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THE MULTIPARTON DISTRIBUTION EQUATIONS IN QCD

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The comparison of QCD predictions with experimental data (in particular the observation of increasing $\langle P_{\perp}^{2}\rangle_{jet}$ with Q^2 and three jet events [1]) has shown in any case QCD legitimacy as perturbative theory if not its correctness [2]. Since all the calculations are made for free quarks and gluons, one needs to be able to describe converting stage of colour objects (quarks and gluons) into actual hadrons to analyse hard process in detail. At present we can describe this stage only phenomenologically, assuming that large transverse momenta of dressed colour fields are immediate source of large transverse momenta of hadrons. However one succeeds in proving the existence [3] of the parton order in a jet at any given stage of its evolution in the leading logarithm approximation (LLA) assuming the validity of $1/N_c$ expansion. Accordingly partons group in a natural way into systems consisting of a quark, an adjacent antiquark and gluons that are between them. Such systems are colour singlets in the $N_c \to \infty$ limit forming clusters with order Q_0 masses independent of initial Q. This result makes it possible to suggest that only these colourless clusters with finite masses will convert into hadrons, i.e., non perturbative confinement forces do not change essentially parton momenta on a converting stage into hadrons. And then one can derive features of hadron jets from multiparton distributions dependent on Q^2 and hadronization functions independent of Q^2 considering their proper convolution. Naturally, multiparton distributions will play decisive role, if we compare these hadron features with experimental data and receive information about mechanism of parton confinement.

In this paper we obtain the equations and solutions for the multiparton distribution functions of deep-inelastic lepton-hadron scattering and multiparton fragmentation functions of e^+e^- -annihilation into hadrons using Lipatov's parton interpretation of perturbative QCD theory diagrams in the LLA [4]. The obtained equations are not identical, but the solutions are the same on the definite initial conditions and coincide with the jet calculus rules [5]. The principal difference of these equations becomes clear when we generalize them for description of parton fragmentation into hadrons.

In Part 1 we give necessary results for the single-parton functions and obtain the equations for the multiparton distribution and fragmentation functions. The solutions of these equations and their connection with the jet calculus rules are discussed in Part 2. In Part 3 we consider the generalization of the obtained results for hadron jets description.

1. The structure functions for ep-scattering and e^+e^--annihilation were calculated in the LLA for vector and pseudoscalar theories in refs.[6,7]. The same calculations were made in QCD in ref.[8]. It was shown[4] that the results of these calculations allowed a simple interpretation within the framework of the parton model with variable cut-off parameter $\Lambda \sim Q^2$ for the transverse momenta. Sets of wave functions Ψ_i^n, $\bar{\Psi}_i^n$ play an important role in this interpretation. $\Psi_i^n(\beta_r, k_{\perp r})$ is the probability amplitude of finding a dressed parton of type i in the state of n bare partons with Sudakov's parameters β_r, $k_{\perp r}$, $r=1,\dots,n$[9]. $\bar{\Psi}_i^n(\beta_r, k_{\perp r})$ is the probability amplitude of finding a bare parton of type i in the state of n dressed partons with Sudakov's parameters β_r, $k_{\perp r}$. These amplitudes satisfy the normalization condition

$$1 = z_i + \sum_{n=2}^{\infty} \int \prod_{r=1}^{n} \frac{d\beta_r}{\beta_r} \theta(\beta_r) d^2 k_{\perp r} |\Psi_i^n|^2 \delta^2(\sum k_{\perp r}) \delta(\sum \beta_r - 1), \qquad (1)$$

where z_i is the wave function renormalization constant of i-type parton. The normalization condition for $\bar{\Psi}_i^n$ is the same. The full expression for Ψ_i^n, $\bar{\Psi}_i^n$ can be obtained by calculating the contribution of all the diagrams for transition $i \to n$ in the infinite momentum frame. Feynman integrals for Ψ_i^n, $\bar{\Psi}_i^n$ and $k_\perp$-integrals in eq.(1) diverge logarithmically in renormalizable theories, they are regularized with the help of cut-off parameter Λ. Then multiparton features of ep-scattering and e^+e^--annihilation processes expressed through sets of wave functions Ψ_i^n, $\bar{\Psi}_i^n$ accordingly become functions of this parameter. And the equations for multiparton distribution and fragmentation functions are obtained by differentiating their definition with respect to Λ and using the equations obtained by differentiating the normalization condition. The differentiation rules are described in ref.[4] in detail. One notes only that in the LLA the main contribution for Ψ_i^n, $\bar{\Psi}_i^n$ is given by tree diagrams that do not interfere between themselves. It allows a classical probability interpretation of each term in eq. (1). Besides transverse momenta are strongly ordered along the branches of the tree: they increase along the tree diagrams for ep-scattering and decrease for e^+e^--annihilation.

The left side is independent of Λ in normalization condition but functions Ψ_i^n (via the z-factors) and the $k_\perp$-integration region depend on Λ in the right side. Therefore differentiation of normalization condition leads to the equations:

$$0 = \Lambda \frac{dz_j}{d\Lambda} z_j^{-1} + \omega_j(g^2_\Lambda), \qquad (2)$$

where

$$\omega_j(g^2_\Lambda) = \frac{\partial \Pi_j(\Lambda/k^2)}{\partial \ln(\Lambda/|k^2|)}\Big|_{k^2=\Lambda} = \frac{g^2_\Lambda}{8\pi^2}\bar{\omega}_j, \qquad (3)$$

$\Pi_j(\frac{\Lambda}{k^2})$ is the one-loop self-energy part of j-type parton, g_Λ is the running coupling constant.

Defining two-parton distribution functions

$$D_i^{j_1 j_2}(x_1, x_2) = z_i \delta_{ij_1} \delta_{j_1 j_2} \delta(x_1 - x_2)\delta(x_1 - 1) + \sum_{n=2}^{\infty} \sum_{r(j_1)} \sum_{r(j_2)} \int \prod_{r=1}^{n} \frac{d\beta_r}{\beta_r} \times \qquad (4)$$

$$\times \theta(\beta_r) d^2 k_{\perp r} |\Psi_i^n|^2 \delta^2(\sum k_{\perp r}) \delta(\sum \beta_r - 1) \delta(\beta_{r(j_1)} - x_1) \delta(\beta_{r(j_2)} - x_2),$$

where $\sum_{r(j)}$ is the sum only over j-type partons in n-parton state, we obtain by differentiating with respect to Λ

$$\Lambda \frac{dD_i^{j_1 j_2}(x_1,x_2)}{d\Lambda} = \frac{g^2_\Lambda}{8\pi^2} \Big\{ \sum_{j_1'} \int_{x_1}^{1-x_2} \frac{dx_1'}{x_1'} D_i^{j_1' j_2}(x_1', x_2) P_{j_1' \to j_1}\left(\frac{x_1}{x_1'}\right) +$$

$$+ \sum_{j_2'} \int_{x_2}^{1-x_1} \frac{dx_2'}{x_2'} D_i^{j_1 j_2'}(x_1, x_2') P_{j_2' \to j_2}\left(\frac{x_2}{x_2'}\right) + \qquad (5.1)$$

$$+ \sum_{j'} D_i^{j'}(x_1 + x_2) \frac{1}{x_1 + x_2} P_{j' \to j_1 j_2}\left(\frac{x_1}{x_1 + x_2}\right) +$$

$$+ \delta_{j_1 j_2} \delta(x_1 - x_2) \sum_{j'} \int_{x_1}^{1} \frac{dx'}{x'} D_i^{j'}(x') P_{j' \to j_1}\left(\frac{x_1}{x'}\right) -$$

$$- D_i^{j_1}(x_1) \frac{1}{x_1} P_{j_1 \to j_2}\left(\frac{x_2}{x_1}\right) - D_i^{j_2}(x_2) \frac{1}{x_2} P_{j_2 \to j_1}\left(\frac{x_1}{x_2}\right) \Big\}.$$

Differentiation of the same definition of fragmentation functions leads to the equations

$$\Lambda \frac{d\bar{D}_i^{j_1 j_2}(x_1,x_2)}{d\Lambda} = \frac{g_\Lambda^2}{8\pi^2}\Big\{ \sum_{i'} \int_{x_1+x_2}^{1} \frac{dx'}{x'^2} \bar{D}_{i'}^{j_1 j_2}\left(\frac{x_1}{x'}, \frac{x_2}{x'}\right) P_{i\to i'}(x') + \tag{5.2}$$

$$+ \sum_{i_1 i_2} \int_{x_1}^{1-x_2} \frac{dx'}{x'(1-x')} P_{i\to i_1 i_2}(x') \bar{D}_{i_1}^{j_1}\left(\frac{x_1}{x'}\right) \bar{D}_{i_2}^{j_2}\left(\frac{x_2}{1-x'}\right)\Big\},$$

where

$$\frac{g_\Lambda^2}{8\pi^2}\frac{1}{x} P_{j\to j_1 j_2}\left(\frac{x_1}{x}\right) = \underset{j_1\le j_2}{\omega_{j\to j_1 j_2}(x\to x_1)} + \underset{j_1\ge j_2}{\omega_{j\to j_1 j_2}(x\to x_1)}, \tag{6.1}$$

$$\frac{g_\Lambda^2}{8\pi^2}\frac{1}{x} P_{j\to j_1}\left(\frac{x_1}{x}\right) = \omega_{j\to j_1}(x\to x_1) - \delta_{jj_1}\delta(x_1-x_2)\,\omega_j(g_\Lambda^2), \tag{6.2}$$

$\omega_{j\to j_1 j_2}(x\to x_1)\,dx_1 \frac{d\Lambda}{\Lambda}$ is as usual the probability that a parton of type j with fraction x of the longitudinal momentum decays into two partons of type j_1, j_2, one of which has fraction x_1 of the longitudinal momentum and the transverse momentum Λ,

$$\omega_{j\to j_1} = \sum_{j_1\le j'} \omega_{j\to j_1 j'} + \sum_{j'\le j_1} \omega_{j\to j' j_1}, \tag{6.3}$$

$$\omega_j = \sum_{j_1\le j_2} \omega_{j\to j_1 j_2},$$

$D_i^j(x)$, $\bar{D}_i^j(x)$ are the single-parton distribution and fragmentation functions accordingly. Their equations were obtained in refs./4,10/

$$\Lambda \frac{dD_i^j(x)}{d\Lambda} = \frac{g_\Lambda^2}{8\pi^2}\sum_{j'} \int_x^1 \frac{dx'}{x'} D_i^{j'}(x') P_{j'\to j}\left(\frac{x}{x'}\right), \tag{7.1}$$

$$\Lambda \frac{d\bar{D}_i^j(x)}{d\Lambda} = \frac{g_\Lambda^2}{8\pi^2}\sum_{i'} \int_x^1 \frac{dx'}{x'} \bar{D}_{i'}^j\left(\frac{x}{x'}\right) P_{i\to i'}(x'). \tag{7.2}$$

It is convenient to rewrite eqs. (5.1), (5.2), (7.1), (7.2) taking moments of $D, \bar{D}$

$$\frac{dM_i^{j_1 j_2}(n_1,n_2,t)}{dt} = \sum_{j_1'} M_i^{j_1' j_2}(n_1,n_2,t)P_{j_1'\to j_1}(n_1) + \sum_{j_2'} M_i^{j_1 j_2'}(n_1,n_2,t)P_{j_2'\to j_2}(n_2) +$$

$$+ \sum_{j'} M_i^{j'}(n_1+n_2,t)\left[P_{j'\to j_1 j_2}(n_1,n_2) + \tilde{P}_{j'\to j_1 j_2}(n_1,n_2)\right], \tag{8.1}$$

$$\frac{d\bar{M}_i^{j_1 j_2}(n_1,n_2,t)}{dt} = \sum_{i'} \bar{M}_{i'}^{j_1 j_2}(n_1,n_2,t) P_{i\to i'}(n_1+n_2) + \tag{8.2}$$

$$+ \sum_{i_1 i_2} \bar{M}_{i_1}^{j_1}(n_1,t)\,\bar{M}_{i_2}^{j_2}(n_2,t)\,P_{i\to i_1 i_2}(n_1,n_2),$$

$$\frac{dM_i^j(n,t)}{dt} = \sum_{j'} M_i^{j'}(n,t) P_{j'\to j}(n), \tag{9.1}$$

$$\frac{d\bar{M}_i^j(n,t)}{dt} = \sum_{i'} \bar{M}_{i'}^j(n,t) P_{i\to i'}(n), \tag{9.2}$$

where

$$M_i^{j_1 j_2}(n_1,n_2,t) = \int_0^1 x_1^{n_1} x_2^{n_2}\,\theta(1-x_1-x_2) D_i^{j_1 j_2}(x_1,x_2)\,dx_1 dx_2,$$

$$\bar{M}_i^{j_1 j_2}(n_1,n_2,t) = \int_0^1 x_1^{n_1} x_2^{n_2}\,\theta(1-x_1-x_2) \bar{D}_i^{j_1 j_2}(x_1,x_2)\,dx_1 dx_2,$$

$$M_i^j(n,t) = \int_0^1 x^n D_i^j(x)\,dx, \quad \bar{M}_i^j(n,t) = \int_0^1 x^n \bar{D}_i^j(x)\,dx, \tag{10}$$

$$P_{i\to i_1}(n) = \int_0^1 x^n P_{i\to i_1}(x)\,dx,$$

$$P_{i\to i_1 i_2}(n_1,n_2) = \int_0^1 x^{n_1}(1-x)^{n_2} P_{i\to i_1 i_2}(x)\,dx,$$

$$\tilde{P}_{j'\to j_1 j_2}(n_1,n_2) = \delta_{j_1 j_2} P_{j'\to j_1}(n_1+n_2) - \delta_{j' j_1} P_{j_1\to j_2}(n_2) - \delta_{j' j_2} P_{j_2\to j_1}(n_1).$$

In these equations we pointed explicitly that the moments of multiparton functions depended on natural variable t which may be expressed through Λ using known in QCD expression for Λ -dependence of the running coupling constant

$$t = \frac{1}{2\pi b}\ln\left[1 + \frac{g^2(\mu^2)}{4\pi} b \ln\left(\frac{\Lambda}{\mu^2}\right)\right], \quad b = \frac{33-2n_f}{12\pi}, \tag{11}$$

where μ^2 is the characteristic virtuality for which the running coupling constant is small and perturbative theory is legitimate.

2. The obtained eqs. (8.1), (8.2) form a set of first-order linear-differential equations with constant coefficients. The equations for distribution functions bind the probabilities of finding bare partons of different types in a dressed parton of type i. The equations for fragmentation functions bind the probabilities of finding definite bare partons of type j_1, j_2 in dressed partons of different types. Naturally, these equations have different solutions in a general case. It is easy to verify directly that the particular solutions of eqs. (8.1), (8.2) may be written as

$$M_i^{j_1j_2}(n_1,n_2,t)=\sum_{jj_1'j_2'}\int_0^t dt' M_i^j(n_1+n_2,t')[P_{j\to j_1'j_2'}(n_1,n_2)+ \tag{12.1}$$

$$+\tilde{P}_{j\to j_1'j_2'}(n_1,n_2)]M_{j_1'}^{j_1}(n_1,t-t')M_{j_2'}^{j_2}(n_2,t-t'),$$

$$\bar{M}_i^{j_1j_2}(n_1,n_2,t)= \tag{12.2}$$

$$=\sum_{jj_1'j_2'}\int_0^t dt'\bar{M}_i^j(n_1+n_2,t-t')P_{j\to j_1'j_2'}(n_1,n_2)\bar{M}_{j_1'}^{j_1}(n_1,t')\bar{M}_{j_2'}^{j_2}(n_2,t').$$

One must use eqs. (9.1), (9.2) for the single-parton functions and their initial conditions $M_i^j(n,t=0)=\delta_{ij}$, $\bar{M}_i^j(n,t=0)=\delta_{ij}$. The difference between these particular solutions is the additional term $\tilde{P}$ in the expression for $M_i^{j_1j_2}$. However if we introduce new functions

$$\tilde{D}_i^{j_1j_2}(x_1,x_2)=D_i^{j_1j_2}(x_1,x_2)-\delta_{j_1j_2}\delta(x_1-x_2)D_i^{j_1}(x_1), \tag{13.1}$$

$$\tilde{\bar{D}}_i^{j_1j_2}(x_1,x_2)=\bar{D}_i^{j_1j_2}(x_1,x_2)-\delta_{j_1j_2}\delta(x_1-x_2)\bar{D}_i^{j_1}(x_1), \tag{13.2}$$

the equations for moments of new fragmentation functions remain unchangeable and a term $\tilde{P}$ vanishes in equations for moments of new distribution functions. And then the particular solutions of type (12) for new functions will be the same.

One notes that these particular solutions satisfy zero initial conditions at $t=0$. They will be the only solutions of corresponding equations if we are interested in solutions with zero initial conditions. We can obtain the same or not the same solutions for the distribution and fragmentation functions depending on the choice of functions M, $\bar{M}$ or $\tilde{M}$, $\tilde{\bar{M}}$ for which we demand zero initial conditions. These initial conditions are obtained unambiguously from definitions of multiparton functions by Lipatov's method: functions $\tilde{D}$, $\tilde{\bar{D}}$ and their moments $\tilde{M}$, $\tilde{\bar{M}}$ satisfy zero initial conditions. Therefore the solutions for moments of two-parton distribution $\tilde{M}_i^{j_1j_2}$ and fragmentation $\tilde{\bar{M}}_i^{j_1j_2}$ functions may be expressed through moments of single-parton functions by eq. (12.2) in the same way. It coincides with the jet calculus rules. There is the same situation in case of n-parton functions. The equations for the multiparton distribution functions were obtained also in ref./11/ by Lipatov's method. However their solutions were compared there with the jet calculus rules proposed for the multiparton fragmentation functions. It is shown in this paper that the distribution and fragmentation functions satisfy essentially different equations but they have the same solutions on the definite initial conditions. The difference becomes crucial when we generalize these equations for hadron jets description. This question is discussed in detail in the next part. We only note that the solutions of the jet calculus rules type may be obtained for correlation distribution and fragmentation functions:

$$\Delta_i^{j_1j_2}(x_1,x_2)=D_i^{j_1j_2}(x_1,x_2)-D_i^{j_1}(x_1)D_i^{j_2}(x_2), \tag{14.1}$$

$$\bar{\Delta}_i^{j_1j_2}(x_1,x_2)=\bar{D}_i^{j_1j_2}(x_1,x_2)-\bar{D}_i^{j_1}(x_1)\bar{D}_i^{j_2}(x_2). \tag{14.2}$$

It is easy to see that the moments of correlation distribution functions $\Delta_i^{j_1j_2}(n_1,n_2,t)$ satisfy the same eq. (8.1) as moments of two-parton distribution functions $M_i^{j_1j_2}(n_1,n_2,t)$. The equations for moments of correlation fragmentation functions $\bar{\Delta}_i^{j_1j_2}(n_1,n_2,t)$ differ from eq. (8.2) by appearance of the additional term $\tilde{P}$

$$\frac{d\bar{\Delta}_i^{j_1j_2}(n_1,n_2,t)}{dt}=\sum_{i'}\bar{\Delta}_{i'}^{j_1j_2}(n_1,n_2,t)P_{i\to i'}(n_1+n_2)+ \tag{15}$$

$$+\sum_{i_1i_2}\bar{M}_{i_1}^{j_1}(n_1,t)\bar{M}_{i_2}^{j_2}(n_2,t)[P_{i\to i_1i_2}(n_1,n_2)+\tilde{P}_{i\to i_1i_2}(n_1,n_2)].$$

The direct substitution shows that the solutions for moments of correlation distribution and fragmentation functions may be expressed through the moments of single-parton functions in the same way by eq. (12.1) unlike the above-considered case of the two-parton distribution and fragmentation functions. It differs from the jet calculus rules by replacement expression $P_{j\to j_1 j_2}(n_1,n_2)$ with one $[P_{j\to j_1 j_2}(n_1,n_2)+\tilde{P}_{j\to j_1 j_2}(n_1,n_2)]$. Again zero initial conditions for correlation functions follow from their definitions.

3. As was noted a present level of our understanding of QCD structure allowed one to work only with free quarks and gluons but the conversion of colour objects into hadrons one must describe phenomenologically using hypothesis of soft colourlessness, i.e., one assumes that nonperturbative hadronization process takes place under small virtualities of partons and does not interfere with the hard process taking place under large virtualities. Then hadron features are obtained by convolution of Q^2-independent phenomenological functions with multiparton fragmentation functions. For example, two-particle fragmentation functions for a parton of type i to fragment into hadrons h_1, h_2 are given by[5]

$$\bar{D}_i^{h_1 h_2}(x_1,x_2,t)=\sum_{j_1 j_2}\int\frac{dx_1' dx_2'}{x_1' x_2'}\bar{D}_{j_1}^{h_1}\left(\frac{x_1}{x_1'},0\right)\bar{D}_{j_2}^{h_2}\left(\frac{x_2}{x_2'},0\right)\tilde{D}_i^{j_1 j_2}(x_1',x_2',t)+ \tag{16}$$
$$+\sum_j\int\frac{dx}{x^2}\bar{D}_j^{h_1 h_2}\left(\frac{x_1}{x},\frac{x_2}{x},0\right)\bar{D}_i^j(x,t),$$

where we pointed explicitly t-dependence of multiparton functions and introduced phenomenological t-independent fragmentation functions $\bar{D}_i^h(x,0)$, $\bar{D}_i^{h_1 h_2}(x_1,x_2,0)$ for a parton of type i to go (inclusively) into one or two hadrons accordingly.

It is easy to obtain the equations for these two-particle fragmentation functions using the equations for functions of parton level

$$\frac{d\bar{D}_i^{h_1 h_2}(x_1,x_2,t)}{dt}=\sum_{i'}\int_{x_1+x_2}^{1}\frac{dx'}{x'^2}\bar{D}_{i'}^{h_1 h_2}\left(\frac{x_1}{x'},\frac{x_2}{x'},t\right)P_{i\to i'}(x')+ \tag{17}$$
$$+\sum_{i_1 i_2}\int_{x_1}^{1-x_2}\frac{dx'}{x'(1-x')}\bar{D}_{i_1}^{h_1}\left(\frac{x_1}{x'},t\right)\bar{D}_{i_2}^{h_2}\left(\frac{x_2}{1-x'},t\right)P_{i\to i_1 i_2}(x').$$

We also used the expression of single-particle fragmentation functions for a parton of type i to fragment into hadron h

$$\bar{D}_i^h(x,t)=\sum_{i'}\int\frac{dx'}{x'}\bar{D}_{i'}^h\left(\frac{x}{x'},0\right)\bar{D}_i^{i'}(x',t). \tag{18}$$

Functions $\bar{D}_i^{h_1 h_2}(x_1,x_2,0)$ introduced phenomenologically define initial conditions for the two-particle fragmentation functions as well as functions $\bar{D}_i^h(x,0)$ define initial conditions for the single-particle fragmentation functions.

Taking moments we obtain

$$\frac{d\bar{M}_i^{h_1 h_2}(n_1,n_2,t)}{dt}=\sum_{i'}\bar{M}_{i'}^{h_1 h_2}(n_1,n_2,t)P_{i\to i'}(n_1+n_2)+ \tag{19}$$
$$+\sum_{i_1 i_2}\bar{M}_{i_1}^{h_1}(n_1,t)\bar{M}_{i_2}^{h_2}(n_2,t)P_{i\to i_1 i_2}(n_1,n_2),$$

where

$$\bar{M}_i^{h_1 h_2}(n_1,n_2,t)=\int_0^1 x_1^{n_1}x_2^{n_2}\theta(1-x_1-x_2)\bar{D}_i^{h_1 h_2}(x_1,x_2,t)dx_1 dx_2, \tag{20}$$
$$\bar{M}_i^h(n,t)=\int_0^1 x^n\bar{D}_i^h(x,t)dx.$$

Again the particular solutions of eq. (19) may be written as

$$\bar{M}_{io}^{h_1 h_2}(n_1,n_2,t)=$$
$$=\sum_j\sum_{j_1 j_2}\int_0^t dt'\bar{M}_i^j(n_1+n_2,t-t')P_{j\to j_1 j_2}(n_1,n_2)\bar{M}_{j_1}^{h_1}(n_1,t')\bar{M}_{j_2}^{h_2}(n_2,t'). \tag{21}$$

However now the fragmentation functions have non-zero initial conditions therefore one must find solutions that satisfy the homogeneous part of eq. (19) and have definite initial conditions to obtain the solutions of eq. (19) with definite initial conditions

$$\bar{M}_i^{h_1 h_2}(n_1,n_2,t)=\bar{M}_{io}^{h_1 h_2}(n_1,n_2,t)+\sum_j\bar{M}_i^j(n_1+n_2,t)\bar{M}_j^{h_1 h_2}(n_1,n_2,0). \tag{22}$$

The eqs. (17), (19) for multiparton fragmentation functions were obtained also in refs.[12,13] but from the jet calculus rules. It was noted[13] that equations obtained in ref.[11] were not applicable for description of parton fragmentation into hadrons though these equations lead to the solutions coinciding with the jet calculus rules. It is clear from the present context that the equations for multiparton distribution functions describe features of ep-scattering and the equations for multiparton fragmentation functions describe features of e^+e^--

annihilation into hadrons. That is why the last equations are easily generalized for description of parton fragmentation into hadrons at standard assumption about hadronization. The equations obtained for multiparton distribution functions may be generalized for description of multiparton distributions in hadrons using the proper convolution of distribution of parton level with t-independent phenomenological distributions of partons in hadrons. For example, the distribution functions of two partons j_1,j_2 in hadron h are defined by convolution

$$D_h^{j_1j_2}(x_1,x_2,t) = \sum_j \int D_h^j(x,0)\,\tilde{D}_j^{j_1j_2}\left(\frac{x_1}{x},\frac{x_2}{x},t\right)\frac{dx}{x^2} + \sum_{j_1'j_2'}\int D_h^{j_1'j_2'}(x_1',x_2',0)\,D_{j_1'}^{j_1}\left(\frac{x_1}{x_1'},t\right)D_{j_2'}^{j_2}\left(\frac{x_2}{x_2'},t\right)\frac{dx_1'}{x_1'}\frac{dx_2'}{x_2'}. \tag{23}$$

Again the equations for these functions are obtained using the equations of parton level. We write only the equations for moments

$$\frac{dM_h^{j_1j_2}(n_1,n_2,t)}{dt} = \sum_{j_1'} M_h^{j_1'j_2}(n_1,n_2,t)P_{j_1'\to j_1}(n_1) + \sum_{j_2'} M_h^{j_1j_2'}(n_1,n_2,t)P_{j_2'\to j_2}(n_2) + \sum_j M_h^j(n_1+n_2,t)P_{j\to j_1j_2}(n_1,n_2). \tag{24}$$

The initial conditions are defined by phenomenological functions as in the case of fragmentation functions. As before the particular solutions of eq. (24) may be written as

$$M_{ho}^{j_1j_2}(n_1,n_2,t) = \sum_{jj_1'j_2'}\int_0^t dt' M_h^j(n_1+n_2,t')P_{j\to j_1'j_2'}(n_1,n_2)\times M_{j_1'}^{j_1}(n_1,t-t')M_{j_2'}^{j_2}(n_2,t-t'). \tag{25}$$

One may find also the solutions that satisfy definite initial conditions introduced phenomenologically

$$M_h^{j_1j_2}(n_1,n_2,t) = M_{ho}^{j_1j_2}(n_1,n_2,t) + \sum_{j_1'j_2'} M_h^{j_1'j_2'}(n_1,n_2,0)M_{j_1'}^{j_1}(n_1,t)M_{j_2'}^{j_2}(n_2,t). \tag{26}$$

In conclusion we emphasize once more that the equations for the multiparton distribution and fragmentation functions have essentially different structure though on the definite initial conditions they lead to the same solutions which coincide with the jet calculus rules. The difference of these equations becomes clear under their generalization for description of parton fragmentation into hadrons.

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Шелест В.П., Снигирев А.М., Зиновьев Г.М. E2-82-57

Уравнения для многопартонных распределений в КХД

Используя партонную интерпретацию диаграмм теории возмущений КХД в главном логарифмическом приближении, мы получаем уравнения для многопартонных функций распределения и фрагментации. Эти уравнения имеют существенно разную структуру, однако, при определенных начальных условиях приводят к одинаковым решениям, совпадающим с правилами исчисления струй. Принципиальное различие этих уравнений проявляется при их обобщении для описания адронных струй.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1982

Shelest V.P., Snigirev A.M., Zinovjev G.M. E2-82-57

The Multiparton Distribution Equations in QCD

The equations for multiparton distribution and fragmentation functions are obtained by using parton interpretation of the leading logarithm diagrams of perturbative QCD theory. These equations have essentially different structute but the solutions are the same on the definite initial conditions and coincide with the jet calculus rules. The difference is crucial when we generalize these equations for hadron jets description.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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