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THE GLASHOW-WEINBERG-SALAM MODEL WITHOUT SCALAR FIELDS

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1. INTRODUCTION

The Higgs effect is the relativistic (and sometimes non-Abelian) version of the Meissner effect (dynamical gauge invariant generation of the photon mass inside the superconductor) described within the phenomenological Ginzburg-Landau theory of superconductivity[1]. If the scalar fields of the GWS model, which seem unlikely at present[2] are to be compared with the order parameter of the Ginzburg-Landau theory, then what are the primary objects, i.e., "electrons" of this phenomenological "superconducting" medium? It is natural to identify them with leptons and quarks.

If we take analogy with superconductivity seriously, then the correct procedure is to ask a physical question, such as what is the primary force which makes the fermion vacuum unstable with respect to the formation of Cooper-like fermion-antifermion pairs. We suggest to introduce for this reason the massive Abelian vector field C as an analog of the phonon field*. Our basic view is then the following. This Abelian theory, being not asymptotically free, becomes the strong coupling theory at small distances, $\Lambda \sim$ TeV. If the interaction is attractive, the non-perturbative formation of the fermion-antifermion condensate is quite plausible[3]. As a consequence, the symmetry $SU(2)_L \times U(1)_Y$ is broken by the fermion mass terms to $U(1)_{em}$. In the absence of standard electroweak interactions (both QCD and the electroweak interactions can be treated perturbatively at the considered momenta) three Goldstone bosons should appear as physical particles. The inclusion of the electroweak interactions in the perturbative way eliminates the "would be" Goldstone bosons in accordance with the general Schwinger mechanism[4]. Unfortunately, it is beyond our ability to solve the strong coupling Abelian theory, so we carry out the analysis phenomenologically in the spirit of the works of Nambu and Jona-Lasinio[5] and Freundlich and Lurie[6].

The paper is organized as follows. In Sec.2 we solve the problem of the dynamical mass generation for one family. In Sec.3 we discuss the conditions imposed on the model by the requirement of the cancellation of the triangular anomalies. The mechanism of the dynamical mass generation is described for the case of

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* For brevity we call the corresponding hypercharge heaviness.

three families including the fermion mixing. Section 4 is devoted to the discussion of the physical implications, as well as of the limitations, of the present approach.

2. ONE FAMILY

The Lagrangian density, we suggest to treat instead of the standard GWS one with the cannonical Higgs doublet, has in the case of one family the following form:

$$\begin{aligned}\mathcal{L} = {}& \bar\psi_L i\gamma^\alpha(\partial_\alpha - ig\tfrac{1}{2}\vec\tau\vec A_\alpha + ig'\tfrac{1}{2}B_\alpha - ih\tfrac{1}{2}Y_H C_\alpha)\psi_L + \\ &+ \bar\nu_R i\gamma^\alpha(\partial_\alpha - ih\tfrac{1}{2}Y_H C_\alpha)\nu_R + \bar e_R i\gamma^\alpha(\partial_\alpha + ig' B_\alpha - ih\tfrac{1}{2}Y_H C_\alpha)e_R + \\ &+ \bar q_L i\gamma^\alpha(\partial_\alpha - ig\tfrac{1}{2}\vec\tau\vec A_\alpha - ig'\tfrac{1}{2}B_\alpha - ih\tfrac{1}{2}Y_H C_\alpha)q_L + \\ &+ \bar u_R i\gamma^\alpha(\partial_\alpha - ig'\tfrac{2}{3}B_\alpha - ih\tfrac{1}{2}Y_H C_\alpha)u_R + \bar d_R i\gamma^\alpha)(\partial_\alpha + ig'\tfrac{1}{3}B_\alpha - ih\tfrac{1}{2}Y_H C_\alpha)d_R \\ &- \tfrac{1}{4}(\partial_\alpha\vec A_\beta - \partial_\beta\vec A_\alpha + g\vec A_\alpha\times A_\beta)^2 - \tfrac{1}{4}(\partial_\alpha B_\beta - \partial_\beta B_\alpha)^2 - \tfrac{1}{4}(\partial_\alpha C_\beta - \partial_\beta C_\alpha)^2 + \tfrac{1}{2}M^2 C_\alpha C^\alpha .\end{aligned} \qquad (2.1)$$

The model (2.1) is clearly $SU(2)_L\times U(1)_Y$ gauge invariant. It is also renormalizable /7/ (off mass shell). The renormalizability is not spoiled by the massive Abelian vector field coupled to the conserved current /8/ provided /9/ the Adler-Bell-Jackiw (ABJ) anomalies are cancelled. Since the $SU(2)_L\times U(1)_Y$ quantum numbers are assigned to fermions in a standard manner, the usual GWS model follows for very large M provided the particles do get dynamically proper masses. Quarks are assumed to be fractionally charged and colored and interact via colored vector gluons. Both the color of the quarks and gluon vertices are considered when treating the ABJ anomalies.

For the momenta squared $\ll M^2$ our system is governed by the effective Lagrangian density*

$$\mathcal{L}_{eff} = \mathcal{L}_{GWS} - \frac{h^2}{M^2} J^C_\alpha J^{C,\alpha} , \qquad (2.2)$$

* The Lagrangian density (2.2) is to be compared with the Gorkow microscopic Lagrangian density of superconductivity. The effects of the "phonon" field C are replaced by the contact four-fermion interaction and the fields $\vec A$ and B will be treated as weak external perturbations. See ref. /10/, Chapter 13.

where J^C_α is the current of heaviness. Although from the naively perturbative point of view the Lagrangian densities (2.1) or (2.2) describe the $SU(2)_L\times U(1)_Y$ gauge invariant interactions of massless fermions and massless vactor bosons $\vec A_\alpha$ and B_α, we know /4,5,6,11,12/ that this needs not be the case.

We proceed to the possibility of the dynamical symmetry breakdown of the $SU(2)_L\times U(1)_Y$ symmetry of the Lagrangian density (2.2) down to $U(1)_{em}$. It is clear that only those terms in $J^C_\alpha J^{C\,\alpha}$ can contribute to the condensation (hence to the fermion masses), which contain fields of opposite chiralities, namely

$$\begin{aligned}\mathcal{L}_{NJL} = {}& -\frac{h^2}{2M^2}y(\psi_L)y(\nu_R)\bar\psi_L\gamma_\alpha\psi_L\cdot\bar\nu_R\gamma^\alpha\nu_R - \frac{h^2}{2M^2}y(\psi_L)y(e_R)\bar\psi_L\gamma_\alpha\psi_L\cdot\bar e_R\gamma^\alpha e_R \\ &- \frac{h^2}{2M^2}y(q_L)y(u_R)\bar q_L\gamma_\alpha q_L\cdot\bar u_R\gamma^\alpha u_R - \frac{h^2}{2M^2}y(q_L)y(d_R)\bar q_L\gamma_\alpha q_L\cdot\bar d_R\gamma^\alpha d_R = \\ &= \frac{h^2}{M^2}y(\psi_L)y(\nu_R)\bar\psi_L\nu_R\cdot\bar\nu_R\psi_L + \frac{h^2}{M^2}y(\psi_L)y(e_R)\bar\psi_L e_R\cdot\bar e_R\psi_L + \\ &+ \frac{h^2}{M^2}y(q_L)y(u_R)\bar q_L u_R\cdot\bar u_R q_L + \frac{h^2}{M^2}y(q_L)y(d_R)\bar q_L d_R\cdot\bar d_R q_L .\end{aligned} \qquad (2.3)$$

For neutrinos, also the following terms are allowed:

$$\begin{aligned}\mathcal{L}_{Majorana} = {}& \frac{h^2}{2M^2}y^2(\psi_L)\bar\nu^c_R\gamma_\alpha\nu^c_R\cdot\bar\nu_L\gamma^\alpha\nu_L + \frac{h^2}{2M^2}y^2(\nu_R)\bar\nu^c_L\gamma_\alpha\nu^c_L\cdot\bar\nu_R\gamma^\alpha\nu_R \\ = {}& -\frac{h^2}{M^2}y^2(\psi_L)\bar\nu^c_R\nu_L\cdot\bar\nu_L\nu^c_R - \frac{h^2}{M^2}y^2(\nu_R)\bar\nu^c_L\nu_R\cdot\bar\nu_R\nu^c_L .\end{aligned} \qquad (2.4)$$

The mechanism of the dynamical fermion mass generation /5/ demands $y(\psi_L)y(\nu_R)$, $y(\psi_L)y(e_R)$, $y(q_L)y(u_R)$, and $y(q_L)y(d_R)$ be all positive (see Eq. (2.7) below). Here y are the eigenvalues of heaviness Y_H. It is important that anomaly free solutions Y_H with the required properties do exist /13/ (for the detailed discussion see Sec.3). Thus, the Dirac fermion masses can be dynamically generated. Majorana neutrino masses cannot be dynamically generated in this one-family approach since, obviously, $y^2(\psi_L)$ and $y^2(\nu_R)$ in Eq. (2.4) cannot be made negative. Notice that the Lagrangian density (2.3) does not contain the potentially dangerous lepton-quark terms, e.g. $y(\psi_L)y(u_R)\bar\psi_L u_R\cdot\bar u_R\psi_L$, which could give rise to the charged condensates $\langle\bar\nu_L u_R\rangle$ or $\langle\bar e_L u_R\rangle$. They would break, against our wish, also the symmetry $U(1)_{em}$. These condensates are all prohibited by the opposite signs of lepton and quark heaviness (see Sec.3).

In rearranging $\mathcal{L}_{NJL}$ we have used the Fiertz transformation to make the correspondence with the standard Higgs mechanism transparent. We identify $\frac{1}{M^2}(\bar{e}_R \psi_L) = \Phi^{(1)}$, $\frac{1}{M^2}(\bar{d}_R q_L) = \Phi^{(2)}$ with two Higgs doublets with the weak hypercharge $y = 1$ and $\frac{1}{M^2}(\bar{\nu}_R \psi_L) = \tilde{\Phi}^{(1)}$, $\frac{1}{M^2}(\bar{u}_R q_L) = \tilde{\Phi}^{(2)}$ with two charge conjugated Higgs doublets with $y = -1$.

To illustrate the dynamical symmetry breakdown of the Lagrangian density (2.2), it is enough to consider only that part of the interaction (2.3), which contains the composite doublet $\Phi^{(1)}$:

$$\mathcal{L}^{(1)}_{NJL} = \frac{h^2}{M^2} y(\psi_L) y(e_R) \bar{\psi}_L' e_R \cdot \bar{e}_R' \psi_L = \qquad (2.5)$$

$$= g_0[\bar{\nu}(1+\gamma_5)e \cdot \bar{e}(1-\gamma_5)\nu + \bar{e}(1+\gamma_5)e \cdot \bar{e}(1-\gamma_5)e],$$

where

$$g_0 = \frac{h^2}{4M^2} y(\psi_L) y(e_R) \qquad (2.6)$$

is taken positive. Other parts of the Lagrangian density (2.3) are treated quite analogously.

The Lagrangian density (2.5) gives rise to the dynamical appearance of the electron mass m, which is "calculable" from the gap equation /5/

$$1 - 8ig_0 \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m^2} = 1 - \frac{g_0}{4\pi^2} \int_{4m^2}^{\Lambda^2} \sqrt{1 - \frac{4m^2}{\kappa^2}}\, d\kappa^2 = 0, \qquad (2.7)$$

which follows from $\mathcal{L}^{(1)}_{NJL}$ as a self-consistency condition of the Hartree-Fock-Bogolubov approximation. This mass breaks spontaneously the gauge $SU(2)_L \times U(1)_Y$ symmetry of the Lagrangian density (2.2). Consequently, three Goldstone bosons, which would be physical in the absence of the gauge fields $\vec{A}_\alpha$ and B_α, must arise. We will find them as massless poles in the fermion-antifermion scattering matrices calculated with the Lagrangian density (2.5) in the chain approximation.

The $\bar{\nu}e$ ($\sim \Phi_-^{(1)}$ in the standard Higgs approach) scattering matrix is given as

$$M_{\bar{\nu} e} = (1+\gamma_5)_i \frac{g_0}{1 - J_{0;m}(q^2)} (1-\gamma_5)_f, \qquad (2.8)$$

in accordance with Fig.1.

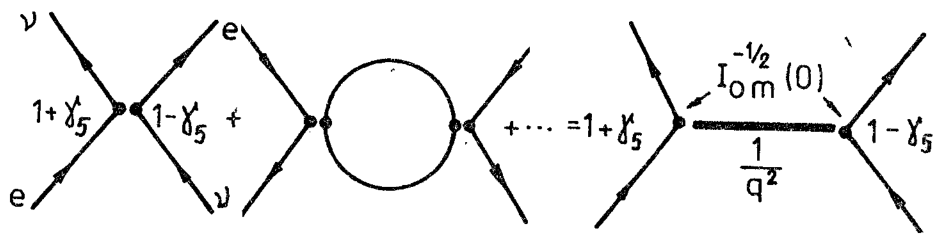

Fig.1. The chain of graphs which gives rise to the charged Goldstone boson.

Here*

$$J_{0;m}(q^2) = ig_0 \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}(1-\gamma_5) S_F^0(p)(1+\gamma_5) S_F^m(p-q) = 1 - q^2 g_0 I_{0;m}(q^2), \qquad (2.9)$$

where

$$I_{0;m}(q^2) = \frac{1}{4\pi^2} \int_{4m^2}^{\Lambda^2} \frac{\sqrt{1 - 4m^2/\kappa^2}\, d\kappa^2}{q^2 + \kappa^2 \frac{1}{4}(1 + \sqrt{1 - 4m^2/\kappa^2})^2}. \qquad (2.10)$$

With the help of (2.9) and (2.10) the $\bar{\nu}e$ scattering matrix (2.8) acquires the desired form

$$M_{\bar{\nu} e} = (1+\gamma_5)_i \frac{1}{I_{0;m}(q^2)} \frac{1}{q^2} (1-\gamma_5)_f. \qquad (2.11)$$

Thus the phenomenological fermion-charged Goldstone boson coupling constant is

$$\frac{G^2_{\bar{\nu} e}}{4\pi} = \frac{1}{I_{0;m}(0)}. \qquad (2.12)$$

Analogously the $\bar{e}e$ ($\sim \Phi_0^{(1)} + \Phi_0^{(1)+}$ in the standard Higgs approach) scattering matrix is given as

$$M_{\bar{e} e} = (i\gamma_5)_i \frac{2g_0}{1 - J_{m;m}(q^2)} (i\gamma_5)_f, \qquad (2.13)$$

in accordance with Fig.2. Here

$$J_{m;m}(q^2) = ig_0 \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\, i\gamma_5 S_F^m(p) i\gamma_5 S_F^m(p-q) = 1 - q^2 g_0 I_{m;m}(q^2), \qquad (2.14)$$

*The neutrino is taken massless here, since the interaction (2.5), which is iterated in the chain approximation, does not give rise to the neutrino mass.

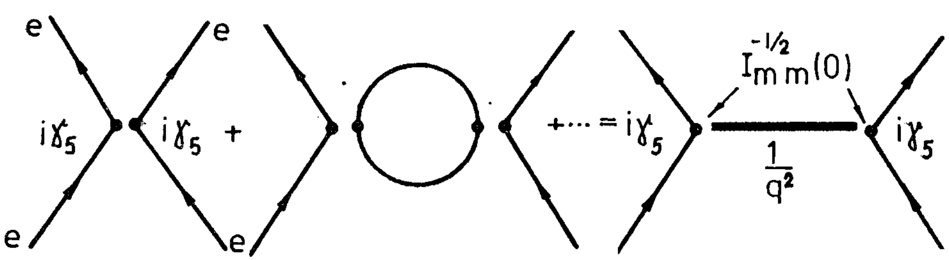

Fig.2. The chain of graphs which gives rise to the neutral Goldstone boson.

where

$$I_{m;m}(q^2)=\frac{1}{4\pi^2}\int_{4m^2}^{\Lambda^2}\frac{\sqrt{1-4m^2/\kappa^2}}{q^2+\kappa^2}\,d\kappa^2 . \tag{2.15}$$

Thus, the $\bar{e}e$ scattering matrix (2.13) can be written with the help of Eqs. (2.14) and (2.15) as

$$M_{\bar{e}e}=(i\gamma_5)_i\,\frac{1}{I_{m;m}(q^2)}\,\frac{2}{q^2}\,(i\gamma_5)_f \tag{2.16}$$

and

$$\frac{G_{\bar{e}e}}{4\pi}=\frac{1}{I_{m;m}(0)} \tag{2.17}$$

can be identified with the electron-neutral Goldstone boson coupling constant.

To evaluate the contribution of the gauge fields $\vec{A}$ and B into the $\bar{\nu}e$ and ee scattering matrices, we need to calculate the fermion-vector boson vertex functions, again in the chain approximation. Starting from the bare $\bar{\nu}e-W$ vertex $\frac{g}{2\sqrt{2}}\gamma_\alpha(1-\gamma_5)$, we obtain by summing the chain of graphs in Fig.3

$$F^{\alpha}_{\bar{\nu}e\text{-}W}=\frac{g}{2\sqrt{2}}\gamma^{\alpha}(1-\gamma_5)+\frac{g}{2\sqrt{2}}(1+\gamma_5)\frac{1}{1-J_{0;m}(q^2)}J^{\alpha}_{0;m}(q) = \frac{g}{2\sqrt{2}}\gamma^{\alpha}(1-\gamma_5)+\frac{g}{2\sqrt{2}}\frac{mq_{\alpha}}{q^2}(1+\gamma_5), \tag{2.18}$$

where

$$J^{\alpha}_{0;m}(q)=g_0\int\frac{d^4p}{(2\pi)^4}\,tr(1-\gamma_5)S^{0}_{F}(p)\gamma^{\alpha}(1-\gamma_5)S^{m}_{F}(p-q) = q^{\alpha}mg_0 I_{0;m}(q^2). \tag{2.19}$$

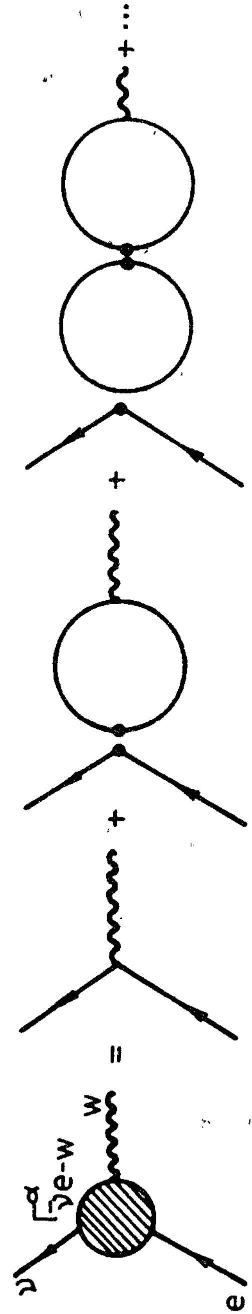

Fig.3. The chain of graphs for the W boson vertex function.

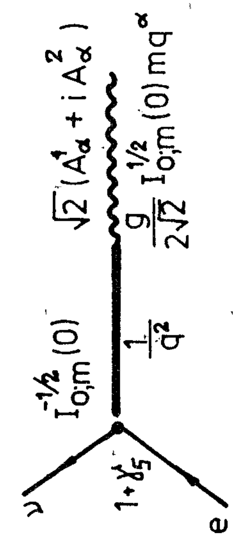

Fig.4. The effective coupling of the charged Goldstone boson with fermions and with the W boson.

Thus, taking into account Fig.1, the second term in Eq. (2.18) corresponds to the effective coupling between the charged Goldstone boson and W boson as shown in Fig.4. This coupling gives rise to the longitudinal part of the polarization tensor of the W boson, singular at $q^2=0$ with the residue equal to $\frac{1}{4}g^2m^2I_{0;m}(0)$. Since the current corresponding to the vertex part (2.18) is conserved, we immediately conclude [4,6,14] that this residue is equal to the squared mass of the W boson:

$$m_W^2=\frac{1}{4}g^2m^2I_{0;m}(0). \tag{2.20}$$

Repeating the same procedure for the case of neutral gauge bosons A_α^3 and B_α, we easily find the effective vertices of the neutral Goldstone boson and the neutral gauge bosons A^3 and B, see Fig.5. The vertex functions $\Gamma^\alpha_{\bar{e}e-A^3}$ and $\Gamma^\alpha_{\bar{e}e-B}$ are given as follows:

$$\Gamma^\alpha_{\bar{e}e-A^3}=\frac{1}{4}g\gamma^\alpha(1-\gamma_5)+\frac{1}{4}g\gamma_5\frac{2}{1-J_{m;m}(q^2)}J^\alpha_{m;m}(q)$$

$$=\frac{1}{4}g\gamma^\alpha(1-\gamma_5)+\frac{1}{4}g\frac{2mq^\alpha}{q^2}\gamma_5,$$

$$\Gamma^\alpha_{\bar{e}e-B}=-\frac{1}{4}g'\gamma^\alpha(1-\gamma_5)-\frac{1}{2}g'\gamma^\alpha(1+\gamma_5)+\frac{1}{4}g'\frac{2mq^\alpha}{q^2}\gamma_5.$$

Here

$$J^\alpha_{m;m}(q)=g_0\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}\gamma_5S_F^m(p)\gamma^\alpha(1-\gamma_5)S_F^m(p-q)=q^\alpha mg_0I_{m;m}(q^2).$$

Hence, the residue at the pole of the longitudinal part of the polarization tensor of the neutral vector bosons is given by the matrix

$$\begin{pmatrix} g^2 & gg' \\ gg' & g'^2\end{pmatrix}\frac{1}{4}m^2I_{m;m}(0)$$

in the (A^3,B) basis [14]. Its diagonalization leads to

$$m_Z^2=\frac{1}{4}(g^2+g'^2)m^2I_{m;m}(0) \tag{2.21}$$

$$m_A^2=0, \tag{2.22}$$

where

$$Z_\alpha=-\cos\theta_W A_\alpha^3+\sin\theta_W B_\alpha$$

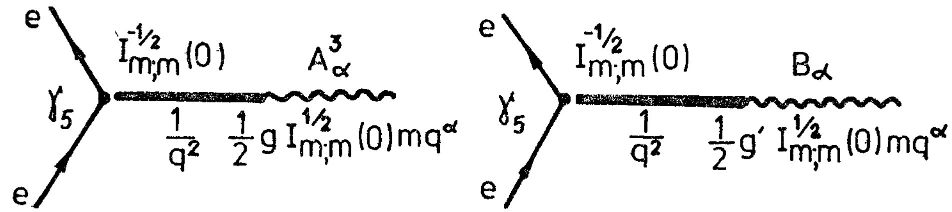

Fig.5. The effective couplings of the neutral Goldstone bosons with fermions and with the neutral gauge bosons A^3 and B.

and

$$A_\alpha=\sin\theta_W A_\alpha^3+\cos\theta_W B_\alpha$$

are the neutral intermediate boson Z with the mass m_Z, Eq. (2.21) and the massless photon, respectively. θ_W is the Weinberg angle, $\mathrm{tg}\,\theta_W=g'/g$.

It is clear that the composite doublets $\Phi^{(2)}=\frac{1}{M^2}(\bar{d}_Rq_L)$, $\tilde{\Phi}^{(1)}=\frac{1}{M^2}(\nu_R\psi_L)$ and $\tilde{\Phi}^{(2)}=\frac{1}{M^2}(\bar{u}_Rq_L)$ give rise to the masses of the d-quark, neutrino and u-quark, respectively via the gap equations, analogous to Eq. (2.7) and that they all contribute in the chain approximation incoherently into the W and Z boson masses. Hence, in the case of one family we find

$$m_W^2=\frac{1}{4}g^2\sum_f m_f^2I_{0;m_f}(0) \tag{2.23}$$

$$m_Z^2=\frac{1}{4}(g^2+g'^2)\sum_f m_f^2I_{m_f;m_f}(0). \tag{2.24}$$

3. THREE FAMILIES

The discussion of anomaly cancellation in the $SU(2)_L\times U(1)\times U(1)'$ gauge models has been already done in ref. [13] and it clearly applies also to our model (2.1). If the $SU(2)_L\times U(1)_Y$ quantum numbers are assigned to leptons and quarks as in the GWS model, three independent sets of heaviness

$$Y_H=(y(q_L),\ y(u_R),\ y(d_R);\ y(\psi_L),\ y(e_R),\ y(\nu_R)) \tag{3.1}$$

exist [13]:

$$Y_H^{(1)}=(1/3,\,4/3,\,-2/3;\,-1,\ -2,\ 0), \tag{3.2}$$

$$Y_H^{(2)} = (0, 1, -1;\ 0, -1, 1), \tag{3.3}$$

$$Y_H^{(3)} = (0, 5, 1; 0, -7, -(35)^{1/3}). \tag{3.4}$$

It is easily verified that any combinations

$$\alpha Y_H^{(1)} + \beta Y_H^{(2)} \tag{3.5}$$

$$\gamma Y_H^{(1)} + \delta Y_H^{(3)} \tag{3.6}$$

also obey the conditions on anomaly cancellation.

In the case of one family we demanded $y(\psi_L)$, $y(e_R)$, $y(\nu_R) < 0$ and $y(q_L)$, $y(u_R)$, $y(d_R) > 0$ (or vice versa). For $\alpha, \gamma > 0$ (for example) any $\beta \in (-4/3\alpha, -2/3\alpha)$ and $\delta > 2/3\gamma$ give the combinations (3.5) or (3.6) with this property.

In the case of more families, in contrast with ref.[13], we relax the requirement of the same Y_H in all families. First, Y_H distinguishes the like fermions in different families for the electroweak interactions switched off. This is desirable. It is also necessary, since equal Y_H of the like fermions in all (or some) families would imply a global SU(n) symmetry ($n \leq$ number of families) in the family space. The dynamically appearing different fermion masses would break spontaneously this symmetry thus generating the unwanted Goldstone bosons. There is, however, nobody to "eat" them. The necessary consequence of such a picture, as shown explicitly below, is the appearance of the terms changing flavor in the neutral current coupled to the field C.

The very existence of three sets of Y_H, (3.2), (3.3) and (3.4) seems to be suggestive to raise the hypothesis of the existence of three fermion families. However, the solution (3.4) cannot be used for embedding the model into a Grand Unified Theory[13]. This problem is not considered in the present paper. We are forced to reject the solution (3.4) anyway, since it produces the gluon-gluon-C boson anomaly, $y(q_L) - y(u_R) \neq y(d_R) - y(q_L)$, unlike the solutions (3.2) and (3.3). Hence, the families will be distinguished by the heaviness

$$Y_{iH} = \alpha_i Y_H^{(1)} + \beta_i Y_H^{(2)}, \tag{3.7}$$

where i runs from one to three without an internal justification. We can only speculate that Y_{iH} will become quantized when the model is embedded into a proper simple gauge group.

The Lagrangian density to be discussed has the same form as the Lagrangian density (2.1), but now the fermion fields represent columns in the family space of the weak interaction

eigenstates and Y_H is the nondegenerate diagonal matrix of the eigenvalues of heaviness (3.7).

As in the case of one family, we will analyze the dynamical breakdown of the gauge $SU(2)_L \times U(1)_Y$ symmetry on the typical (charged lepton) part of that effective four-fermion Lagrangian, which is responsible for the fermion-antifermion condensation:

$$\begin{aligned}\mathcal{L}^{(1)}_{NJL} &= \frac{h^2}{M^2} y_i(\psi_L) y_j(e_R) \bar{\psi}_{iL} e_{jR}\, \bar{e}_{jR} \psi_{iL} \\ &= \frac{h^2}{M^2} y_i(\psi_L) y_j(e_R) [\bar{\nu}_{iL} e_{jR} \cdot \bar{e}_{jR} \nu_{iL} + \bar{e}_{iL} e_{jR} \cdot \bar{e}_{jR} e_{iL}].\end{aligned} \tag{3.8}$$

In fact, the Lagrangian density (3.8) consists of 9 composite Higgs doublets $\frac{1}{M^2} \bar{e}_{iR} \psi_{jL}$ with the weak hypercharge $y = 1$ and it should give rise to 9 condensates $\frac{1}{M^2} \langle \bar{e}_{iR} e_{jL} \rangle$. In order to "calculate" them with the help of the gap equations analogous to Eq. (2.7), we proceed as follows. Let us write

$$\begin{aligned}\nu_{iL} &= U_{ia}(\nu_L)\nu_{aL} \qquad e_{iL} = U_{ia}(e_L) e_{aL} \\ \nu_{iR} &= U_{ia}(\nu_R)\nu_{aR} \qquad e_{iR} = U_{ia}(e_R) e_{aR},\end{aligned} \tag{3.9}$$

where $\nu_a = (\nu_e, \nu_\mu, \nu_\tau)$, $e_a = (e, \mu, \tau)$ are the lepton fields with dynamically generated masses and the matrices $U(\nu_L)$, $U(\nu_R)$, $U(e_L)$ and $U(e_R)$ are unitary matrices. The Lagrangian density (3-8) rewritten in terms of the fields ν_a and e_a (for fixed i, j) gives rise to the masses of charged leptons, which are determined by the HFB selfconsistency equation

$$\begin{aligned} m_a^{ij} \delta_{ab} - 2i \frac{h^2}{M^2} U^+_{ai}(e_L) y_i(\psi_L) U_{ic}(e_L) \int \frac{d^4p}{(2\pi)^4} \frac{m_c \delta_{cd}}{p^2 - m_c^2} \times \\ U^+_{dj}(e_R) y_j(e_R) U_{jb}(e_R) = 0,\end{aligned} \tag{3.10}$$

where m_a^{ij} is the part of the mass of the lepton of sort a which results from the dynamical doublet $\frac{1}{M^2} \bar{e}_{jR} \psi_{iL}$. Hence,

$$m_a = \sum_{i,j} m_a^{ij}.$$

Eq. (3.10) can be also easily rewritten in terms of the nondiagonal condensates

$$\mu_{ij} = U_{ia}(e_L) m_a U^+_{aj}(e_R),$$

which, by definition, determine m_a^{ij}:

$$m_a^{ij} \delta_{ab} = U^+_{ai}(e_L) \mu_{ij} U_{jb}(e_R). \tag{3.11}$$

Simple inspection of the gap equation (3.10) shows that the unitary matrices $U(e_L)$ and $U(e_R)$ must be in fact real orthogonal matrices, i.e.,

$$U(e_L) \to O(e_L), \quad U(e_R) \to O(e_R), \text{ etc.} \tag{3.12}$$

Thus, there is no room for the Kobayashi-Maskawa[15] mechanism of CP violation in this approach. It is, however, clear, that it is merely a consequence of the form of the self-consistency condition (3.10) which we know for sure must be changed in a more accurate approach*.

Generalization of the mechanism of the dynamical symmetry breakdown described in Sec.2 to the case of more (3) fermions with mixing, is straightforward. The iteration of the four-fermion interaction (3-8) rewritten in terms of mass eigenstates (for fixed i, j), gives rise to the massless poles both of charged and neutral Goldstone bosons. Use is made, as before, of the self-consistency equation (3.10) and definition (3.11). The inclusion of the gauge fields W and Z is also the same as in Sec.2. The result is (for $\mathcal{L}^{(1)}_{NJL}$, Eq. (3.8), with (3.9) and (3.12) taken into account)

$$m_W^2 = \frac{1}{4} g^2 \Sigma \frac{O^2_{ka}(e_R) m_a^2 I^2_{0;m_a}(0)}{O^2_{kb}(e_R) I_{0;m_b}(0)}, \tag{3.13}$$

$$m_Z^2 = \frac{1}{4}(g^2+g'^2) \Sigma \frac{O^2_{ka}(e_R) m_a^2 I^2_{m_a;m_a}(0)}{O^2_{kb}(e_R) I_{m_b;m_b}(0)}. \tag{3.14}$$

The final formulas for m_W^2 and m_Z^2 are obtained by summing Eqs. (3.13) and (3.14), respectively, over all fermion types (neutrinos, charged leptons, quarks with the electric charge 2/3 and -1/3) with their respective mixing matrices.

4. CONCLUSIONS

Most of the chiral symmetries, which underlie the gauge theories of the electroweak interactions, do not tolerate the fermion mass terms. Despite this, the elementary fermions, i.e., leptons and quarks, do have masses. A solution to this dilemma has been found, which is based on the assumption of the existence of auxiliary scalar fields with the nonzero vacuum expectation value. By introducting vastly different Yukawa coupling constants, we are able to describe the fermion mass spectrum, but not to explain it. This is a definition of phenomenology.

The role of scalar fields, is however, twofold. They also give masses to the gauge fields in the gauge invariant manner. Notice that these roles are not internally related. We can introduce scalar fields, which give rise to the gauge boson masses, but which cannot be invariantly coupled with fermions to contribute also to their masses. Hence, there is a wide freedom in introducing the scalar fields.

In this work, we have suggested to calculate the fermion masses dynamically as resulting from the strong attraction between left-handed and right-handed components of the originally massless fermion fields due to the exchange of an Abelian field. Such a mechanism points out the deep analogy between the gap of the BCS-Bogolubov superconductivity and the fermion mass[5]. Indeed, the renormalization group argument clearly shows[16] that

$$m = \mu f(\hat{h}) \exp\left[\int_{h_R}^{\hat{h}} \frac{dx}{\beta(x)}\right], \tag{4.1}$$

where h_R is the coupling constant, renormalized at the point μ, $\hat{h}$ is some arbitrary parameter and $\beta(h_R) = \mu\partial/\partial\mu h_R$.

In physical terms, our system governed by the Lagrangian density (2.1), is very similar to the many-body theory of electrons interacting with phonons (to be compared with C) in the presence of the external magnetic field[11] (to be compared with $\vec{A}$ and B). Technically, however, there are great differences. While in the nonrelativistic theory the Cooper phenomenon takes place for arbitrarily weak attractive interaction, there is no signal for analogous effect with small coupling constant in realistic relativistic field theories. Second difference lies in different treatment of the loop integrals. While in superconductivity the gap equation has an immediate physical interpretation and its solution

$$\Delta = 2h\omega_D \exp\left[-\frac{1}{N(0)g}\right] \tag{4.2}$$

exhibits clearly its nonperturbative origin, the analogous senseful equation for the fermion mass is lacking due to our lack of knowledge of solving and renormalizing the strongly interacting theory. If we want to save, in accordance with

*We met an analogous restriction already in the case of one family. Eq. (2.7) offers only the real solution m, although it is clear that any complex solution m would be good as well. It can be always made real with the help of the proper phase transformation of the right-handed fermion field without a physical consequence.

*Here $h\omega_D$ is the mean phonon energy, N(0) is the density of states for one spin projection at the Fermi surface and g is the coupling constant, quite analogous to our g_0, Eq. (2.6).

our intuition, the property of the gap, $\Delta \to 0$ for $g \to 0_+$ also for the fermion mass (4.1) in our approach, we have to assume [16] that h_R has an ultraviolet fixed point, at which the function β develops an essential singularity [17].

What we have done, is only the detailed discussion of the symmetry properties of the starting Lagrangian density (2.1). The anomaly free solutions for heaviness (3.7) should guarante the fermion-antifermion condensation in all desirable channels and forbid it in all undesirable ones. That is, the fermion mass generation with mixing is to be expected.

We have also realized the preliminary program of the dynamical mass generation using the simplified four-fermion interaction, which respects the $SU(2)_L \times U(1)_Y$ electroweak symmetry and which bona fide shares with the original theory its essential physical features, except renormalizability. Hence, our resulting mathematical formulas for both fermion and gauge boson masses are cut-off dependent and thus unqualified to be compared with the experimental numbers. In particular, the phenomenologically important ratio m_W^2/m_Z^2 cannot be safely determined. To be careful, we think of our conclusions as being in the quotation marks:

(i) All fermion masses and mixing angles are calculable in terms of several parameters. For three families our starting Lagrangian density (2.1) contains the following undetermined parameters: $g, g', h, \alpha_i, \beta_i$ and M.

(ii) Masses m_W and m_Z are calculable in terms of fermion masses, fermion-Goldstone boson coupling constants and measurable mixing angles of the fermion right-handed fields.

(iii) Majorana neutrino masses cannot be dynamically generated, as easily checked by writing the gap equation for this case.

(iv) The charged weak current J_α contains the orthogonal mixing matrices $O^T(e_L)O(\nu_L)$ and $O^T(d_L)O(u_L)$ in the lepton and quark sector, respectively. The electromagnetic current J_α^{em} and the weak neutral current J_α^Z remain intact.

(v) The neutral current J becomes flavor nondiagonal,

$$2J_\alpha^C = \bar{\nu}_L O^T(\nu_L) y(\psi_L) O(\nu_L)\gamma_\alpha \nu_L + \bar{e}_L O^T(e_L) y(\psi_L)\gamma_\alpha e_L + \bar{\nu}_R O^T(\nu_R) y(\nu_R) O(\nu_R)\gamma_\alpha \nu_R + \dots, \tag{4.3}$$

i.e., all fermion mixing angles are measurable in principle, in contrast with the canonical GWS model. The interaction, mediated by the current (4.3) is not universal (the mixing matrices in it are not orthogonal). Its appearance imposes the restriction on the ratio h^2/M^2, which must be of order $G_F^2 m^2$ [18], where m is a heavy quark mass.

We consider the "properties" of the model encouraging and hope to put off the quotation marks by repeating essentially the same program in the renormalizable framework in the spirit of the papers in ref. [17] with modifications disctated by the requirement that the calculated masses should be renormalization group invariant [19].

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Хошек И. Модель Глэшоу-Вайнберга-Салама без скалярных полей E2-82-542

Обычная $SU(2)_L \times U(1)_Y$ GWS -модель без скалярных полей дополнена тяжелым абелевым векторным бозоном C, который взаимодействует со всеми фермионами обеих киральностей. Проблема динамического рождения массы фермиона решается в приближении Хартри-Фока-Боголюбова, которое применяется для эффективного четырехфермионного взаимодействия. Вычисляется эффективное взаимодействие между динамическими голдстоуновскими бозонами и калибровочными бозонами, которое приводит стандартным образом к массам векторных бозонов W и Z. Таким образом, массы m_W и m_Z связаны с фермионными массами и с измеренными углами смешивания правых фермионов. Физический ток, с которым связан векторный бозон C, не является диагональным по аромату.

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Hosek J. The Glashow-Weinberg-Salam Model without Scalar Fields E2-82-542

Standard $SU(2)_L \times U(1)_Y$ GWS model without scalar fields is supplemented with the heavy Abelian vector boson C, which interacts with all fermions of both chiralities. The problem of the dynamical fermion mass generation is solved in the Hartree-Fock-Bogolubov approximation applied to the effective four-fermion interaction. The effective interaction between the dynamical Goldstone bosons and the gauge bosons iis calculated, which leads in a standard manner to the masses of W and Z vector bosons. The masses m_W and m_Z are thus related to the fermion masses and to the measurable mixing angles of the right-handed fermions. The physical current, to which the vector boson C couples, is not flavor diagonal.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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