объединенный
институт
ядерных
исследований
дубна

E2-82-598

E.-M.Ilgenfritz, M.Müller-Preussker

SU(3) GLUON CONDENSATE FROM LATTICE MC DATA

Submitted to "Physics Letters B"

1982

Nowadays, Monte-Carlo lattice calculations are the most powerful tool for quantitative studies of nonperturbative effects in gauge theories. Many phenomenologically relevant numbers have been computed in this way: string tension, glueball and a few meson masses, etc. Of special interest are studies of those quantities which characterize the vacuum state, in particular, vacuum expectation values of composite gluon operators. The latter can be calculated in a first approximation within pure Yang-Mills theories leaving the discussion of virtual quark loop corrections to the next step. The most familiar quantity in this respect is the gluon condensate [1]

$$\mathcal{G} = \langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} \rangle \sim .012\ \mathrm{GeV}^4. \qquad (1)$$

All methods used until now to extract this value from MC lattice data rely on the expansion of Wilson loops - in the following rectangular ones of size $Ia \times Ja$ - over expectation values of local operators [2]

$$W(I,J) = 1 - \frac{\pi^2}{12N_c} (\mathcal{G}/\Lambda_L^4)(IJ)^2 (a\Lambda_L)^4 + \ldots \qquad (2)$$

where perturbative corrections due to virtual gluon exchange have been neglected for a moment and where the lattice scale is thought to satisfy the renormalization group behaviour with respect to the bare coupling g_a

$$(a\Lambda_L)^2 = (\beta_0 g_a^2)^{-\frac{\beta_1}{\beta_0^2}} \exp\left(-\frac{1}{\beta_0 g_a^2}\right). \qquad (3)$$

There are essentially two strategies followed by different groups. The first one [3,4] uses the one-plaquette average action values and subtracts the perturbative tail expanded up to some powers of the bare coupling. The second one [5,6] takes into account Wilson loops of different sizes presented in terms of the ratios

$$\chi(I,J) = -\log \frac{W(I,J)\,W(I-1,J-1)}{W(I-1,J)\,W(I,J-1)}. \qquad (4)$$

This way is superior, because somehow larger loops will exhibit a non-perturbative signal more pronounced than single plaquettes. Moreover, the ratios (4) allow one to apply continuum perturbation theory methods and therefore to "optimize" the perturbation expansion in a renormalized coupling g_R. Unpleasant perimeter terms as well as Z factors associated with the corners of the loops cancel out and the one-loop coefficient does not depend on the renormalization point chosen. In Ref. [6] SU(2) data have been investigated and the two-loop expression with an appropriately adjusted effective Λ parameter has been sufficient to end up with an encouraging fit. In contrast, the low temperature expansion for the one-plaquette expectation value in g_a^2 required three additional coefficients to be fitted to the data points in order to get a reliable result [3].

For SU(3), to our knowledge, there exist only first, very rough estimates [4,7] undertaken on the basis of data collected by Pietarinen [8] and Creutz [9]. Recently we have obtained own data for the case of SU(3) [10] on a 8^4 lattice for Wilson loop expectation values applying the heat bath iteration programme [8]. In the meanwhile we have improved their statistics near the weak-to-strong coupling crossover so that averages over 20 sweeps (for $g_a^{-2} = 1.20$) up to 60 sweeps (for $g_a^{-2} = 0.90$) are available by now. Therefore, it seems to be justified to repeat the analysis of Ref. [6] for the SU(3) case. The expansion (2) rests on the assumption that apart from perturbative fluctuations the gauge field varies slowly along distances comparable with the loop size. Therefore, the loops to be considered here should not be too large compared with the correlation length given by the lowest-lying glueball mass gap. In practice we restrict ourselves to loop sizes I, J = 2,3.

The two-loop corrections to Wilson loop ratios have been calculated in Ref. [6] applying dimensional regularization in x-space and by subtracting the singularities within the $\overline{MS}$ scheme. Thus the first two coefficients of the expansion in powers of the renormalized coupling are known

$$\chi(I,J) = K^{(1)}(I,J) g_R^2 + K^{(2)}(I,J;L_0) g_R^4 + K^{(3)}(I,J;L_0) g_R^6 + \ldots$$
$$+ \frac{\pi^2}{36}(2I-1)(2J-1)(\mathcal{G}/\Lambda_L^4)(a\Lambda_L)^4 + \ldots \qquad (5)$$

$K^{(2)}$, $K^{(3)}$ depend on a scale which is conveniently chosen from a quadratic reference loop of size L_0. The numerical values we need in our case are for convenience recollected [6] in the table

$I \times J$	$K^{(1)}$	$K^{(2)}$ for $L_0 = Ia$	$K^{(2)}$ for $L_0 = \sqrt{IJ}a$
2x2	0.0881	-0.00556	-0.00556
2x3	0.0612	-0.00227	-0.00400
3x3	0.0291	-0.00118	-0.00118

In order to compare with lattice data the renormalized coupling must be reexpressed through the bare coupling g_a

$$\frac{1}{g_R^2} = \frac{1}{g_a^2} - \frac{22}{16\pi^2}\log\left(\frac{\Lambda}{\Lambda_L}\,\frac{L_0}{a}\right) - \frac{204}{(16\pi^2)^2}\log\left(\frac{\Lambda}{\Lambda_L}\,\frac{L_0}{a}\right) g_a^2 + \ldots \qquad (6)$$

The ratio Λ/Λ_L turned out in the given scheme to be 25.7. The authors of Ref. /6/ have checked that the coefficients of the resulting expansion in g_a^2 up to two loops are very close to those calculated directly within the lattice low temperature expansion. However, we can easily convince ourselves that the perturbative series in powers of the renormalized coupling better describes the perturbative background seen in the MC data. Nevertheless, a reasonable fit needs the variation of a further parameter, which could be Λ/Λ_L or $K^{(3)}$. In fact, we have fitted the expressions (5) simultaneously to all our data for the three mentioned loop ratios. Using the standard procedure FUMILI for minimizing χ^2 and adopting $L_0=\sqrt{IJ}\,a$ we have obtained the following values:

for $K^{(3)} = 0$ fixed, $\quad \mathcal{G}/\Lambda_L^4 = (2.27 \pm .14)\cdot 10^8$ (A)

with

$I\times J$	2x2	2x3	3x3
Λ/Λ_L	60 ± 4	47 ± 8	18 ± 18

for $\Lambda/\Lambda_L = 25.7$ fixed, $\quad \mathcal{G}/\Lambda_L^4 = (1.61 \pm .22)\cdot 10^8$ (B)

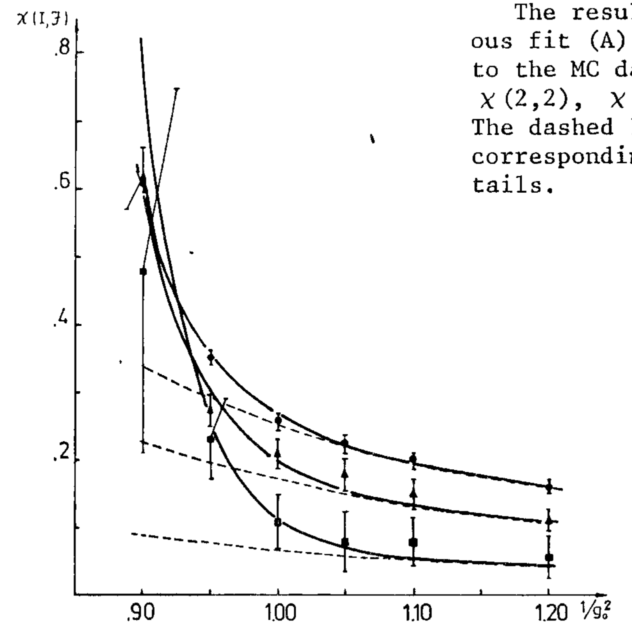
with

$I\times J$	2x2	2x3	3x3
$K^{(3)}$	0.00121 $\pm$0.00012	0.00059 $\pm$0.00015	0.00006 $\pm$0.00023

For $L_0=Ia$ we found the same results within the estimated errors. All these fits ended up with a χ^2 value per data point of the order 0.5...0.6 and a confidence level of more than 80%. The behaviour of the fitted curves (for the case (A)) is shown together with the corresponding data points in the Figure. For comparison, the pure perturbative behaviour has been drawn, too (cf. dashed lines). The small values and large standard deviations for the fitted parameters Λ/Λ_L and $K^{(3)}$ in the case $I\times J = 3\times 3$ indicate that for larger loop sizes the perturbative tail is well-described already at the two-loop level.

Really, we have tried to fit also the ratios $\chi(2,4)$, $\chi(2,5)$ to the available data. These fits show the tendency to lower the gluon condensate remarkably. Since the loop sizes obviously violate the tacit assumptions leading to expansion (2), we did not consider them further. Using the ratio $\Lambda_L = (.007 \pm .001)\sqrt{\sigma}$ extracted in Ref. /10/ and $\sqrt{\sigma} \simeq 400$ MeV the ITEP value for $\mathcal{G}$ is translated into units Λ_L as

$\mathcal{G}/\Lambda_L^4 = (1.9 \pm 1.1)\cdot 10^8$.



The result of our simultaneous fit (A) of expression (5) to the MC data for the ratios $\chi(2,2)$, $\chi(2,3)$ and $\chi(3,3)$. The dashed lines indicate the corresponding perturbative tails.

Thus both fits (A) and (B) are in a very good shape compared with the phenomenological value.

It is often argued that the inclusion of virtual quark loops would considerably suppress the gluon condensate found in pure gluodynamics. From an investigation of the Yang-Mills data, including light quarks within the lowest order of the hopping parameter expansion, i.e., on the basis of an effective one-plaquette action, we expect, however, only small corrections /10/. The results of our fits of the gluon condensate to MC data also point out in this direction.

We would like to express our gratitude to our colleagues of the high energy physics group of the Karl-Marx-Universität Leipzig for providing us with their results prior to publication and acknowledge useful discussions with N.Makhaldiani and V.P.Gerdt.

REFERENCES

1. Shifman M., Vainshtein A., Zakharov V. Nucl.Phys., 1979, B147, p.385, 448.
2. Shifman M. Nucl.Phys., 1980, B173, p.171.
3. Di Giacomo A., Rossi G.C. Phys.Lett., 1981, 100B, p.481; Di Giacomo A., Paffuti G. Preprint IFUP TH/12, Pisa, 1981.

4. Banks T. et al. Nucl.Phys., 1981, B190 /FS3/, p.692.
5. Kripfganz J. Phys.Lett., 1981, 101B, p.169.
6. Kirschner R. et al. Preprint KMU-HEP-82-06, Leipzig, 1982.
7. Kripfganz J. Proc. of the IV Warsaw Symp. on Elementary Particle Physics, Kazimierz, 1981, p.299.
8. Pietarinen E. Nucl.Phys., 1981, B190 /FS3/, p.349.
9. Creutz M. Phys.Rev.Lett., 1980, 45, p.313.
10. Ilgenfritz E.-M., Müller-Preussker M. JINR, E2-82-473, Dubna, 1982.

Received by Publishing Department
on August 5 1982.

Ильгенфриц Э.-М., Мюллер-Пройскер М. E2-82-598
Глюонный конденсат и решеточные расчеты по методу Монте-Карло

Из монте-карловских данных для соотношений Кройтца для вакуумных средних петель Вильсона найдено значение глюонного конденсата в случае калибровочной группы SU(3). Результат хорошо согласуется с феноменологией.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1982

Ilgenfritz E.-M., Müller-Preussker M. E2-82-598
SU(3) Gluon Condensate from Lattice MC Data

The SU(3) gluon condensate is determined from a fit to the Creutz ratios of Wilson loop expectation values taken from new Monte-Carlo data. The result agrees with the phenomenological value very well.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1982