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THE QUADRATIC BUNDLE OF GENERAL FORM AND THE NONLINEAR EVOLUTION EQUATIONS.

EXPANSIONS OVER THE "SQUARED" SOLUTIONS – GENERALIZED FOURIER TRANSFORM

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§1. INTRODUCTION

In the last years the idea to interprete the inverse scattering method (ISM, see refs.[1-4]) as a generalized Fourier transform[5] has been further developed and validified [6-13]. The role of the generalized exponent is played by the "squared" solutions $\{\Psi(x,\lambda)\}$ of the auxiliary linear problem $L(\lambda)\psi(x,\lambda)=0$. Naturally there also appears the operator Λ, for which the elements of $\{\Psi(x,\lambda)\}$ are eigenfunctions.

An important property of $\{\Psi(x,\lambda)\}$, which ensures the applicability of the ISM is its completeness, first formulated in ref.[6] and proved in ref.[8] for the Zakharov-Shabat system $L(\lambda)=i\sigma_3\frac{d}{dx}+Q(x)-\lambda$. The detailed considerations in refs.[7-9, 13] showed, that the set of independent scattering data $\mathcal{T}$ of the problem $L(\lambda)$ and their variations $\delta\mathcal{T}$ appear as coefficients in the expansions of the potential $Q(x)$ and $\sigma_3\,\delta Q(x)$, respectively, over the system $\{\Psi\}$. Starting from these expansions one is able to reproduce in a uniform way most of the important results for the nonlinear evolution equations (NLEE), including: i) the description of the class of NLEE through the operator Λ[5]; ii) their Hamiltonian nature and complete integrability[14]; in proving this it is convenient to make use of the compact expressions for the conservation lows and their variations through the operator Λ[13]; iii) the hierarchies of Hamiltonian structures, generated by the same operator Λ[15]; iv) the explicit calculation of the action-angle variables[7,8,13].

The method of derivation of the completeness relation for the system $\{\Psi(x,\lambda)\}$ propozed in ref.[8] has been applied also for other choices of $L(\lambda)$, see refs. [10-12, 16,17]; the same method is used in the present paper also. For a number of problems $L(\lambda)$ the operators Λ are known explicitly and have been used for the investigation of the corresponding classes of NLEE, see refs. [5, 18-24]. Thus one may conjecture, that: i) the above-mentioned interpretation of the ISM as a Fourier transform is a general one and may be applied to a large class of problems $L(\lambda)$; ii)* for each $L(\lambda)$ one may construct an operator Λ, generating all the important quantities of the NLEE.

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* This has been conjectured earlier in ref. [20].

Let us briefly discuss the Hamiltonian properties of the NLEE; for concreteness let us choose $L(\lambda)$ as a 2x2 matrix polynomial bundle of general form:

$$L(\lambda)\psi(x,\lambda) = [i\sigma_3 \frac{d}{dx} + U(x,t,\lambda) - \lambda^N]\psi(x,\lambda) = 0\,,$$
$$U(x,t,\lambda) = \sum_{k=0}^{N} U_k(x)\lambda^k\,, \qquad \lim_{x\to\pm\infty} U(x,t,\lambda) = 0\,. \tag{1.1}$$

The ISM applied to $L(\lambda)$ (1.1) allows one to solve a whole class of NLEE for the set of potentials $\{U_k\}$*. If they allow Hamiltonian interpretation, then the phase space $\mathfrak{F}$ should be parametrized by the independent elements of the potentials $\{U_k\}$ in $L(\lambda)$. As a Hamiltonian H it is natural to choose an appropriate linear combination of the motion invariants $D^{(m)}$ of the NLEE, which can be constructed by the known methods (see refs. [1-4]) from $L(\lambda)$. Lastly, one should define a symplectic form Ω_0 on $\mathfrak{F}$ such that the Hamiltonian equations of motion defined by (Ω_0, H) coincide with the corresponding NLEE. For a number of important particular choices of $L(\lambda)$ the explicit form of Ω_0 and the global action-angle variables are well known, see the review paper by L.D.Faddeev in ref.[3], p. 339. Using the completeness relation of the system $\{\Psi(x,\lambda)\}$ it is easy to prove the existence of a hierarchy of symplectic forms Ω_m, $m=+1, +2, \ldots$ on $\mathfrak{F}$, pairwize consistent with Ω_0 and between themselves [15] and to construct their mutual Lagrange manifold $\mathfrak{M}$ [10,13,17].

A general approach** for the investigation of the Hamiltonian structure of the NLEE is known, based on the central extension of Lie algebras [25-28]. In our case for $L(\lambda)$ (1.1) the scheme starts by considering the Lie algebra $\mathfrak{A}$ of smooth 2x2 matrix-valued functions $U_k(x)$, vanishing fast enough when $x\to\pm\infty$. Studying an appropriate central extension of the algebra $\hat{\mathfrak{A}} = \mathfrak{A}^+ \oplus \mathfrak{A}^-$ it is possible to write down the Lax representation of the NLEE in explicitly Hamiltonian form. Here the subalgebras $\mathfrak{A}^\pm = \mathfrak{A} \otimes P^\pm(\lambda)$, $P^+(\lambda)$ ($P^-(\lambda)$) being the algebra of polynomials over the non-negative (negative) power of λ. The symplectic structure Ω_0 is given by the Kirrilov-Kostant 2-form. In order that Ω_0 be non-degenerate, one is naturally led to choose as $\mathfrak{F}$, roughly speaking, the orbit of the co-adjoint action in $\mathfrak{A}^+$ with respect to $\mathfrak{A}^-$ (see ref. [26]). This requirements give us the form of $L(\lambda)$

* Here and in what follows we shall omit the dependence of U_k upon the time t.

** Other approaches are presented in refs. [29,30].

up to a gauge transformation [31]. Conveniently fixing the gauge and applying the natural restriction $\mathrm{tr}\,\sigma_3 U(x,t,\lambda) = 0$, the bundle $L(\lambda)$ (1.1) may be cast into:

$$L(\lambda)\psi(x,\lambda) = [i\sigma_3 \frac{d}{dx} + \sum_{k=0}^{N-1} \lambda^k U_k - \lambda^N]\psi(x,\lambda) = 0\,,$$
$$U_k(x) = \begin{pmatrix} r_k\,, & q_k \\ p_k\,, & r_k \end{pmatrix}, \qquad k = 0,1,\ldots,N-1; \quad r_{N-1} = 0. \tag{1-2}$$

In the present paper, applying the scheme of ref.[21] to the polynomial bundle (1.2) we outline the way by which one is able to construct an appropriate system of "squares" $\{\bar{\Psi}(x,\lambda)\}$ and prove its completeness. Thus an attempt is made to bring together the above-mentioned two approaches. The concrete calculations are made for the simplest nontrivial case $N=2$, which exhibits all the peculiarities of the general construction. For $N>2$ the derivation is done analogically by the use of the Green function (3.10), in which $\psi * \phi = \psi \circ \phi \otimes \begin{pmatrix} 1 \\ \lambda \\ \vdots \\ \lambda^{N-1} \end{pmatrix}$ and $A_0 = \begin{pmatrix} 0 & & -i\sigma_2 \\ & \iddots & \\ -i\sigma_2 & & 0 \end{pmatrix}$ see ref.[16]. However the corresponding formulae are very involved and we omit them.

If the operator Λ is not explicitly known, then all the considerations above acquire somewhat abstract character. Λ may be calculated using the fact, that the elements in $\{\Psi(x,\lambda)\}$ are its eigenfunctions, or by solving a certain system of recurrent relations, or as an operator transferring $\delta D^{(m)}$ into $\delta D^{(m+1)}$ and Ω_m into Ω_{m+1}. For the simplest and most important choices of $L(\lambda)$ all these definitions are equivalent and the corresponding Λ-operators are well known. The problem of explicit calculation of the Λ-operator for general polynomial bundles has been considered in ref.[22] and reduced (using the first of the above-mentioned definitions) to the solution of an algebraic equation of power $2N$ with matrix operator-valued coefficients. On these grounds it has been concluded in ref.[22], that there exist $2N$ different operators Λ related to a bundle of the type (1.1). More detailed study of this algebraic equation for $L(\lambda)$ (1.2) based on the scheme of ref.[21] allows one to calculate explicitly Λ as a $2N\times 2N$ - matrix integro-differential operator, see ref.[16]. From the completeness relation for $\{\bar{\Psi}(x,\lambda)\}$ it follows, that relations (3.15) uniquely determine $\Lambda = \Lambda_+$; all "the other Λ-operators" with the same system of eigenfunctions $\{\Psi(x,\lambda)\}$ are functions of Λ_+.

In the next §2 of the present paper we present the necessary facts from the direct and inverse scattering problem for the system:

$$L(\lambda)\psi(x,\lambda) = [i\sigma_3 \frac{d}{dx} + Q_0 + \lambda Q_1 + r_0 - \lambda^2]\psi(x,\lambda) = 0\,,$$

$$Q_i(x) = \begin{pmatrix} 0 & q_i \\ p_i & 0 \end{pmatrix}, \quad i = 0,1, \quad r_0 = -\frac{1}{2} q_1 p_1\,, \tag{1.3}$$

which is obtained from (1.2) with $N=2$ and $r_0 = -\frac{1}{2} q_1 p_1$. This last restriction is not crucial for our considerations; its origin and importance for the NLEE will be discussed in ref./32/. In §3 we prove the completeness relation for the "squared" solution $\{\bar{\Psi}(x,\lambda)\}$ of (1.3) and calculate the operator Λ_+. In the last, fourth paragraph compact expressions for the trace identities (see ref. /1/) of (1.3) through the operator Λ are obtained.

The application of these results to the NLEE are considered in our next paper /32/.

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§2. DIRECT AND INVERSE SCATTERING PROBLEM

Below we give the necessary facts from the direct and inverse scattering problem for the system (1.3). All of them are simple generalizations of the results, contained in refs./32,33/. For simplicity we shall assume, that the potentials $Q_i(x)$ are complex-valued functions of Schwartz type and such, that the discrete spectrum of the system (1.3) consists of a finite number of simple eigenvalues. Under these conditions the Jost solutions, defined by:

$$\lim_{x\to\infty} \psi(x,\lambda) e^{i\lambda^2\sigma_3 x} = 1\,, \quad \lim_{x\to-\infty} \phi(x,\lambda) e^{i\lambda^2\sigma_3 x} = 1\,, \tag{2.1}$$
$$\psi(x,\lambda) = \|\psi^-, \psi^+\|, \quad \phi(x,\lambda) = \|\phi^+, \phi^-\|$$

exist, the columns $\psi^+(x,\lambda)$, $\phi^+(x,\lambda)$, $(\psi^-(x,\lambda), \phi^-(x,\lambda))$ being analytic functions of λ in the regions $\mathrm{Im}\lambda^2 > 0$ $(\mathrm{Im}\lambda < 0)$. The transition matrix is introduced as usual:

$$\phi(x,\lambda) = \psi(x,\lambda) S(\lambda)\,, \quad S(\lambda) = \begin{pmatrix} a^+ & -b^- \\ b^+ & a^- \end{pmatrix}, \tag{2.2}$$
$$\det S(\lambda) = 1\,,$$

$a^+(\lambda)\,(a^-(\lambda))$ also being analytic functions of λ for $\mathrm{Im}\lambda^2 > 0$ $(\mathrm{Im}\lambda^2 < 0)$. The resolvent of the system (1.3) is expressed through the fundamental solutions

$$\chi^+(x,\lambda) = \|\phi^+, \psi^+\|, \quad \chi^-(x,\lambda) = \|\psi^-, \phi^-\| \tag{2.3}$$

as follows:

$$R(x,y,\lambda) = R^{\pm}(x,y,\lambda), \qquad \mathrm{Im}\lambda^2 \gtrless 0\,,$$
$$R^{\pm}(x,y,\lambda) = \pm i\chi^{\pm}(x,\lambda)\,\Theta(\pm(x-y))\,\chi^{\pm}(y,\lambda)^{-1}\sigma_3\,, \tag{2.4}$$
$$\Theta(x) = \mathrm{diag}(\theta(-x), -\theta(x)).$$

Obviously $R(x,y,\lambda)$ is analytic in λ for all $\mathrm{Im}\lambda^2 \neq 0$ except the points, where $\det\chi^{\pm}(x,\lambda) = a^{\pm}(\lambda) = 0$. The supposition that the discrete spectrum Δ is finite and simple means that $a^{\pm}(\lambda)$ have only finite number of simple zeroes:

$$\Delta = \Delta^+ \cup \Delta^-\,, \quad \Delta^{\pm} = \{\lambda_{a\pm},\ \mathrm{Im}\lambda^2_{a\pm} \gtrless 0,\ a^{\pm}(\lambda_{a\pm}) = 0,\ a = 1,\ldots,N\}.$$

The inverse scattering problem for the system (1.3) is readily formulated as Riemann problem for the solutions χ^+, χ^-:

$$\check{\chi}^+(x,\lambda) = \check{\chi}^-(x,\lambda)\,G(x,\lambda)\,, \quad \check{\chi}^{\pm}(x,\lambda) = \chi^{\pm}(x,\lambda)\,e^{i\lambda^2\sigma_3 x}\,,$$
$$G(x,\lambda) = e^{-i\lambda^2\sigma_3 x}\,G_0(\lambda)\,e^{i\lambda^2\sigma_3 x}\,, \quad G_0(\lambda) = \frac{1}{a^+}\begin{pmatrix} 1 & -b^- \\ -b^+ & 1 \end{pmatrix}, \tag{2.5}$$
$$\lim_{x\to\infty} \check{\chi}^+(x,\lambda) = 1\,.$$

with canonical normalization* for $\lambda\to\infty$. For our purposes it is somewhat more convenient to use the following representations** for the Jost solutions ψ^+, ψ^- /32,33/:

$$\check{\psi}^-(x,\lambda) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \sum_{a=1}^{N} \frac{c_a^+ e_{a+}^2 \psi_a^+(x)}{\lambda_{a+} - \lambda} + \frac{1}{2\pi i}\int_\Gamma \frac{d\mu}{\mu - \lambda}\rho^+(\mu)\check{\psi}^+(x,\mu)\,e^{2i\mu^2 x}, \quad \mathrm{Im}\lambda^2 < 0,$$
$$\check{\psi}^+(x,\lambda) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \sum_{a=1} \frac{c_a^- e_{a-}^2 \psi_a^-(x)}{\lambda_{a-} - \lambda} + \frac{1}{2\pi i}\int_\Gamma \frac{d\mu}{\mu - \lambda}\rho^-(\mu)\check{\psi}^-(x,\mu)\,e^{-2i\mu^2 x}, \quad \mathrm{Im}\lambda^2 > 0, \tag{2.6}$$

*The choice $r_0 = -\frac{1}{2} q_1 p_1$ in (1.3) ensures the consistency of the Riemann problem normalization with the asymptotic of the solution $\chi^+(x,\lambda)$ for $\lambda\to\infty$.

**By performing Fourier transformation and introducing appropriate transformation operators one obtains from (2.6) the Gel'fand-Levitan-Marchenko equation for the system (1.3)/32,33/, where $\check{\psi}^{\pm}(x,\lambda) = \psi^{\pm}(x,\lambda)e^{\mp i\lambda^2 x}$, $\check{\psi}^{\pm}_a(x) = \check{\psi}^{\pm}(x,\lambda_{a\pm})$, $e_{a\pm} = \exp(\pm i\lambda^2_{a\pm}x)$ and the contour Γ is given on fig.1. If the set of scattering data

$$\mathcal{T} \equiv \{\rho^{\pm}(\lambda),\ \lambda\in\Gamma,\ c^{\pm}_a,\ \lambda_{a\pm},\ \mathrm{Im}\lambda^2_{a\pm} \gtrless 0,\ a=1,\dots,N\},$$

$$\rho^{\pm}(\lambda) = b^{\pm}/a^{\pm}(\lambda),\quad c^{\pm}_a = b^{\pm}_a/\dot{a}^{\pm}_a,\quad \dot{a}^{\pm}_a = \frac{da^{\pm}}{d\lambda}\Big|_{\lambda=\lambda_{a\pm}}, \tag{2.7}$$

$$b^{\pm}_a:\ \phi^{\pm}_a(x) = b^{\pm}_a\psi^{\pm}_a(x),$$

is given, then from (2.6) one is able to obtain a system of singular integral equations for the functions $\check{\psi}^{\pm}(x,\lambda)$, $\lambda\in\Gamma$ and $\lambda\in\Gamma$.Solving it we can find $\psi^{\pm}(x,\lambda)$ for all $\mathrm{Im}\lambda^2 \gtrless 0$. Then the potentials $Q_i(x)$ are reconstructed from the first few terms in the asymptotics of $\check{\psi}(x,\lambda)$ for $\lambda\to\infty$:

$$\frac{1}{2}\sigma_3\{\sigma_3, \check{\psi}(x,\lambda)\} = 1 - \frac{i}{2\lambda}\sigma_3 u(x) + O(\frac{1}{\lambda^2}),$$

$$u(x) = \int_x^{\infty} dy(q_1p_0 + q_0p_1), \tag{2.8}$$

$$\frac{1}{2}\sigma_3[\sigma_3, \check{\psi}(x,\lambda)] =$$

$$= \frac{1}{2\lambda}Q_1(x) + \frac{1}{2\lambda^2}[Q_0(x) - \frac{i}{2}Q_1(x)\sigma_3 u(x)] + O(\frac{1}{\lambda^3}).$$

In particular, for $\rho^{\pm}(\lambda) = 0$ (2.6) gives us a system of algebraic equations, which is easily solved explicitly. The corresponding solutions lead to the reflectionless potentials, the simplest of which has the form:

$$q_1^{(1s)}(x) = -\frac{2c_1^- e_{1-}}{e_{1+}d(x)},\quad p_1^{(1s)} = \frac{2e_{1+}c_1^+}{e_{1-}d(x)},\quad d(x) = \frac{1}{e_{1+}e_{1-}} - \omega_1 e_{1+}e_{1-},$$

$$q_0^{(1s)}(x) = \frac{2c_1^- e_{1-}}{e_{1+}d^2(x)}[\lambda_{1+}\omega_1 e_{1+}e_{1-} - \frac{\lambda_{1-}}{e_{1+}e_{1-}}],\quad \omega_1 = c_{1+}c_{1-}(\lambda_{1+}-\lambda_{1-})^{-2},$$

$$p_0^{(1s)}(x) = \frac{2c^+_{1+}e_{1+}}{e_{1-}d^2(x)}[\frac{\lambda_{1+}}{e_{1+}e_{1-}} - \lambda_{1-}\omega_1 e_{1+}e_{1-}],\quad e_{1\pm} = \exp(\pm i\lambda^2_{1\pm}x).$$

The transition matrix $S(\lambda)$ is reconstructed from the set $\mathcal{T}$ by the use of the dispersion relation:

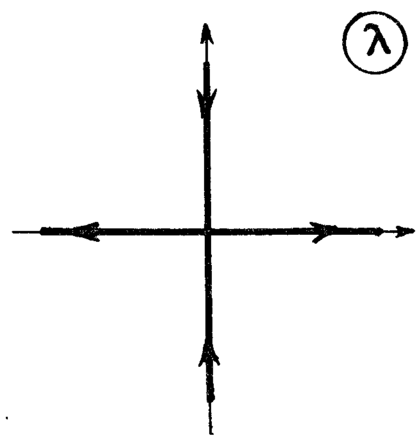

Fig.1. The contour Γ.

$$D(\lambda) = \frac{i}{2\pi}\int_{\Gamma}\frac{d\mu}{\mu-\lambda}\ln[1+\rho^+\rho^-(\mu)] + \sum_{a=1}^{N}\ln\frac{\lambda-\lambda_{a+}}{\lambda-\lambda_{a-}}, \tag{2.9}$$

$$D(\lambda) = \ln a^+(\lambda),\quad \mathrm{Im}\lambda^2 > 0,$$

$$D(\lambda) = -\ln a^-(\lambda),\quad \mathrm{Im}\lambda^2 < 0.$$

The set, $\mathcal{T}$ corresponding to the simplest reflectionless potential is given by $\{\rho^{\pm}(\lambda) = 0,\ c_1^+, c_1^-, \lambda_{1+}, \lambda_{1-}\}$, and the transition matrix equals to $S(\lambda) = \mathrm{diag}(\frac{\lambda-\lambda_{1+}}{\lambda-\lambda_{1-}}, \frac{\lambda-\lambda_{1-}}{\lambda-\lambda_{1+}})$.

§3. THE INTERRELATION BETWEEN THE SCATTERING DATA AND THE POTENTIAL - GENERALIZED FOURIER TRANSFORM

Let us investigate in greater detail the interrelation between the set of potentials $\{Q_i(x)\}$ in (1.3) and the set of scattering data $\mathcal{T}$ (2.7). For this we start from the relations:

$$A^{\pm}(\lambda) = i(\chi^{\pm})^{-1}\sigma_3\chi^{\pm}(x,\lambda)\Big|_{x=-\infty}^{\infty} =$$

$$= -2\int_{-\infty}^{\infty} dx(\chi^{\pm})^{-1}(Q_0 + \lambda Q_1)\chi^{\pm}(x,\lambda),\qquad \mathrm{Im}\lambda^2 \gtrless 0. \tag{3.1}$$

Using (2.1)-(2.3), the l.h.s. of (3.1) is easily expressed by the transition matrix, and hence - by the scattering data $\mathcal{T}$. The matrix elements in the r.h.s. of (3.1) are rewritten conveniently by using the following skew-scalar product in the space $X, Y \in \mathcal{S}(\mathbf{C}^4)$:

$$[X, Y] = \int_{-\infty}^{\infty} dx X^T(x)A_0 Y(x),$$

$$A_0 = \begin{pmatrix} 0 & -i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix},\quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{3.2}$$

in the form:

$$A_{12}(\lambda) = \frac{2}{a^+}[\bar{w}, \Psi^+], \quad A^+_{21}(\lambda) = -\frac{2}{a^+}[\bar{w}, \Phi^+], \quad A^-_{12}(\lambda) = \frac{2}{a^-}[\bar{w}, \Phi^-],$$

$$A_{21}(\lambda) = -\frac{2}{a^-}[\bar{w}, \Psi^-], \quad A^{\pm}_{11}(\lambda) = -A^{\pm}_{22}(\lambda) = \frac{2}{a^{\pm}}[\bar{w}, \phi^{\pm} * \psi^{\pm}]. \tag{3.3}$$

Here we have used the following notations:

$$\bar{w} = \begin{pmatrix} w_1 \\ w_0 \end{pmatrix}, \quad w_i = \begin{pmatrix} q_i \\ p_i \end{pmatrix}, \quad i = 0,1, \quad \Psi^{\pm}(x,\lambda) = \psi^{\pm} * \psi^{\pm}(x, \lambda),$$

$$\Phi^{\pm}(x, \lambda) = \phi^{\pm} * \phi^{\pm}(x, \lambda), \quad \phi * \psi = \begin{matrix} \phi \circ \psi \\ \lambda \phi \circ \psi \end{matrix} \equiv \phi \circ \psi \otimes \binom{1}{\lambda}, \phi \circ \psi = \begin{pmatrix} \phi_1 \psi_1 \\ \phi_2 \psi_2 \end{pmatrix}. \tag{3.4}$$

The quantities (3.3) can be considered as Fourier coefficients of the potential $\bar{w}$ with respect to the system of "squared" solutions $\Psi^{\pm}(x,\lambda)$ or $\Phi^{\pm}(x, \lambda)$. When investigating the NLEE and their Hamiltonian structure it would be important to find analogical relations between the variations of the potentials δQ_i and the corresponding variations of $\delta \mathcal{T}$. Such relations follow from:

$$B^{\pm}(\lambda) = i(\chi^{\pm})^{-1} \delta \chi^{\pm}(x, \lambda)\Big|_{x=-\infty}^{\infty} =$$

$$= -\int_{-\infty}^{\infty} dx (\chi^{\pm})^{-1} \sigma_3 (\delta r_0 + \delta Q_0 + \lambda \delta Q_1) \chi^{\pm}(x, \lambda), \qquad \mathrm{Im}\lambda^2 \gtrless 0. \tag{3.5}$$

Using (2.1)-(2.3) $B^{\pm}(\lambda)$ is easily expressed through the variations of the scattering data. The r.h.s. of (3.5), after some algebra, is cast into the form:

$$B^+_{12}(\lambda) = -\frac{1}{a^+}[\bar{\sigma}_3 \delta \bar{w}, N_+^{-1} \Psi^+], \quad B^+_{21}(\lambda) = \frac{1}{a^+}[\bar{\sigma}_3 \delta \bar{w}, N_-^{-1} \Phi^+],$$

$$B^-_{12}(\lambda) = -\frac{1}{a^-}[\bar{\sigma}_3 \delta \bar{w}, N_-^{-1} \Phi^-], \quad B^-_{21}(\lambda) = \frac{1}{a^-}[\bar{\sigma}_3 \delta \bar{w}, N_+^{-1} \Psi^-], \tag{3.6}$$

where $\bar{\sigma}_3 = \mathrm{diag}(\sigma_3, \sigma_3)$ and the integro-differential operators $N_{\pm}^{-1}$ have the form:

$$N_{\pm}^{-1} = \begin{pmatrix} 1, & 0 \\ Z^{\pm}_{10}, & 1 + Z^{\pm}_{11} \end{pmatrix} \qquad \begin{aligned} Z^{\pm}_{ik} &= -i w_i \int_x^{\pm\infty} dy\, \tilde{w}_k(y), \\ \tilde{w}_k &= w_k^T(-i\sigma_2), \quad i,k = 0,1. \end{aligned} \tag{3.7}$$

Taking into account the fact, that $[\bar{w}, N_{\pm}^{-1} X] = [\bar{w}, X]$ we choose more convenient "squares" of the from $\bar{\Psi}^{\pm}(x, \lambda) = N_+^{-1}\Psi^{\pm}(x,\lambda)$ and $\bar{\Phi}^{\pm}(x, \lambda) = N_-^{-1}\Phi^{\pm}(x, \lambda)$.

The completeness relation for the systems of vector-functions

$$\{\bar{\Psi}\} \equiv \{\bar{\Psi}^{\pm}(x,\lambda), \quad \lambda \in \Gamma, \quad \bar{\Psi}^{\pm}_{a}(x), \quad \dot{\bar{\Psi}}^{\pm}_{a}(x), \quad a = 1,\dots,N\},$$

$$\{\bar{\Phi}\} \equiv \{\bar{\Phi}^{\pm}(x,\lambda), \quad \lambda \in \Gamma, \quad \bar{\Phi}^{\pm}_{a}(x), \quad \dot{\bar{\Phi}}^{\pm}_{a}(x), \quad a = 1,\dots,N\},$$

$$\bar{\Psi}^{\pm}(x, \lambda) = N_+^{-1}\Psi^{\pm}(x, \lambda), \quad \bar{\Phi}^{\pm}(x, \lambda) = N_-^{-1}\Phi^{\pm}(x, \lambda), \tag{3.8}$$

$$\dot{F}^{\pm}(x, \lambda) = \frac{d}{d\lambda} F^{\pm}(x, \lambda)\Big|_{\lambda = \lambda_{a\pm}},$$

has the form:

$$\delta(x-y) = -\frac{1}{\pi}\int_{\Gamma} d\lambda \left[\frac{\bar{\Psi}^+(x,\lambda)\bar{\Phi}^{+T}(y,\lambda)}{(a^+(\lambda))^2} - \frac{\bar{\Psi}^-(x,\lambda)\bar{\Phi}^{-T}(y,\lambda)}{(a^-(\lambda))^2}\right] A_0 + \sum_{a=1}^{N}(X^+_a + X^-_a)(x,y),$$

$$X^{\pm}_a(x,y) = \frac{2i}{(\dot{a}^{\pm}_a)^2}\left[\bar{\Psi}^{\pm}_a(x)\dot{\bar{\Phi}}^{\pm T}_a(y) + \dot{\bar{\Psi}}^{\pm}_a(x)\bar{\Phi}^{\pm T}_a(y) - \frac{\ddot{a}^{\pm}_a}{\dot{a}^{\pm}_a}\bar{\Psi}^{\pm}_a(x)\bar{\Phi}^{\pm T}_a(y)\right] A_0. \tag{3.9}$$

In deriving (3.9) we have applied the contour integration method to the integral

$$\frac{1}{2\pi i}\oint_{\gamma_1 \cup \gamma_3} d\lambda\, G^+(x, y, \lambda) - \frac{1}{2\pi i}\oint_{\gamma_2 \cup \gamma_4} d\lambda\, G^-(x, y, \lambda),$$

where the contours γ_i, $i = 1,\dots,4$ are given on fig.2, and the functions $G^{\pm}(x, y, \lambda)$ for $\mathrm{Im}\lambda^2 \gtrless 0$ are equal to:

$$G^{\pm}(x, y, \lambda) = \frac{2i}{(a^{\pm}(\lambda))^2}\{\Psi^{\pm}(x, \lambda)\Phi^{\pm T}(y, \lambda)\theta(x-y) +$$

$$+ [2(\phi^{\pm} * \psi^{\pm})(x, \lambda)(\phi^{\pm} * \psi^{\pm})^T(y, \lambda) - \Phi^{\pm}(x,\lambda)\Psi^{\pm T}(y,\lambda)]\theta(y-x)\} A_0. \tag{3.10}$$

Thus one obtains the completeness relation for the systems $\{\Psi\}$ and $\{\Phi\}$, which differs from (3.9) by: i) in the l.h.s. one gets $\Lambda_1^{\pm}\delta(x-y)$, where

$$\Lambda_1^{\pm} = M_{\pm}N_{\pm} = N_{\pm}M_{\pm}, \quad M_{\pm} = \begin{pmatrix} 1 - Z^{\pm}_{11}, & 0 \\ -Z^{\pm}_{01}, & 1 \end{pmatrix} \tag{3.11}$$

and ii) in the r.h.s. everywhere $\bar{\Psi}^{\pm}(x, \lambda)$ and $\bar{\Phi}^{\pm}(y,\lambda)$ should be replaced by $\Psi^{\pm}(x, \lambda)$ and $\Phi^{\pm}(y, \lambda)$, respectively. In order to obtain (3.9) one should use (3.11) and the relations:

$$[X, M_\pm Y] = [N_\mp X, Y], \qquad X, Y \in \mathcal{S}(\mathbf{C}^4), \tag{3.12}$$

which follow from (3.2), (3.7) and (3.11) with integration by parts.

Let us write down also the symplectic form of the completeness relation:

$$\delta(x-y) = \int_\Gamma d\lambda\, [Q(x,\lambda) P^T(y,\lambda) - P(x,\lambda) Q^T(y,\lambda)] A_0 +$$
$$+ \sum_{a=1}^N [Q_a^+(x) P_a^{+T}(y) - P_a^+(x) Q_a^{+T}(y) + Q_a^-(x) P_a^{-T}(y) - P_a^-(x) Q_a^{-T}(y)] A_0, \tag{3.13}$$

where $P(x,\lambda)$ and $Q(x,\lambda)$ are given by:

$$P(x,\lambda) = \frac{1}{\pi}(\rho^+ \bar{\Psi}^+ + \rho^- \bar{\Psi}^-)(x,\lambda) = \frac{1}{\pi}(\sigma^+ \bar{\Phi}^+ + \sigma^- \bar{\Phi}^-)(x,\lambda),$$
$$Q(x,\lambda) = \frac{1}{2b^+ b^-}(\sigma^+ \bar{\Phi}^+ - \rho^+ \bar{\Psi}^+)(x,\lambda) = \frac{1}{2b^+ b^-}(\rho^- \bar{\Psi}^- - \sigma^- \bar{\Phi}^-)(x,\lambda),$$
$$P_a^\pm(x) = \mp 2i c_a^\pm \bar{\Psi}_a^\pm(x), \quad Q_a^\pm(x) = \mp \frac{1}{2}[c_a^\pm \dot{\bar{\Psi}}_a^\pm(x) - d_a^\pm \dot{\bar{\Phi}}_a^\pm(x)], \tag{3.14}$$
$$\sigma^\pm(\lambda) = \frac{b^\mp}{a^\pm(\lambda)}, \quad \lambda \in \Gamma; \quad d_a^\pm = (b_a^\pm \dot{a}_a^\pm)^{-1}.$$

It can be checked, that the systems $\{\bar{\Psi}\}$ and $\{\bar{\Phi}\}$ are eigen- and adjoint-functions

$$(\Lambda_+ - \lambda)\bar{\Psi}^\pm(x,\lambda) = 0, \quad \lambda \in \Gamma \cup \Delta; \quad (\Lambda_+ - \lambda_{a\pm})\dot{\bar{\Psi}}_a^\pm(x) = \bar{\Psi}_a^\pm(x),$$
$$(\Lambda_- - \lambda)\bar{\Phi}^\pm(x,\lambda) = 0, \quad \lambda \in \Gamma \cup \Delta; \quad (\Lambda_- - \lambda_{a\pm})\dot{\bar{\Phi}}_a^\pm(x) = \bar{\Phi}_a^\pm(x) \tag{3.15}$$

of the following integro-differential operators Λ_+ and Λ_-:

$$\Lambda_\pm = \begin{pmatrix} -Z_{10}^\pm, & 1 - Z_{11}^\pm \\ \hat{D} - Z_{00}^\pm, & -Z_{01}^\pm \end{pmatrix} \qquad \hat{D} = \frac{i}{2}\sigma_3 \frac{d}{dx} + r_0(x). \tag{3.16}$$

As a domain of definition of the operators $\Lambda_\pm$ we shall consider the space of complex-valued vector-functions of Schwartz type $\mathcal{S}(\mathbf{C}^4)$. Obviously if $X \in \mathcal{S}(\mathbf{C}^4)$, then $\Lambda_\pm X \in \mathcal{S}(\mathbf{C}^4)$ also. The operators Λ_+ and Λ_- satisfy conjugation-like relations with respect to the skew-scalar product [,] (3.2) in $\mathcal{S}(\mathbf{C}^4)$:

$$[X, \Lambda_+ Y] = [\Lambda_- X, Y]. \tag{3.17}$$

(3.17) is derived like (3.12) with integration by parts.

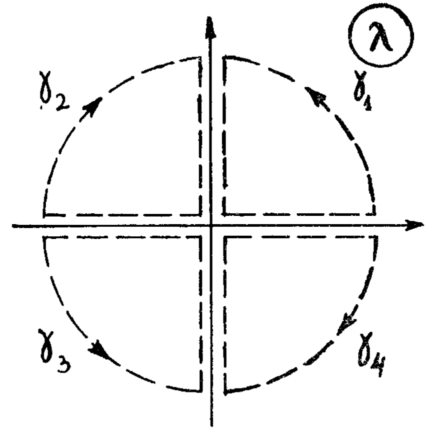

Fig.2. The contours γ_i, i =1,...,4.

Using (3.9) and (3.13) one is able to expand the potential $\bar{w}$ and its variation $\bar{\sigma}_3\delta\bar{w}$ over the systems $\{\bar{\Psi}\}$ and $\{P, Q\}$ (3.14). The expansion coefficients are explicitly calculated in terms of the scattering data $\mathfrak{T}$ (2.7) and their variations by use of (3.3) and (3.6). Thus one obtains:

$$\bar{w}(x) = \frac{i}{\pi}\int_\Gamma d\lambda\, (\rho^+ \Psi^+ + \rho^- \Psi^-)(x,\lambda) + 2\sum_{a=1}^N (c_a^+ \Psi_a^+(x) - c_a^- \Psi_a^-(x)) =$$
$$= i\int_\Gamma d\lambda\, P(x,\lambda) + i\sum_{a=1}^N (P_a^+(x) + P_a^-(x)) \tag{3.18}$$

and

$$\bar{\sigma}_3 \delta\bar{w}(x) = -\frac{i}{\pi}\int_\Gamma d\lambda\, (\delta\rho^+ \bar{\Psi}^+ - \delta\rho^- \bar{\Psi}^-)(x,\lambda) - 2\sum_{a=1}^N (\bar{U}_a^+(x) + \bar{U}_a^-(x)) = \tag{3.19}$$
$$= \int_\Gamma d\lambda\, [Q(x,\lambda)\delta\hat{p}(\lambda) - P(x,\lambda)\delta\hat{q}(\lambda)] + \sum_{a=1}^N (\bar{V}_a^+(x) + \bar{V}_a^-(x)),$$
$$\bar{U}_a^\pm(x) = \delta c_a^\pm \bar{\Psi}_a^\pm(x) + c_a^\pm \delta\lambda_{a\pm} \dot{\bar{\Psi}}_a^\pm(x),$$
$$\bar{V}_a^\pm(x) = Q_a^\pm(x)\delta\hat{p}_a^\pm - P_a^\pm(x)\delta\hat{q}_a^\pm.$$

In (3.19) we have introduced the notations:

$$\hat{p}(\lambda) = \frac{i}{\pi}\ln[1 + \rho^+\rho^-(\lambda)], \qquad \hat{q}(\lambda) = \frac{i}{2}\ln(b^+(\lambda)/b^-(\lambda)), \quad \lambda \in \Gamma,$$
$$\hat{p}_a^\pm = \pm 2\lambda_{a\pm}, \qquad \hat{q}_a^\pm = \pm i \ln b_a^\pm. \tag{3.20}$$

Thus from (3.18) it becomes obvious, that the minimal set of scattering data $\mathfrak{T}$ (2.7) may be interpreted as Fourier expansion coefficients of $\bar{w}(x)$ over the system $\{\Psi\}$. Analogically one can expand $\bar{w}(x)$ and $\sigma_3\delta\bar{w}(x)$ over the system $\{\Phi\}$; the corresponding set of expansion coefficients consists of:

$$\tilde{\mathcal{T}} \equiv \{\sigma^{\pm}(\lambda),\ \lambda\in\Gamma,\ d_a^{\pm},\ \lambda_{a\pm},\ a=1,\dots,N\}, \qquad (3.21)$$

where the notations have been introduced in (3.14).

Making use of the dispersion relations (2.9) one readily verifies that the sets $\mathcal{T}, \tilde{\mathcal{T}}$ and $\{\hat{p}, \hat{q}\}$ (3.20) are mutually equivalent. As we have already noted in §2, they uniquely reproduce both the transition matrix $S(\lambda)$ and the potentials $Q_i(x)$.

§4. TRACE IDENTITIES

The trace identities [1] have been widely used in the literature to construct the conserved quantities of the NLEE, see refs. [1-5]. For polynomial bundles of general form, and also for rational bundles with finite rank divisors the recurrent formulae for calculating the conservation laws have been given in refs. [22,35].

In this last paragraph we shall derive compact formulae, expressing the regularized functional determinant of (1.3) through the operator Λ_+. Let us start by showing that:

$$\eta(\lambda)\frac{d}{d\lambda}\ln\mathrm{Det}[L(\lambda)L_0^{-1}(\lambda)] = \frac{d}{d\lambda}D(\lambda), \qquad (4.1)$$

$$\eta(\lambda) = \begin{cases} 1, & \mathrm{Im}\lambda^2 > 0, \\ -1, & \mathrm{Im}\lambda^2 < 0, \end{cases}$$

where $D(\lambda)$ is introduced in (2.9), and $L_0(\lambda)$ is the operator (1.3) with $Q_0 = Q_1 = 0$. To do this we represent the r.h.s. of (4.1) in the form:

$$\frac{d}{d\lambda}D(\lambda) = \frac{1}{2}\{\mathrm{tr}[(\chi^{\pm})^{-1}\dot{\chi}^{\pm}(x,\lambda)\sigma_3] + 4i\lambda x\}\Big|_{x=-\infty}^{\infty} =$$

$$= \int_{-\infty}^{\infty} dx\{\mathrm{tr}[\frac{i}{2}(\chi^{\pm})^{-1}\sigma_3(Q_1-2\lambda)\chi^{\pm}(x,\lambda)\sigma_3] + 2i\lambda\} \qquad (4.2)$$

by making use of (1.1), (2.1) and (2.2). For the l.h.s. of (4.1) we have

$$\eta(\lambda)\frac{d}{d\lambda}\ln\mathrm{Det}[L(\lambda)L_0^{-1}(\lambda)] = \eta(\lambda)\frac{d}{d\lambda}\mathrm{Tr}\ln[L(\lambda)L_0^{-1}(\lambda)] = \qquad (4.3)$$

$$= \int_{-\infty}^{\infty} dx\{\mathrm{tr}[\eta(\lambda)R(x,x,\lambda)(Q_1(x)-2\lambda)] + 2i\lambda\}.$$

Incerting in (4.3) the explicit formulae for

$$R(x,x,\lambda) = \frac{1}{2}[R(x,x+0,\lambda) + R(x+0,x,\lambda)]$$

from (2.4) it is easy to check that the last lines in (4.3) and (4.2) coincide. Thus (4.1) is proved.

Now by use of (1.1) and (2.3), we rewrite (4.2) in the form:

$$\frac{dD(\lambda)}{d\lambda} = 4\lambda\int_{-\infty}^{\infty}dx\int_x^{\infty}dy\,\bar{w}^T(y)A_0E(y,\lambda) - i\int_{-\infty}^{\infty}dx(\widetilde{\sigma_3 w_1(x)},0)E(x,\lambda), \qquad (4.4)$$

$$E(x,\lambda) = E^{\pm}(x,\lambda),\quad \mathrm{Im}\lambda^2 \gtrless 0,\quad E^{\pm}(x,\lambda) = \frac{\phi^{\pm} * \phi^{\pm}}{a^{\pm}}(x,\lambda).$$

Applying the contour integration method to the integral

$$\frac{1}{2\pi i}\oint_{\gamma_1\cup\gamma_3}\frac{d\mu}{\mu-\lambda}E^+(x,\mu) - \frac{1}{2\pi i}\oint_{\gamma_2\cup\gamma_4}\frac{d\mu}{\mu-\lambda}E^-(x,\mu),\quad \mathrm{Im}\lambda^2\neq 0,\ \lambda\,\bar{\in}\,\Gamma\ \Delta,$$

one obtains:

$$E(x,\lambda) = -\frac{i}{2\pi}\int_{\Gamma}\frac{d\mu}{\mu-\lambda}(\rho^+\Psi^+ + \rho^-\Psi^-)(x,\mu) -$$

$$-\sum_{a=1}^{N}\left[\frac{c_a^+\Psi_a^+(x)}{\lambda_{a+}-\lambda} - \frac{c_a^-\Psi_a^-(x)}{\lambda_{a-}-\lambda}\right] + \frac{1}{2}\begin{pmatrix}0\\ w_1\end{pmatrix}(x) = \qquad (4.5)$$

$$= N_+[-\frac{1}{2}(\Lambda_+-\lambda)^{-1}\bar{w}(x) + \frac{1}{2}\begin{pmatrix}0\\ w_1\end{pmatrix}(x)].$$

The last line in (4.5) follows from (3.18) and (3.15). Incerting (4.5) into (4.4) one immidiately obtains:

$$\frac{dD(\lambda)}{d\lambda} = -2\lambda\int_{-\infty}^{\infty}dx\int_x^{\infty}dy\,\bar{w}^T(y)A_0(\Lambda_+-\lambda)^{-1}\bar{w}(y) +$$

$$+\frac{i}{2}\int_{-\infty}^{\infty}dx((\sigma_3 w_1(x))^T, 0)A_0(\Lambda_+-\lambda)^{-1}\bar{w}(x). \qquad (4.6)$$

From the definition (2.8) there follows, that $D(\lambda)$ is an analytic function of λ, and therefore it has an asymptotic expansion in the neignbourhood of $\lambda\to\infty$: $D(\lambda) = \sum_{m=1}^{\infty}\lambda^{-m}D^{(m)}$. Expanding the r.h.s. of (4.6) over the inverse power of λ we get:

$$D^{(m)} = -\frac{2}{m}\int_{-\infty}^{\infty}dx\int_x^{\infty}dy\,\bar{w}^T(y)A_0\Lambda_+^{m+1}\bar{w}(y) + \qquad (4.7)$$

$$+\frac{i}{2m}\int_{-\infty}^{\infty}dx((\sigma_3 w_1)^T, 0)A_0\Lambda_+^m\bar{w}(x),$$

i.e., $D^{(m)}$ is expressed as a functional of the potentials of (1.3). Analogically from (2.8)

$$D^{(m)} = -\frac{i}{2\pi}\int_{-\infty}^{\infty} d\mu\,\mu^{m-1}\ln[1+\rho^{+}\rho^{-}(\mu)] - \frac{1}{m}\sum_{a=1}^{N}(\lambda_{a+}^{m} - \lambda_{a-}^{m}), \qquad (4.8)$$

$D^{(m)}$ is expressed as a functional of the scattering data $\mathcal{T}$. Equating the r.h.sides of (4.7) and (4.8) we obtain the so-called trace identities for (1.3).

In our next paper [32] we shall also need analogicall expressions for the variations $\delta D^{(m)}$. Their derivation is based on the relations [14]:

$$\delta D(\lambda) = \frac{1}{2}\mathrm{tr}[(\chi^{\pm})^{-1}\,\delta\chi^{\pm}(x,\lambda)\sigma_3\,]\Big|_{x=-\infty}^{\infty} =$$

$$= \frac{i}{2}\int_{-\infty}^{\infty} dx\,\mathrm{tr}[(\chi^{\pm})^{-1}\sigma_3(\delta Q + \delta r_0)\chi^{\pm}(x,\lambda)\sigma_3], \qquad (4.9)$$

which is obtained from (1.3) and (2.1)-(2.3). We conveniently rewrite (4.9) in the form:

$$\delta D(\lambda) = -\frac{i}{2}\int_{-\infty}^{\infty} dx\,\delta(q_1p_1) - i[\bar{\sigma}_3\,\delta\bar{w},\ N_{+}^{-1}E^{\pm}(x,\lambda)]$$

and incerting $E^{\pm}(x,\lambda)$ from (4.5) find:

$$\delta D(\lambda) = \frac{i}{2}[\bar{\sigma}_3\,\delta\bar{w},\ (\Lambda_{+}-\lambda)^{-1}\,\bar{w}], \qquad (4.10)$$

i.e.,

$$\delta D^{(m)} = -\frac{i}{2}[\bar{\sigma}_3\,\delta\bar{w},\ \Lambda_{+}^{m-1}\ \bar{w}], \qquad (4.11)$$

where the skew-scalar product [,] is given in (3.2).

Thus we have obtained compact expressions for $D^{(m)}$ (4.7) and $\delta D^{(m)}$ (4.11), which are convenient in the derivation of the Hamiltonian structures of the NLEE, see ref. [32].

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Гerджиков В.С., Иванов М.И. E2-82-545

Квадратичный пучок общего вида и нелинейные эволюционные уравнения. Разложения по "квадратам" решений - обобщенные преобразования Фурье

Доказано соотношение полноты для системы "квадратов" решений $\{\Psi(x,\lambda)\}$ квадратичного пучка общего вида $L(\lambda)$. Явно вычислен интегродифференциальный оператор Λ_+, для которого элементы $\{\Psi(x,\lambda)\}$ являются собственными и присоединенными функциями. Показано, что отображение множества потенциалов $L(\lambda)$ на множество данных рассеяния имеет смысл преобразования Фурье по системе $\{\Psi(x,\lambda)\}$. Получены компактные выражения для формул следов через оператор Λ_+.

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Gerdjikov V.S., Ivanov M.I. E2-82-545

The Quadratic Bundle of General Form and the Nonlinear Evolution Equations. Expansions over the "Squared" Solutions - Generalized Fourier Transform

The completeness relation for the system of "squared" solutions $\{\Psi(x,\lambda)\}$ of the quadratic bundle $L(\lambda)$ of general form is proved. The integro-differential operator Λ_+, for which the elements of $\{\Psi(x,\lambda)\}$ are eigen and adjoint-functions is explicitly calculated. The interrelation between the set of potentials of $L(\lambda)$ and the set of scattering data is shown to have the meaning of a generalized Fourier transform over the system $\{\Psi(x,\lambda)\}$. Compact expressions for the trace identities of $L(\lambda)$ through the operator Λ_+ are obtained,

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Препринт Объединенного института ядерных исследований. Дубна 1982