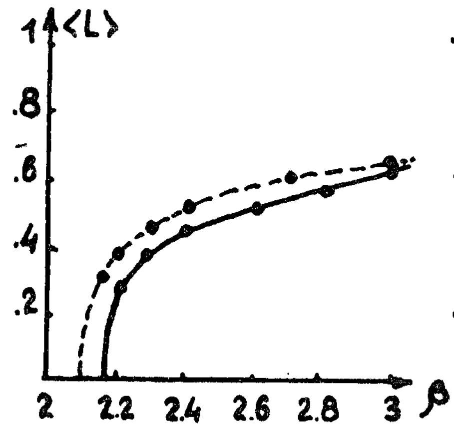

Fig.1

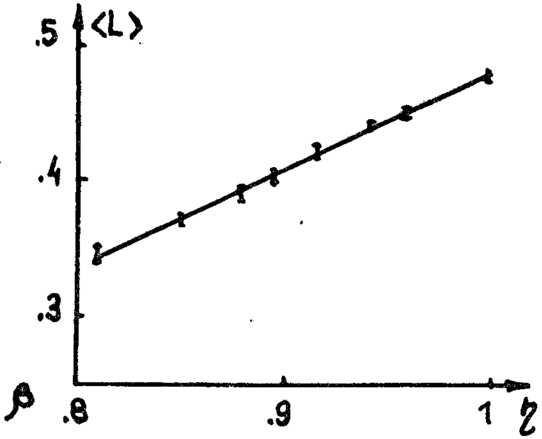

Fig.2

mula

$$\langle L \rangle = \frac{r^\alpha}{r^\alpha + \gamma} , \tag{12}$$

where

$$r = \frac{\beta - \beta_c}{\beta_c} .$$

The values of α and γ coincide within the limits of errors and equal

$$\alpha \simeq 0{,}5 ; \quad \gamma \simeq 0{,}4 . \tag{13}$$

At the same time the values of β_{cE} and β_{cH} differ from each other

$$\beta_{cE} \simeq 2.15 , \qquad \beta_{cH} \simeq 2.1 . \tag{14}$$

Using expressions (12)-(14), one can easily get that the values of β_E and β_H are related by

$$\beta_E - \beta_H = \Delta\beta = \beta_{cE} - \beta_{cH} . \tag{15}$$

To study the role of quantum corrections to the Hamiltonian, we have investigated the dependence of the Wilson loop on η. Figure 2 shows the dependence of $\langle L \rangle$ on η for $\beta = 2.4$ on lattice 15×7^3 with $\xi = 5$. The calculations performed in refs.[4,5] in the one-loop approximation predict for η the values $\eta \simeq 0.88$. With this value of η we can calculate the ratio Λ_H/Λ_E. Using (9), we get

$$\Lambda_H/\Lambda_E = 0{,}87 \pm 0{,}04$$

that is in agreement with refs.[4,5].

Thus, our calculations show that in the region of intermediate and weak coupling β_E, $\beta_H \sim 2$ on lattice with nonsymmetric spacing, one should take into account quantum corrections leading to the difference of the second Hamiltonian constant $\eta = \sqrt{\frac{g_s^2}{g_t^2}}$ from unity, the difference being of 10-20%. In this case the dependence of β_E and β_H on the lattice specific conforms with the formulae of asymptotic freedom behaviour and the ratio of renormalization constants Λ_H/Λ_E obtained as a result agrees astonishingly well with the values for Λ_H/Λ_E which have been calculated in the one-loop approximation in the case of weak coupling in refs.[4,5].

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REFERENCES

1. Wilson K.G. Phys.Rev., 1974, D10, p.2445.
2. Kogut J., Susskind L. Phys.Rev., 1975, D11, p.395.
3. Dashen R., Gross D.J. Phys.Rev., 1981, D23, p.2340.
4. Hasenfratz A., Hasenfratz P. Nucl.Phys., 1981, B193, p.210.
5. Karsch F. BI-TP 81/34, 1981.
6. Polyakov A.M. Phys.Lett., 1978, 72B, p.477.
7. Susskind L. Phys.Rev., 1979, D20, p.2610.
8. McLerran L.D., Svetitsky B. Phys.Lett., 1981, 98B, p.195; Phys.Rev., 1981, D24, p.450.
9. Kuti J., Polonyi J., Szlachanyi K. Phys.Lett., 1981, 98B, p.199.
10. Engeles J. et al. Phys.Lett., 1981, 102B, p.332.

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Гердт В.П., Митрюшкин В.К. E2-82-671
Фазовые переходы в евклидовом и гамильтоновом подходах в калибровочных теориях на решетке при конечной температуре

В работе изучались термодинамические свойства глюонного газа при конечных температурах в рамках калибровочной теории поля на решетке с SU(2) -симметрией. Методом Монте-Карло исследовалась температурная зависимость вильсоновской струны на симметричной ($a_t = a_s$) и несимметричной ($a_t \neq a_s$) решетках. Найдена связь между константами взаимодействия в обоих подходах, а также отношение констант перенормировки.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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REFERENCES

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