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THE QUADRATIC BUNDLE OF GENERAL FORM AND THE NONLINEAR EVOLUTION EQUATIONS.

HIERARCHIES OF HAMILTONIAN STRUCTURES

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1. INTRODUCTION

The application of the inverse scattering method (ISM) [1] to quadratic bundles of special type allowed one the integrate some physically important nonlinear evolution equation (NLEE). The first in this direction was the paper [2], where starting from a bundle of the type*:

$$L_1(\lambda)\psi^{(1)} \equiv [i\sigma_3 \frac{d}{dx} + \frac{1}{2}|u_1^2| + \lambda Q^{(1)} - \lambda^2]\psi^{(1)}(x,\lambda) = 0,$$

$$Q_1^{(1)} = \begin{pmatrix} 0 & u_1 \\ -u_1^* & 0 \end{pmatrix}, \qquad (1.1)$$

an exhaustive study of the massive Thirring model has been presented. In the papers [3,4] it has been shown, that the bundle:

$$L_2(\lambda)\psi^{(2)} \equiv [i\sigma_3 \frac{d}{dx} + \lambda Q^{(2)} - \lambda^2]\psi^{(2)}(x,\lambda) = 0, \quad Q^{(2)} = \begin{pmatrix} 0 & q \\ p & 0 \end{pmatrix} \qquad (1.2)$$

allows one to solve the modified nonlinear Schrodinger equation (NLS eq.):

$$iq_t + q_{xx} + i\epsilon(|q^2|q) = 0, \qquad p = \epsilon q^*. \qquad (1.3)$$

The complete integrability and the construction of the hierarchy of Hamiltonian structures [5,6] for the NLEE, related to (1.2), has been proved in [7]. The considerations there have been based on the method of expansions over the "squared" solutions [7-10]. Another NLEE, related to (1.2) is the Mikhailov model:

$$h_{1xt} - 2ih_1h_2h_{1x} + m^2h_1 = 0, \quad h_1 = q_x,$$

$$h_{2xt} + 2ih_1h_2h_{2x} + m^2h_2 = 0, \quad h_2 = p_x, \qquad (1.4)$$

a relativistically invariant field theory model, equivalent to the massive Thirring model [7]. In refs. [11-13] the following bundles have been considered:

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* Here we have used characteristics variables; thus the bundle $L_1(\lambda)$ equals the summ $T(\lambda) + X(\lambda)$ of the bundles, introduced in ref. [2].

$$L_3(\lambda)\psi^{(3)} \equiv [i\sigma_3 \frac{d}{dx} + \lambda Q^{(2)} - (\lambda^2 + 1)]\psi^{(3)}(x,\lambda) = 0, \qquad (1.5)$$

$$L_4(\lambda)\psi^{(4)} \equiv [i\sigma_3 \frac{d}{dx} + (\bar{a}\lambda + \bar{\beta})Q^{(2)} + (\bar{a}\lambda^2 + 2\bar{\beta}\lambda)]\psi^{(4)}(x,\lambda) = 0. \qquad (1.6)$$

They allow one to obtain the Lax representation for another variant of the NLS eq.:

$$iq_t + q_{xx} + 2\bar{\beta}^2|q^2|q - i\bar{a}\frac{\partial}{\partial x}(|q^2|q)_x = 0. \qquad (1.7)$$

Note, that both variants of the modified NLS equation are applicable in plasma physics [14-17].

On the other hand, the standard ways of the ISM have been applied to the polynomial bundle of maximally general type [18], which in the 2x2 case has the form:

$$L_5(\lambda)\psi^{(5)} \equiv [i\sigma_3 \frac{d}{dx} + \sum_{k=0}^{N} \lambda^k U_k(x)]\psi^{(5)}(x,\lambda) = 0. \qquad (1.8)$$

The analysis, based on the study of the central extensions of Lie algebras (see refs [18,19] and the references therein) lead to explicitly Hamiltonian form of the NLEE. Unfortinately the corresponding Kirillov-Kostant syplectic form is degenerated. The natural solution of the problem consists, roughly speaking, in the following: one should somewhat restrict the form of the linear problem (1.8) so, that the phase space of the corresponding NLEE coincides with the orbit of the co-adjoint action in our algebra (more rigorously - see ref. [18]). In particular, for the quadratic in λ bundles this restriction, together with an appropriate choice of the gauge [20] and the co-adjoint action, leads to:

$$L(\lambda)\psi \equiv [i\sigma_3 \frac{d}{dx} + Q_0 + \lambda Q_1 + r_0 - \lambda^2]\psi(x,\lambda) = 0, \quad Q_1 = \begin{pmatrix} 0 & q_1 \\ p_1 & 0 \end{pmatrix}. \qquad (1.9)$$

Therefore, from general arguments it follows, that the NLEE, related to (1.9) possess a hierarchy of Hamiltonian structures, the corresponding symplectic forms Ω_m, $m = 0, \pm 1, \pm 2, \ldots$ being nondegenerate. But generically both the NLEE and even the simplest of the forms Ω_0 depend nonlocally on q_1, p_1, r_0.

As a conjecture, allowing one to obtain local NLEE we propose the following. It is well known, that the solution of the inverse scattering problem for a large number of linear problems $L(\lambda)$ is equivalent to the solution of a Riemann problem [1]. This last problem has an unique solution with fixed norm, i.e., with fixed value of the solution for $\lambda \to \infty$. Obviously the normalization of the Riemann problem should correlate with the asymptotics of the solutions $L(\lambda)\psi = 0$ of the linear problem for $\lambda \to \infty$. Our conjecture consists in that one should require canonical normalization for the Riemann problem. In particular, for the system (1.9) the corresponding Riemann problem is written down in ref. [21], and the requirement of canonical normalization is fulfilled if:

$$r_0 = -\frac{1}{2} q_1 p_1. \qquad (1.10)$$

As we shall see below, the "restriction" (1.10) leads to local NLEE, and the simplest symplectic form Ω_0 becomes canonical. The same conjecture, applied to the 2x2 polynomial bundles of power $N \geq 3$ also allows one to obtain local NLEE and simple 2-form Ω_0 [22]. In this case from the 4N initially independent elemets of $L_5(\lambda)$ (1.8) there remain only $2N-2$ independent ones, i.e., this conjecture has the sence of reduction for the NLEE [23].

Let us now explain why the condition (1.10) is not essentially a restriction for the system (1.9). Really, applying the gauge transformation [20]:

$$L \to L'(\lambda) = e^{-i\sigma_3\varphi(x)} L(\lambda) e^{i\sigma_3\varphi(x)}, \qquad \varphi(x) = \int_x^\infty dy\,(r'_0 - r_0), \qquad (1.11)$$

the system (1.9) goes into the system (1.9'), with Q'_1 and r'_0 instead of Q_1 and r_0, where:

$$Q'_1 = \begin{pmatrix} 0 & q'_1 \\ p'_1 & 0 \end{pmatrix}, \quad q'_1 = q_1 e^{-2i\varphi}, \quad p'_1 = p_1 e^{2i\varphi}. \qquad (1.12)$$

Obviously the transformation (1.11) is equivalent to the change of variables (1.12) in the NLEE.

In the present paper the expansions of $\bar{w} = \begin{pmatrix} w_1 \\ w_0 \end{pmatrix}$, $w_1 = \begin{pmatrix} q_1 \\ p_1 \end{pmatrix}$ and $\bar{\sigma}_3\delta\bar{w}$, $\bar{\sigma}_3 = \mathrm{diag}(\sigma_3, \sigma_3)$ over the "squared" solutions $\{\bar{\Psi}\}$ of (1.9), (1.10) obtained in ref. [21] are applied for the study of corresponding NLEE. The main result consists in the explicit construction of the Hamiltonian structures of these NLEE and the calculation of their action-angle variables. The appropriate choice of the system of "squares" $\{\bar{\Psi}\}$ in ref. [21] makes all the proofs in Sec.3 analogical to the simpler cases [7,9]. This once more leads up to the conjecture, that the expansion over the "squares" method is of more universal character and may be applied to a much larger class of linear problems $L(\lambda)$, see [22]. In Sec.4 we display a new 2-component variant of the modified NLS eq. Equations (1.3) and (1.7) may be obtained from it by special reductions, after which the symplectic forms Ω_{2k} with even indices become degenerate. Therefore the corresponding action-angle variables are calculated using one of the forms Ω_{2k+1}, e.g., Ω_{-1} or Ω_1. We also give a generalization of the Mikhailov model (1.4). In Sec. 2 we formulate some of the results in ref. [21], which are needed in Secs. 3 and 4.

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2. PRELIMINARIES

Let the potentials $Q_i(x)$ in $L(\lambda)$ (1.9), (1.10) be complex-valued functions of Schwartz type such, that the discrete spectrum Δ of the operator $L(\lambda)$ is simple and finite. The Jost solutions are given by their asymptotics:

$$\lim_{x\to\infty} \psi(x,\lambda)\, e^{i\lambda^2\sigma_3 x} = 1, \qquad \lim_{x\to-\infty} \phi(x,\lambda)\, e^{i\lambda^2\sigma_3 x} = 1, \tag{2.1}$$

$$\psi(x,\lambda) = \|\psi^-, \psi^+\|, \qquad \phi(x,\lambda) = \|\phi^+, \phi^-\|,$$

their columns ψ^+, ϕ^+ (ψ^-, ϕ^-) being analytic in λ for $\mathrm{Im}\lambda^2 > 0$ ($\mathrm{Im}\lambda^2 < 0$), The transfer matrix is introduced by:

$$\phi(x,\lambda) = \psi(x,\lambda) S(\lambda), \qquad S(\lambda) = \begin{pmatrix} a^+, & -b^- \\ b^+, & a^- \end{pmatrix}. \qquad \det S(\lambda) = 1. \tag{2.2}$$

The diagonal elements $a^{\pm}(\lambda)$ are also analytic in λ for $\mathrm{Im}\lambda^2 \gtrless 0$, and satisfy the dispersion relations:

$$D(\lambda) = \frac{i}{2\pi}\int_\Gamma \frac{d\mu}{\mu-\lambda} \ln[1+\rho^+\rho^-(\mu)] + \sum_{a=1}^{N} \ln\frac{\lambda-\lambda_{a+}}{\lambda-\lambda_{a-}},$$
$$D(\lambda) = \pm\ln a^{\pm}(\lambda), \quad \mathrm{Im}\lambda^2 \gtrless 0, \quad \Gamma \equiv \boldsymbol{R} \oplus i\boldsymbol{R}, \tag{2.3}$$

where $\rho^{\pm}(\lambda) = b^{\pm}/a^{\pm}$, $\lambda_{a\pm} \in \Delta$ are the discrete eigenvalues of $L(\lambda)$ and the contour Γ is introduced in fig. 1 of [21].

The main result of ref. [21] consists in the proof of the completeness relations for the systems $\{\bar\Psi\}$ and $\{\bar\Phi\}$ of "squared" solutions of $L(\lambda)$, where:

$$\{\bar\Psi\} \equiv \{\bar\Psi^{\pm}(x,\lambda),\ \lambda\in\Gamma,\ \bar\Psi^{\pm}_a(x),\ \dot{\bar\Psi}^{\pm}_a(x),\ a=1,\dots,N\},$$

$$\{\bar\Phi\} \equiv \{\bar\Phi^{\pm}(x,\lambda),\ \lambda\in\Gamma,\ \bar\Phi^{\pm}_a(x),\ \dot{\bar\Phi}^{\pm}_a(x),\ a=1,\dots,N\},$$

$$\bar\Psi^{\pm} = N_+^{-1}\Psi^{\pm}(x,\lambda), \qquad \bar\Phi^{\pm} = N_-^{-1}\Phi^{\pm}(x,\lambda),$$

$$\dot{\bar\Psi}^{\pm}_a(x) = \frac{d}{d\lambda}\bar\Psi^{\pm}(x,\lambda)\Big|_{\lambda_{a+}}, \quad \Psi^{\pm} = \psi^{\pm} * \phi^{\pm}(x,\lambda), \quad \Phi^{\pm} = \phi^{\pm} * \phi^{\pm}(x,\lambda),$$

$$\psi * \phi = \begin{pmatrix} \psi\circ\phi \\ \lambda\psi\circ\phi \end{pmatrix}, \qquad \psi\circ\phi = \begin{pmatrix} \psi_1 & \phi_1 \\ \psi_2 & \phi_2 \end{pmatrix},$$

$$N_{\pm}^{-1} = \begin{pmatrix} 1 & 0 \\ Z^{\pm}_{10}, & 1+Z^{\pm}_{11} \end{pmatrix}, \quad Z^{\pm}_{ik} = -iw_i\int_x^{\infty} dy\, \tilde w_k, \quad \tilde w_k = (q_k, -p_k).$$

Here we shall write down only the symplectic variant of the completeness relation:

$$\delta(x-y) = \int_\Gamma d\lambda\, [Q(x,\lambda)P^T(y,\lambda) - P(x,\lambda)Q^T(y,\lambda)]A_0 + \\ + \sum_{a=1}^{N} [Q^+_a(x)P^{+T}_a(y) - P^+_a(x)Q^{+T}_a(y) + Q^-_a(x)P^{-T}_a(y) - P^-_a(x)Q^{-T}_a(y)]A_0, \tag{2.4}$$

where

$$P(x,\lambda) = \frac{1}{\pi}(\rho^+\bar\Psi^+ + \rho^-\bar\Psi^-)(x,\lambda) = \frac{1}{\pi}(\sigma^+\bar\Phi^+ + \sigma^-\bar\Phi^-)(x,\lambda),$$

$$Q(x,\lambda) = \frac{1}{2b^+b^-}(\sigma^+\bar\Phi^+ - \rho^+\bar\Psi^+)(x,\lambda) = \frac{1}{2b^+b^-}(\rho^-\bar\Psi^- - \sigma^-\bar\Phi^-)(x,\lambda), \tag{2.5}$$

$$P^{\pm}_a(x) = \mp 2ic^{\pm}_a\bar\Psi^{\pm}_a(x), \quad Q^{\pm}_a(x) = \mp\frac{1}{2}[c^{\pm}_a\dot{\bar\Psi}^{\pm}_a - d^{\pm}_a\dot{\bar\Phi}^{\pm}_a(x)],$$

$$\sigma^{\pm}(\lambda) = b^{\mp}/a^{\pm}, \quad \lambda\in\Gamma, \quad d^{\pm}_a = (b^{\pm}_a\dot a^{\pm}_a)^{-1}, \quad c^{\pm}_a = b^{\pm}_a/\dot a^{\pm}_a,$$

$$b^{\pm}_a:\ \phi^{\pm}_a(x) = b^{\pm}_a\psi^{\pm}_a(x), \quad A_0 = \begin{pmatrix} 0 & -i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

These relations allow one to expand any function $f^T = (f_1, \ldots, f_4) \in \mathcal{S}(\mathbf{C}^4)$ over the systems $\{\Psi\}$, $\{\Phi\}$ or $\{P, Q\}$. The corresponding expansion coefficients have the form:

$$[\bar{\Psi}^{\pm}, f], \quad [\bar{\Phi}^{\pm}, f], \quad [P, f], \quad [Q, f],$$

where $[\ ,\ ]$ is the following skew-scalar product in $\mathcal{S}(\mathbf{C}^4)$:

$$[f, g] = \int_{-\infty}^{\infty} dx\, f^T(x) A_0 g(x), \quad A_0 = \begin{pmatrix} 0 & -i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix}. \tag{2.6}$$

In particular, for $f = \bar{w}(x)$ and $f = \bar{\sigma}_3 \delta \bar{w}(x)$ the corresponding expansion coefficients are expressed through the scattering data of the system $L(\lambda)$ and their variations:

$$\begin{aligned} &[\bar{\Phi}^{\pm}, \bar{w}] = \mp i a^{\pm} b^{\pm}, \qquad [\bar{\Phi}^{\pm}, \bar{\sigma}_2 \delta \bar{w}] = -i a^{\pm 2} \delta\rho^{\pm}, \\ &[\bar{\Psi}^{\pm}, \bar{w}] = \pm i a^{\pm} b^{\pm}, \qquad [\bar{\Psi}^{\pm}, \bar{\sigma}_3 \delta \bar{w}] = -i a^{\pm 2} \delta\sigma^{\pm}, \end{aligned} \tag{2.7}$$

The expansions themselves have the form:

$$\begin{aligned} \bar{w}(x) &= \frac{i}{\pi} \int_{\Gamma} d\lambda (\rho^+ \bar{\Psi}^+ + \rho^- \bar{\Psi}^-)(x, \lambda) + \\ &+ 2 \sum_{\alpha=1}^{N} [c_\alpha^+ \bar{\Psi}_\alpha^+(x) - c_\alpha^- \bar{\Psi}_\alpha^-(x)] = i \int_{\Gamma} d\lambda\, P(x, \lambda) + i \sum_{\alpha=1}^{N} [P_\alpha^+(x) + P_\alpha^-(x)], \end{aligned} \tag{2.8}$$

and

$$\begin{aligned} \bar{\sigma}_3 \delta \bar{w}(x) &= -\frac{i}{\pi} \int_{\Gamma} d\lambda (\delta\rho^+ \bar{\Psi}^+ - \delta\rho^- \bar{\Psi}^-)(x,\lambda) - 2 \sum_{\alpha=1}^{N} [\bar{U}_\alpha^+(x) + \bar{U}_\alpha^-(x)] = \\ &= \int_{\Gamma} d\lambda [Q(x, \lambda)\, \delta \hat{p}(\lambda) - P(x, \lambda) \delta \hat{q}(\lambda)] + \sum_{\alpha=1}^{N} [\bar{V}_\alpha^+(x) + \bar{V}_\alpha^-(x)], \end{aligned} \tag{2.9}$$

$$\bar{U}_\alpha^{\pm}(x) = \delta c_\alpha^{\pm} \bar{\Psi}_\alpha^{\pm}(x) + c_\alpha^{\pm} \delta\lambda_{\alpha\pm} \dot{\bar{\Psi}}_\alpha^{\pm}(x), \quad \bar{V}_\alpha^{\pm}(x) = Q_\alpha^{\pm}(x) \delta \hat{p}_\alpha^{\pm} - P_\alpha^{\pm}(x)\, \delta \hat{q}_\alpha^{\pm}.$$

In (2.9) we have introduced the following notations (compare with (2.5) and (2.7)):

$$\begin{aligned} &\delta \hat{p}(\lambda) = [P, \bar{\sigma}_3 \delta \bar{w}], \qquad \delta \hat{q}(\lambda) = [Q, \bar{\sigma}_3 \delta \bar{w}], \\ &\hat{p}(\lambda) = \frac{i}{\pi} \ln[1 + \rho^+ \rho^-(\lambda)], \qquad \hat{q}(\lambda) = \frac{i}{2} \ln \frac{b^+}{b^-}(\lambda), \quad \lambda \in \Gamma, \end{aligned} \tag{2.10}$$

$$\hat{p}_\alpha^{\pm} = \pm 2\lambda_{\alpha\pm}, \qquad \hat{q}_\alpha^{\pm} = \pm i \ln b_\alpha^{\pm}.$$

Note, that the set of expansion coefficients for $\bar{w}(x)$ over the system $\{\Psi\}$ coincides with the set of scattering data T introduced in ref. /21/, formula (2.7). Thus the formulae (2.8) and (2.9) allow one to interprete the transfer from $\bar{w}(x)$ to T and the ISM itself as a generalized Fourier transform.

Let us give also the explicit form of the operators $\Lambda_{\pm}$:

$$\Lambda_{\pm} = \begin{pmatrix} -Z_{10}^{\pm}, & 1 - Z_{11}^{\pm} \\ \hat{D} - Z_{00}^{\pm}, & -Z_{01}^{\pm} \end{pmatrix} \qquad \hat{D} = \frac{i}{2} \sigma_3 \frac{d}{dx} - \frac{1}{2} q_1 p_1, \tag{2.11}$$

for which the elements of $\{\bar{\Psi}\}$ and $\{\bar{\Phi}\}$ are eigen- and adjoint-functions:

$$\begin{aligned} &(\Lambda_+ - \lambda) \bar{\Psi}^{\pm}(x, \lambda) = 0, \quad \lambda \in \Gamma \cup \Delta; \quad (\Lambda_+ - \lambda_{\alpha\pm}) \dot{\bar{\Psi}}_\alpha^{\pm}(x) = \bar{\Psi}_\alpha^{\pm}(x), \\ &(\Lambda_- - \lambda) \bar{\Phi}^{\pm}(x, \lambda) = 0, \quad \lambda \in \Gamma \cup \Delta; \quad (\Lambda_- - \lambda_{\alpha\pm}) \dot{\bar{\Phi}}_\alpha^{\pm}(x) = \bar{\Phi}_\alpha^{\pm}(x). \end{aligned} \tag{2.12}$$

Analogically the operator* $\Lambda = \frac{1}{2}(\Lambda_+ + \Lambda_-)$ is naturally related to the system $\{P, Q\}$, since:

$$(\Lambda - \lambda) P(x, \lambda) = 0, \quad (\Lambda - \lambda) Q(x, \lambda) = 0, \quad \lambda \in \Gamma \cup \Delta. \tag{2.13}$$

The operators $\Lambda_{\pm}$ and Λ satisfy conjugation-like relations with respect to the skew-scalar product (2.6):

$$\begin{aligned} &[f, \Lambda_+ g] = [\Lambda_- f, g], \\ &[f, \Lambda g] = [\Lambda f, g], \end{aligned} \qquad f, g \in \mathcal{S}(\mathbf{C}^4). \tag{2.14}$$

With the help of the operators $\Lambda_{\pm}$ and Λ one is able to obtain compact expressions for the expansion coefficients $D^{(m)}$ of $D(\lambda)$ (2.3):

$$D(\lambda) = \sum_{m=1}^{\infty} \lambda^{-m} D^{(m)}, \quad |\lambda| \gg 1; \qquad D(\lambda) = -\sum_{m=0}^{\infty} \lambda^m D^{(-m)}, \quad |\lambda| \ll 1,$$

and their variations as functionals of $\bar{w}(x)$:

$$\begin{aligned} D^{(m)} &= -\frac{2}{m} \int_{-\infty}^{\infty} dx \int_{x}^{\infty} dy\, \bar{w}^T(y) A_0 \Lambda_+^{m+1} \bar{w}(y) + \\ &+ \frac{i}{2m} \int_{-\infty}^{\infty} dx\, (\widetilde{\sigma_3 w_1(x), 0})\, \Lambda^m\, \bar{w}(x), \end{aligned} \tag{2.15}$$

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* The authors are gratefull to S.V.Manakov, who called their attention to the operator Λ.

$$\delta D^{(m)} = -\frac{i}{2}[\bar{\sigma}_3 \delta \bar{w}, \Lambda_+^{m-1} \bar{w}] . \qquad (2.16)$$

It is also easy to check, that from the expansion of $\bar{w}(x)$ over the sympletic basis $\{P, Q\}$ and from (2.12), (2.13) there follows:

$$F(\Lambda_+)\bar{w} = F(\Lambda_-)\bar{w} = F(\Lambda)\bar{w} . \qquad (2.17)$$

Thus in the formulae (2.14) and (2.15) one may replace the operator Λ_+ by Λ_- or Λ. This we shall use below, and here we write down the explicit form of the first few $D^{(m)}$:

$$D^{(1)} = \frac{i}{2}\int_{-\infty}^{\infty} dx (q_0 p_1 + q_1 p_0) ,$$

$$D^{(2)} = \frac{i}{2}\int_{-\infty}^{\infty} dx [q_0 p_0 + \frac{1}{4} q_1^2 p_1^2 + \frac{i}{4}(p_1 q_{1x} - q_1 p_{1x})] ,$$

$$D^{(3)} = -\frac{1}{4}\int_{-\infty}^{\infty} dx (p_0 q_{1x} - q_0 p_{1x}) , \qquad (2.18)$$

$$D^{(4)} = \frac{i}{8}\int_{-\infty}^{\infty} dx [p_{1x} q_{1x} + i(p_0 q_{0x} - q_0 p_{0x}) + (p_1 q_0 + p_0 q_1)^2 - \frac{i}{4} q_1 p_1 (p_1 q_{1x} - q_1 p_{1x})] .$$

3. THE DESCRIPTION OF THE NLEE AND THEIR HAMILTONIAN STRUCTURES

Starting from the results, listed in Sec.2, it is not difficult to construct the theory of the NLEE related to $L(\lambda)$ (1.9), (1.10). In the proofs below we shall follow the ideas of ref. /9,7/, but will prefer to use the expansions over the symplectic basis $\{P, Q\}$ rather than those over the system $\{\Psi\}$, because the operator Λ is "selfadjoint" with respect to the skew-scalar product, see (2.14).

Theorem. Let the potential of the linear problem (1.9), (1.10) $\bar{w}(x, t)$ and the meromorphic function $F(\lambda)$ are such that $0 \neq F(\lambda_{a\pm}) \neq \infty$ and the integrals $\int_\Gamma d\lambda \frac{d\hat{p}(\lambda)}{dt}$ and $\int_\Gamma d\lambda [i\frac{d\hat{q}(\lambda)}{dt} - F(\lambda)]$ are absolutely convergent for all $0 < t < \infty$. Then $\bar{w}(x, t)$ satisfies the NLEE:

$$i\bar{\sigma}_3 \frac{d\bar{w}}{dt} + F(\Lambda)\bar{w} = 0 \qquad (3.1)$$

if and only if the set $\{\hat{p}(\lambda), \hat{q}(\lambda)\}$ (2.10) satisfies the following linear equations:

$$\frac{d\hat{p}}{dt} = 0, \qquad i\frac{d\hat{q}}{dt} = F(\lambda) ,$$
$$\frac{d\hat{p}_a^{\pm}}{dt} = 0, \qquad i\frac{d\hat{q}_a^{\pm}}{dt} = F(\lambda_{a\pm}) . \qquad (3.2)$$

The proof is obtained directly, incerting in the l.h.s. of (3.1) the expansions (2.8) for $\bar{w}(x, t)$ and (2.9) for the variation of the form $\bar{\sigma}_3 \delta \bar{w} = \bar{\sigma}_3 \frac{d\bar{w}}{dt}\delta t + O(\delta t)^2$ over the system $\{P, Q\}$.

Remark 1. If $F(\lambda) = F_2(\lambda)/F_1(\lambda)$, where $F_2(\lambda)$ and $F_1(\lambda)$ are polynomials in λ, then the NLEE (3.1) should be understood as

$$iF_1(\Lambda)\bar{\sigma}_3 \frac{d\bar{w}}{dt} + F_2(\Lambda)\bar{w} = 0 . \qquad (3.3)$$

Remark 2. From (2.16) it is obvious, that the NLEE (3.1) may be written in two more forms, equivalent to (3.1):

$$i\bar{\sigma}_3 \frac{d\bar{w}}{dt} + F(\Lambda_+)\bar{w} = 0, \qquad i\bar{\sigma}_3 \frac{d\bar{w}}{dt} + F(\Lambda_-)\bar{w} = 0 .$$

Remark 3. From the expansions of $\bar{w}(x,t)$ and $\bar{\sigma}_3 \bar{w}_t$ over the system $\{\Psi\}$ there follows, that the NLEE (3.1) is equivalent to the following set of linear equations for the scattering data T:

$$i\frac{d\rho^{\pm}}{dt} = \pm F(\lambda)\rho^{\pm}(\lambda, t), \quad i\frac{dc_a^{\pm}}{dt} = \pm F(\lambda_{a\pm})c_a^{\pm}(t), \quad \frac{d\lambda_{a\pm}}{dt} = 0 . \qquad (3.4)$$

From (3.2) and (2.3) it follows, that:

$$\frac{dD(\lambda)}{dt} = 0 , \qquad (3.5)$$

i.e., the quantities $D^{(m)}$ are integrals of motion for the NLEE (3.1).

Let us go now to the Hamiltonian structures of the NLEE (3.1). The corresponding phase space is naturally parametrized by the independent elements of the potentials of $\bar{w}$. The Hamiltonian H should be constructed as a linear combination of the integrals of motion $D^{(m)}$. One should also find a symplectic form Ω such, that the Hamiltonian equations of motion, given by H and Ω :

$$\Omega(\bar{\sigma}_3 \frac{d\bar{w}}{dt}, \cdot) = \delta H(\cdot) \tag{3.6}$$

coincide with the NLEE (3.1). Let $F(\lambda) = \Sigma F_k \lambda^k$ be a polynomial over the positive and negative powers of λ ,

$$H_F = i \Sigma F_k D^{(k+1)} \tag{3.7}$$

and $\Omega = \Omega_0$, where Ω_0 is a canonical 2-form:

$$\Omega_0 = -\frac{i}{2}[\bar{\sigma}_3 \delta \bar{w} \wedge \bar{\sigma}_3 \delta \bar{w}] . \tag{3.8}$$

Here the sign $\wedge$ means exterior product. From (3.7), (2.16) and (2.17) there immidiately follows that $\delta H_f = \frac{1}{2}[\bar{\sigma}_3 \delta \bar{w}, F(\Lambda)\bar{w}]$.

Thus it is easy to check that (3.6) with $H = H_F$ and $\Omega = \Omega_0$ coincides with the NLEE (3.1).

Note, that the choice of the pair Ω_0, H_F leading to eq.(3.1) or to an eq. equivalent to (3.1) is by no means unique [4,5]. Thus the choice:

$$\Omega = \Omega_m = -\frac{i}{2}[\bar{\sigma}_3 \delta \bar{w} \wedge \Lambda^m \bar{\sigma}_3 \delta \bar{w}] , \tag{3.9}$$

$$H = H_F^{(m)} = i \Sigma F_k D^{(k+m+1)} \tag{3.10}$$

leads to equations, differing from (3.1) by a left multiplication with the operator Λ^m. The corresponding linear equations for the scattering data may differ from (3.2) only for $\lambda = 0$, i.e., on a manifold with measure zero. Thus the pairs Ω_m, $H_F^{(m)}$ $m = 0, \pm 1, \pm 2, \ldots$ give us a hierarchy of Hamiltonian structures for the NLEE (3.1). Sometimes for the NLEE in the form (3.3) it is convenient to choose:

$$\Omega_{F_2} = -\frac{i}{2}[\bar{\sigma}_3 \delta \bar{w} \wedge F_2(\Lambda)\bar{\sigma}_3 \delta \bar{w}] , \tag{3.11}$$

$$H_{F_1} = i \Sigma F_{1,k} D^{(k+1)}, \quad F_1(\lambda) = \Sigma F_{1,k} \lambda^k . \tag{3.12}$$

The pairwise compatibility of the 2-forms Ω_m is most easily established by recalculating them in terms of the scattering data variations. This is conveniently done with the help of the symplectic completeness relation (2.4). Incerting in into (3.8) and (3.9) and using the first line in (2.10) we readily obtain:

$$\Omega_m = i \int_\Gamma d\lambda \lambda^m \delta\hat{p}(\lambda) \wedge \delta\hat{q}(\lambda) + i \sum_{a=1}^N [\lambda_{a+}^m \delta \hat{p}_a^+ \wedge \delta \hat{q}_a^+ + \lambda_{a-}^m \delta \hat{p}_a^- \wedge \delta \hat{q}_a^-] . \tag{3.13}$$

Thus in terms of $\{\hat{p}(\lambda), \hat{q}(\lambda)\}$ all Ω_m simultaneously become canonical, and therefore all Ω_m are pairwise compatible between themselves.

Let us express now the Hamiltonian H_F in terms of the scattering data. From (3.9) and (2.3) we obtain, that:

$$H = -\frac{i}{2}\int_\Gamma d\mu F(\mu) \hat{p}(\mu) - i \sum_{a=1} [\tilde{F}(\frac{\hat{p}_{a+}}{2}) - \tilde{F}(-\frac{\hat{p}_{a-}}{2})], \quad \tilde{F}(\lambda) = \int^\lambda d\lambda' F(\lambda'), \tag{3.14}$$

i.e., H_F depends only on the half of the canonical variables $\{\hat{p}, \hat{q}\}$ (2.10). Thus the complete integrability of all the NLEE (3.1) is established; the corresponding action-angle variables are given by (2.10).

Let us finish this paragraph by the remark, that the symplectic basis $\{P, Q\}$ allows one to define explicitly the Lagarange manifold of the NLEE $\mathfrak{M}$ by:

$$\mathfrak{M} \equiv \{ f \in \mathfrak{M} ; \ [f, P] = 0, \ [f, P_a^\pm] = 0 \} . \tag{3.15}$$

We list without proofs all the important properties of $\mathfrak{M}$,see refs.[9,7]:

i) If $f \in \mathfrak{M}$, then $\Lambda f = \Lambda_- f = \Lambda_+ f \in \mathfrak{M}$;
ii) $\bar{w} \in \mathfrak{M}$, and therefore $F(\Lambda_+)\bar{w} = F(\Lambda_-)\bar{w} = F(\Lambda)\bar{w} \in \mathfrak{M}$;
iii) if $\bar{w}(x,t)$ satifies the NLEE (3.1), then $\bar{\sigma}_3 \bar{w}_t \in \mathfrak{M}$.
iv) $\dim \mathfrak{M} = \operatorname{codim} \mathfrak{M}$.

Let us only verify that $\mathfrak{M}$ is indeed the Lagrange manifold, i.e., that $\Omega_m|_{\mathfrak{M}} \equiv 0$. Indeed, from $\bar{\sigma}_3 \delta \bar{w} \in \mathfrak{M}$ and (3.15) we have

$[\bar{\sigma}_3 \delta \bar{w}, P] = \delta\hat{p}(\lambda) = 0$, , $\lambda \in \Gamma \cup \Delta$, and from (3.13) it is obvious, that $\Omega_m|_{\mathfrak{M}} \equiv 0$.

4. EXAMPLES OF NLEE

Here we shall consider some interesting from our point of view NLEE (3.1). Choosing $F(\lambda)=-4\lambda^4$ in (3.1) we obtain the system:

$$\begin{gathered} i\sigma_3 w_{1t} + w_{1xx} + iB_1 + u_1 w_1 - 2u_0 w_0 = 0\,, \\ i\sigma_3 w_{0t} + w_{0xx} - iB_0 - 2iu_0\sigma_3 w_{1x} + u_1 w_0 + q_1 p_1 u_0 w_1 = 0\,, \\ B_0 = \begin{pmatrix} q_1^2 & p_{0x} \\ -p_1^2 & q_{0x} \end{pmatrix}, \qquad B_1 = \begin{pmatrix} q_1^2 & p_{1x} \\ -p^2 & q_{1x} \end{pmatrix}, \\ u_0 = q_0 p_1 + q_1 p_0, \quad u_1 = \frac{1}{2} q_1^2 p_1^2 - 2q_0 p_0\,, \end{gathered} \tag{4.1}$$

which after the involution

$$p_0 = \epsilon_0 q_0^*\,, \quad p_1 = \epsilon_1 q_1^*\,, \quad \epsilon_0^2 = \epsilon_1^2\,, \quad \epsilon_1^* = \epsilon_1 \tag{4.2}$$

goes into the following 2-component modified NLS eq.:

$$\begin{gathered} iq_{1t} + q_{1xx} + i\epsilon_1 q_1^2 q_{1x}^* + v_1 q_1 - 2v_0 q_0 = 0\,, \\ iq_{0t} + q_{0xx} - i\epsilon_0 q_1^2 q_{0x}^* - 2iv_0 q_{1x} + \\ + \epsilon_1 |q_1|^2 v_0 q_1 + v_1 q_0 = 0\,. \\ v_0 = \epsilon_1 q_0 q_1^* + \epsilon_0 q_1 q_0^*\,, \quad v_1 = \frac{\epsilon_1^2}{2} |q_1^4| - 2\epsilon_0 |q_0^2|\,. \end{gathered} \tag{4.3}$$

This system contains as particular cases the NLS equation, and also the modified NLS eqs. (1.3) and (1.7). Indeed the NLS equation $iq_{0t} + q_{0,xx} - 2\epsilon_0 |q_0^2| q_0 = 0$ is obtained from (4.3) with $q_1 = 0$. This is to be expected, since for $q_1 = 0$ the linear problem (1.3) goes into the Zakharov-Shabat system, which has been investigated in detail earlier, see refs. [1,6-10].

For $q_0 = 0$ (4.3) is reduced to the NLEE:

$$iq_{1t} + q_{1xx} + i\epsilon_1 q_1^2 q_x^* + \frac{\epsilon_1^2}{2} |q_1|^4 q_1 = 0, \tag{4.4}$$

which after the change of variables (1.12) with $\varphi(x) = \frac{\epsilon_1}{2} \int_x^\infty dy\, |q_1^2|$ goes into the modified NLS eq. (1.3). This change of variables is closely related to the gauge transformation (1.11), after which the linear problem (1.9), (1.10) with $Q_0 = 0$ becomes equal to $L_2(\lambda)$ (1.2). This may be used to check, that in this way all the results related to the system (1.2) are reproduced, see ref. [7].

Another possible reduction, leading to equations of NLS'type has the form:

$$q_0 = a q_1\,, \quad p_0 = -a p_1\,, \quad a = -\eta a^*\,, \quad \eta = \frac{\epsilon_0}{\epsilon_1} = \pm 1\,. \tag{4.5}$$

Then (4.3) becomes:

$$iq_{1t} + q_{1xx} + i\epsilon_1 q_1^2 q_{1x}^* - 2|a^2|\epsilon_0 |q_1^2|\, q_1 + \frac{\epsilon_1^2}{2} |q_1^4|\, q_1 = 0 \tag{4.6}$$

and after the change of variables (1.12) with $\varphi(x) = \frac{\epsilon_1}{2} \int_x^\infty dy\, |q_1^2|$ one obtains the following modified NLS eq.:

$$\begin{gathered} iu_t + u_{xx} + i\epsilon_1 (|u^2| u)_x - 2a^2 \epsilon_0 |u^2| u = 0\,, \\ u = q_1(x) e^{-i\varphi} \end{gathered} \tag{4.7}$$

This with $\epsilon_1 = -\bar{a}$, $\epsilon_0 = -\frac{\bar{\beta}^2}{a^2}$ coinsides with (1.7).

The involution (4.2) imposes the following restrictions on the set of scattering data **T**:

$$\begin{gathered} \rho^+(\lambda) = -\epsilon_0 \rho^{-*}(\lambda^* \eta), \qquad b_a^+ = -\epsilon_0 b_a^{-*}, \\ \lambda_{a-} = \eta \lambda_{a+}^*\,, \qquad \eta = \frac{\epsilon_0}{\epsilon_1}, \end{gathered} \tag{4.8}$$

Analogically (4.5) leads to

$$\begin{gathered} \rho^\pm(\lambda) = -\rho^\pm(-\lambda), \qquad \lambda \in \Gamma \\ a^\pm(\lambda) = a^\pm(-\lambda), \qquad \operatorname{Im}\lambda^2 \gtrless 0 \end{gathered} \tag{4.9}$$

Note, that the restrictions (4.9) do not depend on a and coincide with the properties of the sctteering data for $L_2(\lambda)$ (1.2); the former is gauge equivalent (see (1.11)) to $L(\lambda)$ with $Q_0 = 0$.

The Hamiltonian structure of the NLS' eq. (4.3) is most simply given by $\Omega = \Omega_0$, $H = -4D^{(5)}$, which after taking into consideration the involution (4.2) are equal to:

$$\Omega_0 = i \int_{-\infty}^{\infty} dx\, [\epsilon_0 \delta q_0^* \wedge \delta q_1 - \epsilon_1 \delta q_0 \wedge \delta q_1^*] =$$

$$= i \int_\Gamma d\lambda \delta \hat{p}'(\lambda) \quad \delta \hat{q}'(\lambda) \; +$$

$$+ 2i(1+\eta) \sum_{a=1}^{N} [\delta\lambda^{\circ}_{a+} \wedge \delta\beta^{\circ}_{a} - \delta\lambda^{1}_{a+} \wedge \delta\beta^{1}_{a}] \; + \tag{4.10}$$

$$+ 2i(1-\eta) \sum_{a=1}^{N} [\delta\lambda^{1}_{a+} \wedge \delta\beta^{\circ}_{a} + \delta\lambda^{\circ}_{a+} \wedge \delta\beta^{1}_{a}] \; ,$$

$$\hat{p}'(\lambda) = \frac{1}{\pi} \ln[1 - \epsilon_0 \rho^+(\lambda) \rho^{+*}(\eta\lambda^*)] \; ,$$

$$\hat{q}'(\lambda) = \frac{i}{2} \ln [b^+(\lambda) / b^{+*}(\lambda^*\eta)] \; , \quad \lambda \in \Gamma \; ,$$

$$\lambda_{a+} = \lambda^{\circ}_{a+} + i\lambda^{1}_{a+} \; , \quad \ln b^{+}_{a} = \beta^{\circ}_{a} + i\beta^{1}_{a} \; , \quad a = 1, \dots, N \; ,$$

$$H = \frac{1}{2} \int_{-\infty}^{\infty} dx \{ -(\epsilon_0 q_{1x} q^*_{0x} + \epsilon_1 q^*_{1x} q_{0x}) \; +$$

$$+ \frac{3i\epsilon_1}{5} |q_1^2| (\epsilon_0 q_{1x} q^*_0 - \epsilon_1 q_0 q^*_{1x}) + v_0 v_1 \} = \tag{4.11}$$

$$= 2i \int_\Gamma d\mu \, \mu^4 \hat{p}'(\mu) + \frac{4i}{\mu} \sum_{a=1}^{N} [(\lambda_{a+})^5 - (\eta\lambda^*_{a+})^5] \; .$$

From (4.3) it is easy to obtain the explicit form of the action-angle variables.

As it has been noted, the modified NLS eq. (4.7) is obtained from (4.1) by imposing the two involutions (4.2) and (4.5). From (4.9) and (3.13) it follows, that all 2-forms $\Omega_{2k} \equiv 0$; analogically $D^{(2k+1)} \equiv 0$, see (3.14). The Hamiltonian structure of (4.7) may be given, e.g., by $\Omega = \tilde{\Omega}_{-1}$, $H = -4\tilde{D}^{(4)}$ or by $\Omega = \tilde{\Omega}_1$, $H = -4D^{(6)}$, where the sign ~ here means, that in the expressions for Ω_k and $D^{(m)}$ one should impose the involutions (4.2) and (4.5). The explicit calculation of Ω_{-m}, $m > 0$ in terms of the potential $\bar{w}(x,t)$ is related to the calculation of the inverse operator Λ^{-1}, which for $a \neq 0$ is difficult. In this case we can make use of the corresponding Poisson brackets:

$$\{F, G\}_m = [\bar{\nabla} F \bar{\sigma}_3 \; , \; A_0 \Lambda^m \bar{\sigma}_3 A_0 \bar{\nabla} G] \; ,$$

$$\bar{\nabla} F = \begin{pmatrix} \nabla_1 F \\ \nabla_0 F \end{pmatrix} , \quad (\nabla_k F)^T = \left(\frac{\delta F}{\delta q_k}, \frac{\delta F}{\delta p_k}\right), \quad k = 0,1. \tag{4.12}$$

Incerting $F = G = \bar{w}$ in (4.12) we obtain the corresponding Poisson brackets between the elements of the potentials Q_i in

$$\{\bar{\sigma}_3 \bar{w} \overset{\otimes}{,} \bar{\sigma}_3 A_0 \bar{w}\}_m = \Lambda^m \delta(x-y) \; , \tag{4.13}$$

where the i,j-th element of the matrix $\{F \overset{\otimes}{,} G\}_m$ is equal to $\{F_i, G_j\}_m$. Now it is easy to check, that the Hamiltonian equations of motion:

$$i\bar{\sigma}_3 \bar{w}_t = \{\bar{\sigma}_3 \bar{w}, H\}_1 \tag{4.14}$$

with $H = -4D^{(4)}$ directly lead to the system (4.1). In order to impose the involutions (4.2), (4.5) on the Poisson brackets $\{,\}_1$ one should calculate the corresponding Dirack brackets, which in our case is difficult. Therefore we shall only write down $\tilde{\Omega}_1$

$$\tilde{\Omega}_1 = \frac{i\epsilon_1}{2} \int_{-\infty}^{\infty} dx [\delta q_1^* \wedge \frac{d}{dx} \delta q_1 - \epsilon_1 |q_1^2| \delta q_1^* \wedge \delta q_1] \; ,$$

which together with $H = -4D^{(6)}$ generates (4.3).

In terms of the scattering data the expression for $\tilde{\Omega}_{-1}$ has the same form as that of Ω_0, given in ref. [7], and we omit it. Thus the NLS eqs. (1.3), and (4.7) have an equivalent sets of action-angle variables.

Let us give one more example of a NLEE, generalizing the Mikhailov system:

$$\begin{aligned} & w_{1xt} + iq_1 p_1 \sigma_3 w_{1t} + 2a^2 w_1 I - 2c_1 w_1 = 0 \; , \\ & I_x + (q_1 p_1)_t = 0 \; . \end{aligned} \tag{4.15}$$

This system is obtained from (3.3) with $F_1(\lambda) = \lambda^2$ and $F_2(\lambda) = c = \text{const}$, after imposing the involution (4.5); (4.14) survives also the involution (4.2). The Hamiltonian structure of (4.14) may be given, e.g., by $\Omega = \tilde{\Omega}_3$ and $H = c\tilde{D}^{(2)}$.

In conclusion letus make a few remarks.

i) Considering block bundles of the form (1.2), (1.3), (1.6) it is possible to solve the multicomponent (vector and matrix) analog of the NLEE's, considered above, see refs. [4,13]. The corresponding expansions over the "squared" solutions may be derived analogously to ref. [25]. The difficulties with the explicit calculation of the operator Λ have been noted in ref. [22].

ii) The polynomial bundles (1.8) are easily written down as eigenvalue problems of the type $[J\frac{d}{dx} + U - \lambda]\tilde{\psi} = 0$, where $\tilde{\psi}^T = (\psi^T, \lambda\psi^T, \dots, \lambda^{N-1}\psi^T)$, J is a degenerate constant matrix and the potential U is a specific 2Nx2N matrix, whose matrix elements are expressed by those of U_k, see, e.g., ref.[13]. Thus the study of the polynomial bundles and their possible involutions is directly related to the reduction problem of the NLEE, ref.[23].

iii) Other variants of the modified NLS equations have been considered in refs.[23,24].

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Квадратичный пучок общего вида и нелинейные эволюционные уравнения. Иерархия гамильтоновских структур

При использовании метода разложения по "квадратам" решений описан класс нелинейных эволюционных уравнений, связанных с квадратичным пучком общего вида. Доказано, что эти уравнения являются вполне интегрируемыми гамильтоновскими системами и обладают иерархией гамильтоновских структур.

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The Quadratic Bundle of General Form and the Nonlinear Evolution Equations. Hierarchies of Hamiltonian Structures

Using the method of expansions over the "squared" solutions of the auxiliary linear problem, the class of nonlinear evolution equations related to the quadratic bundle of general form is described. It is proved, that these equations are completely integrable Hamiltonian systems, possessing hierarchies of Hamiltonian structures.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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