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2512/82

31/V-82

E2-82-52

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ON THE LIGHT-CONE EXPANSION IN GAUGE FIELD THEORIES (QED)

1982

1. INTRODUCTION

The light-cone expansion (LCE) for the product of two local operators (currents) was derived and proved for scalar field theories by S.A.Anikin and O.I.Zavialov [1]. All questions concerning the existence of the expansion, the renormalization of the occurring light-cone operators, the relation between local and nonlocal expansions and so on were extensively discussed in papers [2,3,4]. The nonlocal LCE for the scalar current $j(x) = :\phi(x)\phi(y):$ has in leading order in the scalar ϕ^4-theory the form

$$
\begin{aligned}
& j(x)\, j(y) \underset{x^2 \to 0}{\sim} \int_0^1 d\kappa_1 \int_0^1 d\kappa_2\, f(x^2, \underline{\kappa})\, \hat{O}(\underline{\kappa}) \;, \\
& \hat{O}(\kappa) = :\phi(\kappa_1 \tilde{x})\phi(\kappa_2 \tilde{x}): \;,
\end{aligned} \tag{1.1}
$$

where $\phi(x)$ are the scalar field operators and $f(x^2, \underline{\kappa})$ are c-number coefficient functions of the LCE. The corresponding local LCE has the form

$$
\begin{aligned}
& j(x) j(0) \underset{x^2 \to 0}{\sim} \sum_{n_1, n_2 \geq 0} \frac{1}{n_1! n_2!} f_{(n)}\, (x^2)\, \hat{O}_{(n)} \,, \\
& \hat{O}_{(n)} = (\tilde{x}\partial_{y_1})^{n_1} (\tilde{x}\partial_{y_2})^{n_2} \; :\phi(y_1)\phi(y_2):|_{y_1 = y_2 = 0} \,.
\end{aligned} \tag{1.2}
$$

where $f_{(n)}(x^2)$ are also c-number coefficient functions. Both expansions are connected by the Mellin-transform [3]. The aim of this paper is the extension of (1.1) and (1.2) to gauge theories.

Formally it is easy to do this by a straightforward expansion of the technique applied in the scalar case. In doing so more accurately, in theories containing vector fields there arise two questions. First, one has to include into the set of light-cone operators also operators with arbitrary many powers of the vector field operators in accordance with the fact that the canonical light-cone singularity is in this case determined by the twist rather than by the dimension of the operators. The expressions containing a sum over the powers of the vector-field operators can be summed up to get a closed expression [5]. It is however simpler to solve the

Ward-identities[6] at first and apply its solution as starting point for further investigations.

Second, it is meaningful to require that the LCE respects the invariances of the underlying theory, especially the gauge invariance. Because the LCE is derived starting with the definition of a special subtraction operator $\mathfrak{M}$, it is a priori not clear, if this condition is satisfied. In this paper we consider the LCE for scalar, gauge invariant operators

$$j(x) = \mathrm{Tr}\colon \bar{\psi}(x)\psi(y)\colon \tag{1.3}$$

in QED. We show that the simplest choice of the subtraction operator $\mathfrak{M}$ leads automatically to a gauge invariant form of the leading order LCE. For this it was necessary to employ a special solution of the Ward-identities of the QED[6]. The paper is organized as follows. In the second section we introduce the necessary notation. In the third we apply a special solution of the Ward-identities to derive the nonlocal LCE. The results are discussed in section 4. In the Appendix we prove the smallness of the remainder.

2. NOTATION

Let $\frac{1}{i}S(\bar{\psi},\psi,A)$ be the action of the QED with $\bar{\psi}(x)$, $\psi(x)$ the electron field and $A_\mu(x)$ the photon field. All quantities under consideration are functionals of $\bar{\psi}$, ψ, and A_μ:

$$F = \sum_{\ell,\ell'} \frac{1}{\ell!\,\ell'!} \int F_{\ell\ell'\mu_1\ldots\mu_\ell}(p_1,p_1',\ldots,p_\ell,p_{\ell'}',k_1,\ldots,k_\ell) \times \colon\bar{\psi}(p_1)\psi(p_1')\ldots\bar{\psi}(p_\ell)\psi(p_{\ell'}')A_{\mu_1}(k_1)\ldots A_{\mu_\ell}(k_\ell)\colon d\underline{p}\,d\underline{p}'\,d\underline{k}\,. \tag{2.1}$$

Here the summation over the corresponding spinor indices is assumed. All further notations are the same as in the scalar case (see refs.[1] or [3]):

$$RE_0(S) \equiv R\exp S \tag{2.2}$$

is the functional of the S-matrix[1]

$$Rj(x)E_0(S)\,, \tag{2.3}$$

$$Rj(x)j(y)E_0(S) \tag{2.4}$$

generate the coefficient functions of all graphs with one, resp., two insertions of the operator j(x) (1.3).

* Following [1] we omit the symbol of the time ordering everywhere.

An essential role for the derivation of the LCE there plays the subtraction operator $\mathfrak{M}^a$. For the QED we choose (cf. [1])

$$\mathfrak{M}^a F(x) = \sum_{\ell,\ell'} \frac{1}{\ell!\,\ell'!} M_\sigma^{a+2-\ell-3\ell'+s}\sigma^s \times F^{x\text{-prop}}_{\ell\ell'(\mu)}\left(\frac{x_\sigma}{\sigma},\sigma\underline{p},\sigma\underline{p}',\sigma\underline{k}\right) \times \colon\bar{\psi}(p_1)\psi(p_1')\ldots\bar{\psi}(p_{\ell'})\psi(p_{\ell'}')A_{\mu_1}(k_1)\ldots A_{\mu_\ell}(k_\ell)\colon d\underline{p}\,d\underline{p}'\,d\underline{k} \tag{2.5}$$

with

$$M_\sigma^b f(\sigma) = \sum_{k=0}^{b} \frac{1}{k!}\left(\frac{\partial}{\partial\sigma}\right)^k f(\sigma)\Big|_{\sigma=0}\,, \tag{2.6}$$

where F(x) is a functional with two insertions of the operator j(x), x-prop means the contribution of graphs, which become 1PI after identification of the vertices corresponding to the operator insertions. The vector x_σ is given by

$$x_\sigma = \tilde{x} + \eta\,\frac{\tilde{x}\eta}{\eta^2}\left[\sqrt{1+\sigma^2\frac{x^2\eta^2}{(\tilde{x}\eta)^2}} - 1\right] \tag{2.7}$$

with

$$\tilde{x} = \frac{1}{\eta^2}\left[x\eta^2 - \eta(x\eta)\right] + \frac{\eta}{\eta^2}\sqrt{(x\eta)^2 - x^2\eta^2}\,, \qquad \tilde{x}^2 = 0 \tag{2.8}$$

(cf. [3]). Further in (2.5) a is a number $a \geq 0$ and s is a number large enough, so that

$$\sigma^s F^{x\text{-prop}}_{\ell\ell'(\mu)}\left(\frac{x_\sigma}{\sigma},\sigma\underline{p},\sigma\underline{p}',\sigma\underline{k}\right) \tag{2.9}$$

is regular for $\sigma\to 0$. The sum over ℓ and ℓ' in (2.8) is bounded by the condition $a+2-\ell-3\ell'+s>0$.

It should be remarked that a bigger choice of s does not change anything, because $M_\sigma^{b+a}\sigma^a f(\sigma) = M_\sigma^b f(\sigma)$ for $f(\sigma)$ regular for $\sigma\to 0$. It is easy to find the minimal s. Inverse powers of s comes only from x_σ/σ. Because coefficient functions of graphs are polynomials of x (with vector and spinor indices), besides the dependence of scalar products like $\left(\frac{x_\sigma}{\sigma}\right)^2 = x^2$, $\frac{x_\sigma}{\sigma}\cdot\sigma k = x_\sigma\cdot k$ inverse powers of σ can occur only from factors $\frac{x_{\sigma\mu}}{\sigma}$, $\frac{\hat{x}_\sigma}{\sigma}$. The maximal number of such factors is $\ell+\ell'$. So we can choose

$$s = \ell + \ell' . \tag{2.10}$$

With this the sum over ℓ and ℓ' is bounded by $\ell \leq (a+2)/2$. If we are interested only in the leading order LCE, that means we choose $a=0$, only terms with $\ell'=0$ and $\ell'=1$ will contribute to (2.8). The sum over ℓ remains unbounded. The LCE comes from the identity [1]

$$R(j(x)j(0)E_0(S)) = E_0(S_r)\frac{1}{1+\mathfrak{M}^a E_1(S_r)}\mathfrak{M}^a R(j(x)j(0)E_0(S)) + Q^a(x), \tag{2.11}$$

where $Q^a(x)$ is the remainder with the property

$$Q^a(x) \underset{x^2 \to 0}{\sim} (x^2)^{[a/2]+1}\,{}^{*}. \tag{2.12}$$

This property is correctly proved in [1] for scalar theories. The proof for nonscalar theories is given in the appendix. The first term in the r.h.s. of (2.11) contains the LCE. One has to compute

$$\mathfrak{M}^a R(j(x)j(0)E_0(S)) \tag{2.13}$$

to get a sum of operators (the light-cone operators) with c-number coefficient functions. The remaining operation $E_0(S_1)/(1+\mathfrak{M}^a E_1(S_r))$ means then the renormalization of the light-cone operators.

Now let $\bar F^{x\text{-prop}}(x,y=0\,|\,\bar\psi,\psi,A)$ denote the x-prop part of the functional (2.5). Due to the Ward-identities its coefficient functions $F^{x\text{-prop}}_{\ell,\ell',\mu_1,\dots,\mu_\ell}(x,p_1,p_1',\dots,p_{\ell'},p_{\ell'}',k_1,\dots,k_\ell)$ (compare (2.1)) are not independent. In ref. [6] it was shown, that the coefficient functions with $\ell'=0,1$ obey such a set of relations that the corresponding functional, which we denote by $\Gamma^{x\text{-prop}}_{j(x)j(0),0}(A)$ for $\ell'=0$ and $\int \Gamma^{x\text{-prop}}_{j(x)j(0),1}(x_1,x_2\,|\,A)\bar\psi(x_1)\psi(x_2)dx_1dx_2$ for $\ell'=1$, can be expressed in the form

$$\Gamma^{x\text{-prop}}_{j(x)j(0),0}(A) = \Gamma^{tr,x\text{-prop}}_{j(x)j(y),0}(A) + \frac{4}{a}\int A_\mu(z)\frac{\partial^2}{\partial z^2}A_\mu(z)dz, \tag{2.14}$$

$$\Gamma^{x\text{-prop}}_{j(x)j(0),1}(x_1,x_2\,|\,A) = \Gamma^{tr,x\text{-prop}}_{j(x)j(0),1}(x_1,x_2\,|\,A)\times \exp\Big(ig\int_{x_1}^{x_2} A_\mu(\xi)d\xi_\mu\Big). \tag{2.15}$$

The functionals $\Gamma^{tr,x\text{-prop}}_{j(x)j(0),\ell'}(A)$, $(\ell'=0,1)$ have the property

$$\frac{\partial}{\partial z_\mu}\frac{\delta}{\delta A_\mu(z)}\Gamma^{tr,x\text{-prop}}_{j(x),j(0),\ell'}(A) = 0 . \tag{2.16}$$

*$[a/2]$ means the integer part of $[a/2]$.

So they are transversal ones. The integral in the exponential in (2.15) goes along a path γ connecting points x_1 and x_2. This path is arbitrary. A change of the path means a change of the functional $\Gamma^{tr,x\text{-prop}}_{j(x)j(0),1}(x_1,x_2\,|\,A)$ by transversal terms.

3. THE NON-LOCAL LCE

Let us compute expression (2.13) which has now the form

$$\begin{aligned}&\mathfrak{M}^\circ R(j(x)j(0)E_0(S)) = \\ &= \mathfrak{M}^\circ \Gamma^{x\text{-prop}}_{j(x)j(0),0}(A) + \mathfrak{M}^\circ \int \Gamma^{x\text{-prop}}_{j(x)j(0),1}(x_1,x_2\,|\,A)\bar\psi(x_1)\psi(x_2)dx_1dx_2 .\end{aligned} \tag{3.1}$$

For the first term of the r.h.s. we have with (2.5) and (2.14)

$$\begin{aligned}&\mathfrak{M}^\circ \Gamma^{x\text{-prop}}_{j(x)j(0),0}(A) = \\ &= \sum_\ell \frac{1}{\ell!}M^2_\sigma \sigma^\ell \int \Big[\Gamma^{tr,x\text{-prop}}_{\ell,0,\mu_1,\dots\mu_\ell}\Big(\frac{x_\sigma}{\sigma},\sigma k_1,\dots,\sigma k_\ell\Big) - \\ &- \delta_{\ell,2}\frac{4}{a}(\sigma k_1)^2\delta(\sigma(k_1+k_2))g_{\mu_1\mu_2}\Big]A_{\mu_1}(k_1)\dots A_{\mu_\ell}(k_\ell)dk_1\dots dk_\ell ,\end{aligned} \tag{3.2}$$

where $\Gamma^{tr,x\text{-prop}}_{\ell,0,\mu_1\dots\mu_\ell}$ are the coefficient functions of $\Gamma^{tr,x\text{-prop}}_{j(x)j(0),0}$ and $A_\mu(z) = \int e^{ikz}A_\mu(k)dk$ is the Fourier transform.

Eq. (2.16) means for the coefficient functions

$$k_s^{\mu_s}\Gamma^{tr,x\text{-prop}}_{\ell,0,\mu_1\dots\mu_\ell}(x,k_1,\dots,k_\ell) = 0 .$$

So, we can write down the identity

$$\begin{aligned}&\Gamma^{tr,x\text{-prop}}_{\ell,0,\mu_1\dots\mu_\ell}(x,k_1,\dots,k_\ell) = \\ &= \prod_{s=1}^{\ell}\frac{g_{\mu_s\lambda_s}(xk_s) - x_{\mu_s}k_{s\lambda_s}}{(xk_s)}\bar\Gamma^{tr,x\text{-prop}}_{\ell,0,\lambda_1,\dots,\lambda_\ell}(x^2, xk_i, k_1,\dots,k_s),\end{aligned} \tag{3.3}$$

where $\bar\Gamma^{tr,x\text{-prop}}_{\ell,0,\lambda_1\dots\lambda_\ell}$ is that part of $\Gamma^{tr,x\text{-prop}}_{\ell,0,\mu_1\dots\mu_s}$, which contains x in scalar products only. We see that

$$\Gamma^{tr,x-prop}_{\ell,0,\mu_1,\ldots,\mu_s}\left(\frac{x_\sigma}{\sigma},\sigma k_1,\ldots,\sigma k_\ell\right) =$$

$$= \prod_{s=1}^{\ell} \frac{g_{\mu_s\lambda_s}(x_\sigma k_s) - x_{\sigma\mu_s} k_{s\lambda_s}}{(x_\sigma k_s)} \bar{\Gamma}^{tr,x-prop}_{\ell,0,\lambda_1,\ldots,\lambda_\ell}(x^2, x_\sigma k_i, \sigma k_1,\ldots,\sigma k_\ell)$$

is a regular function of σ for $\sigma \to 0$ because it does not contain σ in the denominator. Consequently, in (3.2) the term with $\ell = 2$ survives only. Taking into account that the longitudinal contribution in (3.2) vanishes, we get

$$\mathfrak{M}\circ\Gamma^{x-prop}_{j(x)j(0),0}(A) =$$

$$= \frac{1}{2}\int \prod_{s=1}^{2} \frac{g_{\mu_s\lambda_s}(\tilde{x}k_s) - \tilde{x}_{\mu_s} k_{s\lambda_s}}{(\tilde{x}k_s)} g_{\lambda_1\lambda_2} \times \tag{3.4}$$

$$\times \bar{\Gamma}^{tr,x-prop}_{2,0}(x^2, \tilde{x}k_i) : A_{\mu_1}(k_1)A_{\mu_2}(k_2) : dk_1 dk_2 ,$$

where $\bar{\Gamma}^{tr,x-prop}_{2,0}$ is the contribution to $\bar{\Gamma}^{tr,x-prop}_{\ell,2,\lambda_1\lambda_2}$, which is proportional to $g_{\lambda_1\lambda_2}$. All other contributions are proportional to $(\sigma k_s)_{\lambda_s}$ and vanish.

Like in the scalar case, we introduce now the Fourier transform with respect to $\tilde{x}k_1$, $\tilde{x}k_2$:

$$f_{2,0}(x^2, \underline{\kappa}) =$$

$$= \int dz_1 dz_2 e^{iz_1\kappa_1 + iz_2\kappa_2} \left. \frac{\bar{\Gamma}^{tr,x-prop}_{2,0}(x^2, \tilde{x}k_i)}{(\tilde{x}k_1)(\tilde{x}k_2)} \right|_{\tilde{x}k_i = z_i} . \tag{3.5}$$

Its support property follows from the fact that $\Gamma^{tr,x-prop}_{\ell,0,\mu_1\ldots\mu_\ell}(x, k_1,\ldots,k_\ell)$ is an analytical function of xk_i. On the other hand, this can be shown in a-representation/1,2/. So, we get finally

$$\mathfrak{M}\circ\Gamma^{x-prop}_{j(x)j(0),0}(A) = \frac{1}{2}\int_0^1 d\kappa_1 \int_0^1 d\kappa_2 \, f_{2,0}(x^2,\underline{\kappa})\hat{O}_{2,0}(\underline{\kappa}) \tag{3.6}$$

with the light-cone operators

$$\hat{O}_{2,0}(\underline{\kappa}) = \int e^{i\kappa_1\tilde{x}k_1 + i\kappa_2\tilde{x}k_2}(g_{\mu_1\lambda_1}(\tilde{x}k_1) - \tilde{x}_{\mu_1}k_{1\lambda_1}) \times$$

$$\times (g_{\mu_2\lambda_2}(\tilde{x}k_2) - \tilde{x}_{\mu_2}k_{2\lambda_2}) g_{\lambda_1\lambda_2} : A_{\mu_1}(k_1)A_{\mu_2}(k_2) : dk_1 dk_2 = \tag{3.7}$$

$$= \tilde{x}^\mu \tilde{x}^\nu : F_{\mu\rho}(\tilde{x}\kappa_1) F_{\nu\rho}(\tilde{x}\kappa_2) :$$

They depend on the field-strength tensor

$$F_{\mu\rho}(y) = \partial_{y_\mu} A_\nu(y) - \partial_{y_\nu} A_\mu(y)$$

only and are bilocal operators localized at two points on the light-cone like in the scalar case.

Now we have to calculate the second term in the r.h.s. of (3.1). Here we take into account the representation (2.15). Consider the first term in braces in (2.15) that means the 1PI contribution. We have

$$\mathfrak{M}\circ\int\Gamma_{j(x)j(0),1}(x_1, x_2|A)\bar{\psi}(x_1)\psi(x_2)dx_1dx_2 =$$

$$= M^\circ_\sigma \int \sum_{n\geq 0} \frac{\sigma^n}{n!} \prod_{s=1}^{n} \left(ig\int_{x_1}^{x_2} e^{i\sigma k_s z_s} dz_{s\mu_s} A_{\mu_s}(k_s)dk_s\right) \times$$

$$\times \sum_{\ell\geq 0} \frac{\sigma^{\ell+1}}{\ell!} \Gamma^{tr}_{\ell,0,\mu_1\ldots\mu_\ell}\left(\frac{x_\sigma}{\sigma}, q_1, q_2, \sigma k_1,\ldots,\sigma k_\ell\right) \times \tag{3.8}$$

$$A_{\mu_1}(k_1)\cdots A_{\mu_\ell}(k_\ell)dk_1\cdots dk_\ell \times$$

$$\times \prod_{r=1}^{2}(e^{ix_r(q_r - \sigma p_r)} dq_r dx_r)\bar{\psi}(p_1)\psi(p_2)dp_1dp_2$$

with the Fourier transform

$$\psi(x) = \int e^{ipx}\psi(p)dp .$$

The functions

$$\Gamma^{tr}_{\ell,1,\mu_1,\ldots,\mu_\ell}(x, q_1, q_2, k_1,\ldots,k_\ell) =$$

$$= \int e^{ix_1p_1 + ix_2p_2}\,\Gamma^{tr}_{\ell,1,\mu_1,\ldots,\mu_\ell}(x, x_1, x_2, k_1,\ldots,k_\ell)dx_1dx_2$$

are the coefficient functions of the functional $\Gamma^{tr}_{j(x)j(0),1}(x_1,x_2|A)$. Because of (2.16) they are transversal

$$k_{s\mu_s}\Gamma^{tr}_{\ell,1,\mu_1\ldots\mu_\ell}(x,q_1,q_2,k_1,\ldots,k_\ell) = 0 .$$

For the application of the definition (2.5) of the subtraction operator $\mathfrak{M}$ one must take into account that the number ℓ of vector fields in (2.5) has to be rewritten in (3.8) to $\ell+n$, where ℓ is the number of vector fields in the expansion of the functional $\Gamma^{tr}_{j(x)j(0),1}(x_1,x_2|A)$ and n is the number of vector fields in the expansion of

$$\exp(ig\int_{x_1}^{x_2} A_\mu(z)dz_\mu).$$

Substituting in (3.8)

$$z_\mu \to z_\mu/\sigma, \quad x_i \to x_i/\sigma, \quad q_i \to \sigma q_i$$

we see, that the remaining σ-dependence has the form

$$M^\circ_\sigma \sum_{\ell\geq 0} \frac{\sigma^{\ell+1}}{\ell!} \Gamma^{tr}_{\ell,1,\mu_1,\dots,\mu_\ell}\left(\frac{x_\sigma}{\sigma}, \sigma q_1, \sigma q_2, \sigma k_1, \dots, \sigma k_\ell\right). \tag{3.9}$$

Because of the transversality it is easy to see that

$$\sigma\Gamma^{tr}_{\ell,1,\mu_1,\dots,\mu_\ell}\left(\frac{x_\sigma}{\sigma}, \sigma q_1, \sigma q_\ell, \sigma k_1, \dots, \sigma k_\ell\right)$$

is regular for $\sigma\to 0$. This can be seen in the same way as for (3.3) whereby an additional factor, $\hat{x}_\sigma/\sigma$, can occur only. So, in (3.9) we have the contribution of $\ell=0$ which is proportional to $\hat{x} \equiv x^\mu\gamma_\mu$

$$\Gamma^{tr}_{0,1}(x,q_1,q_2) = \hat{x}\bar{\Gamma}^{tr}_{0,1}(x^2,xq_1,xq_2,q_1,q_2) + \dots$$

only and get

$$M^\circ_\sigma \sum_{\ell\geq 0} \frac{\sigma^{\ell+1}}{\ell!} \Gamma^{tr}_{\ell,1,\mu_1,\dots,\mu_\ell}\left(\frac{x_\sigma}{\sigma}, \sigma q_1, \sigma q_2, \sigma k_1, \dots, \sigma k_\ell\right) = \tag{3.10}$$

$$= \hat{\tilde{x}}\,\bar{\Gamma}^{tr}_{0,1}(x^2, \tilde{x}q_1, \tilde{x}q_2, 0, 0).$$

Because $\bar{\Gamma}^{tr}_{0,1}(x^2,\tilde{x}q_1,\tilde{x}q_2,0,0)$ depends on scalar products only, it must be proportional to the unity matrix. Introducing once more the Fourier transform

$$f^{1PI}_{0,1}(x^2, \underline{\kappa}) = \tag{3.11}$$

$$= \int dz_1 dz_2\, e^{i\kappa_1 z_1 + i\kappa_2 z_2}\,\bar{\Gamma}^{tr}_{0,1}(x^2, \tilde{x}q_1, \tilde{x}q_2, 0, 0)\big|_{\tilde{x}q_i = z_i}$$

we get finally

$$\mathfrak{M}^\circ \int \Gamma_{j(x)j(0),1}(x_1,x_2|A)\bar{\psi}(x_1)\psi(x_2)dx_1 dx_2 =$$

$$= \int_0^1 d\kappa_1 \int_0^1 d\kappa_2\, f_{0,1}(x^2,\underline{\kappa})\,\hat{O}_{0,1}(\underline{\kappa}) \tag{3.12}$$

with the light-cone operators

$$\hat{O}_{0,1}(\underline{\kappa}) = :\bar{\psi}(\tilde{x}\kappa_1)\,\hat{\tilde{x}}\,\psi(\tilde{x}\kappa_2)\exp(ig\int_{\tilde{x}\kappa_1}^{\tilde{x}\kappa_2} A_\mu(z)dz_\mu): \tag{3.13}$$

The coefficient functions $f^{1PI}_{0,1}(x^2,\underline{\kappa})$ are obtained by (3.11) from that of the functional $\Gamma_{j(x)j(0),1}(x_1,x_2|A)$ for $A=0$. The contribution of x-prop rather than 1PI functionals (the second and third term in braces in (2.15)) can be calculated analogously. For this reason one must assume that the length of the path γ is proportional to the distance between its endpoints, so that

$$\int_{\sigma z}^{\sigma z'} A_\mu(\zeta)d\zeta_\mu \to 0 \qquad \text{for } \sigma\to 0.$$

Finally, we get

$$\mathfrak{M}^\circ \int \Gamma^{x\text{-prop}}_{j(x)j(0),1}(x_1,x_2|A)\bar{\psi}(x_1)\psi(x_2)dx_1 dx_2 =$$

$$= \int_0^1 d\kappa_1 \int_0^1 d\kappa_2\, f_{0,1}(x^2,\underline{\kappa})\,\hat{O}_{0,1}(\underline{\kappa}), \tag{3.14}$$

where one gets $f_{0,1}(x^2,\underline{\kappa})$ by (3.11) from the functional $\Gamma^{x\text{-prop}}_{j(x)j(0),1}(x_1,x_2|A)$ for $A=0$.

4. SUMMARY

From the calculations in the previous sections it follows that the nonlocal light-cone expansion for scalar currents

$$j(x) = \mathrm{Sp}:\bar{\psi}(x)\psi(x): \tag{4.1}$$

has in the leading order the form

$$R(j(x)j(0)E_0(S)) \underset{x^2\to 0}{\tilde{=}} \int_0^1 d\kappa_1 \int_0^1 d\kappa_2\, f_{0,1}(x^2,\underline{\kappa})\tilde{R}(\hat{O}_{0,1}(\underline{\kappa})E_0(S)) + \int_0^1 d\kappa_1 \int_0^1 d\kappa_2\, f_{1,0}(x^2,\underline{\kappa})\tilde{R}(\hat{O}_{2,0}(\underline{\kappa})E_0(S)), \tag{4.2}$$

where the operators are given by

$$\hat{O}_{0,1}(\underline{\kappa}) = :\bar{\psi}(\tilde{x}\kappa_1)\,\hat{\tilde{x}}\,\psi(\tilde{x}\kappa_2)\exp(ig\int_{\tilde{x}\kappa_1}^{\tilde{x}\kappa_2} A_\mu(z)dz_\mu): \tag{4.3}$$

$$\hat{O}_{2,0}(\underline{\kappa}) = :\tilde{x}^\mu\tilde{x}^\nu F_{\mu\rho}(\tilde{x}\kappa_1)F_{\nu\rho}(\tilde{x}\kappa_2):. \tag{4.4}$$

The coefficient functions are connected by the Fourier transform

$$f_{0,1}(x^2,\underline{\kappa}) = \int dz_1 dz_2\, e^{i\kappa_1 z_1 + i\kappa_2 z_2}\, \Gamma^{x\text{-prop}}_{0,1}(x^2, xq_i, q_i q_j)\Big|_{\substack{q_i q_j = 0 \\ xq_i = z_i}}$$

$$f_{2,0}(x^2,\underline{\kappa}) = \int \frac{dz_1}{z_1}\frac{dz_2}{z_2}\, e^{i\kappa_1 z_1 + i\kappa_2 z_2}\, \Gamma^{x\text{-prop}}_{2,0}(x^2, xq_i, q_i q_j)\Big|_{\substack{q_i q_j = 0 \\ xq_i = z_i}}$$

with some parts of the Green functions with two insertions of the operator j. Namely, $\Gamma^{x\text{-prop}}_{0,1}(x^2, xq_i, q_i q_j)$ is the x-prop contribution to the part of the Green function with two external electron lines that is proportional to $\hat{x}$, and $\Gamma^{x\text{-prop}}_{2,0}(x^2, xq_i, q_i q_j)$ is the x-prop contribution to the part of the Green function with two external photon lines that is proportional to $g_{\mu\nu}$.

This light-cone expansion (4.2) is very similar to that for the scalar theory (1.1). The operators $\hat{O}_{0,1}(\underline{\kappa})$ and $\hat{O}_{2,0}(\underline{\kappa})$ are gauge invariant and, as in the scalar case, are concentrated on the light-cone. Both operators $\hat{O}_{0,1}$ and $\hat{O}_{2,0}$ have dimension 2 so that the coefficient functions $f_{0,1}$ and $f_{2,0}$ behave as $(x^2)^{-2}$ whereas the remainder behaves as x^2 (cf. (2.12)).

It is easy to get a local LCE from (4.2) by means of the Mellin transform as in the scalar case [3]. Define the local light-cone operators by

$$\hat{O}_{(n_1,n_2)} = \left(\frac{\partial}{\partial i\kappa_1}\right)^{n_1}\left(\frac{\partial}{\partial i\kappa_2}\right)^{n_2} \hat{O}(\underline{\kappa})\Big|_{\underline{\kappa}=0} \tag{4.5}$$

Then the coefficient functions are given by

$$f_{(n_1,n_2)}(x^2) = \int_0^1 d\kappa_1 \int_0^1 d\kappa_2\, (i\kappa_1)^{n_1} (i\kappa_2)^{n_2} f(x^2,\underline{\kappa}). \tag{4.6}$$

In our case we get as local operators

$$\hat{O}_{0,1(n_1,n_2)} = :(\tilde{x}D_{y_1})^{n_1} \bar{\psi}(y_1)\hat{\tilde{x}}(\tilde{x}D_{y_2})^{n_2} \psi(y_2):\Big|_{y_1=y_2=0} \tag{4.7}$$

and

$$\hat{O}_{2,0(n_1,n_2)} = :(\tilde{x}\partial_{y_1})^{n_1} \tilde{x}^{\mu} F_{\mu\rho}(y_1)(\tilde{x}\partial_{y_2})^{n_2} \tilde{x}^{\nu} F_{\nu\rho}(y_2):\Big|_{y_1=y_2=0}, \tag{4.8}$$

where D_y is the covariant derivative. Formulae (4.7) is easy to prove by induction.

We see that the local operators have a form similar to that in the scalar case (1.2) with substituting the ordinary derivatives by the covariant ones for (4.7). In general, we see in this way that the LCE in a nonscalar theory is very similar to that in the scalar case and all points essential

for the LCE remain unchanged. Difficulties connected with the gauge invariance of the theory will not arise when using the solutions (2.14) and (2.15) of the Ward-identities.

Of course, there are some questions which are not answered in this paper. So it is not shown that no infrared difficulties occur. Further one has to shift the subtraction point in the subtraction operators and to prove the renormalization properties of the operators $\hat{O}$.

We thank D.Robaschik for useful discussions.

APPENDIX

Here we will prove the property (2.12) of the remainder. The proof is a straightforward generalization of the one given for scalar theories by A.S.Anikin and O.I.Zavialov [1], and we are able to employ many details from ref. [1]. The proof will be given here in a general manner valid not only for QED. For this reason we must introduce some additional notation.

Let us consider a field theory with sorts of fields ϕ_i and propagators

$$\Delta(P) = i\int_0^\infty d\alpha\, P\left(\frac{1}{2i}\partial_\xi, \alpha\right) \exp[i\alpha(p^2 - m^2 + i\epsilon) + 2i\xi p]\Big|_{\xi=0}, \tag{A.1}$$

where $P(\frac{1}{2i}\partial_\xi, \alpha)$ is a polynomial in ∂_ξ and α. Let n be the highest power of ∂_ξ in P and n′ the lowest power of α in P. Derivative couplings will be included in the propagators of the lines incident with the corresponding vertices. In this manner any operator insertion can be treated as a usual vertex. The object under consideration $Q^a(x)$ is a sum of graphs Γ having two operator insertions j(x) and j(y). The corresponding vertices are denoted by V_x and V_y *. Other external vertices with external momenta are denoted by V_i. The graphs contributing to $Q^a(x)$ are renormalized by the R-operation $\tilde{R}$ which acts on graphs with no or one insertion of the operator j with the subtraction operator M and on subgraphs with two insertions of the operator j with the subtraction operator $\mathfrak{M}^a$ (cf. [1], [2]). As the divergence index of a graph γ we choose

$$\omega_\gamma = 4\vec{\mathfrak{K}}_\gamma - 2\ell_\gamma + \sum_{\ell\in\gamma}(n_\ell - 2n'_\ell), \tag{A.2}$$

where $\vec{\mathfrak{K}}_\gamma$ is the number of loops of γ; and ℓ_γ, the number of lines in γ. The sum runs over all lines of γ. As subtraction operator M acting on functionals of the fields ϕ_i without

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* Due to translation invariance we put $y=0$.

insertion of the operator j we choose

$$MF = \sum_{i_1,\dots,i_\ell} M_\sigma^\omega \int F^{1PI}_{i_1\dots i_\ell}(\sigma k_1,\dots,\sigma k_\ell)\delta(\Sigma k);\ \phi_{i_1}(k_1)\dots\phi_{i_\ell}(k_\ell):d\underline{k}. \quad (A.3)$$

and for functionals with one insertion of j(x)

$$MF(k) = \sum_{i_1\dots i_\ell} M_\sigma^\omega \int F^{1PI}_{i_1\dots i_\ell}(\frac{x}{\sigma}, \sigma k_1,\dots,\sigma k_\ell):\phi_{i_1}(k_1)\dots\phi_{i_\ell}(k_\ell):d\underline{k}. \quad (A.4)$$

The sums are bounded by the condition $\omega \geq 0$ *. As subtraction operator $\mathfrak{M}^a$ acting on functionals with two insertions of the operator j we choose

$$\mathfrak{M}^a F(x) = \sum_{i_1,\dots,i_\ell} M_\sigma^{a+\omega+s}\sigma^s \int F^{x\text{-prop}}_{i_1\dots i_\ell}(\frac{x_\sigma}{\sigma},\sigma k_1,\dots,\sigma k_\ell):\phi_{i_1}(k_1)\dots\phi_{i_\ell}(k_\ell):d\underline{k}, \quad (A.5)$$

where the sum is bounded by $a+\omega+s \geq 0$. It is clear that (A.5) coincides with (2.5) for QED.

Under these assumptions we show that the coefficient function $F_\Gamma(x, \underline{k})$ of any graph Γ contributing to $Q^a(x)$ has the property

$$F_\mu(x,\underline{k}) \underset{x^2\to 0}{\sim} (x)^{[a/2]+1}, \quad (A.6)$$

where $[a/2]$ means the integer part of $a/2$. To show this we have to introduce some further notation. Let γ be a subgraph given by an arbitrary set of lines and all vertices incident with them. $\mathfrak{L}$ is the set of all such subgraphs. $\mathfrak{L}_c$ is the subset of $\mathfrak{L}$ consisting of subgraphs which contain the vertices V_x and V_y in one component of connectivity. A nest $\mathfrak{N}$ is a subset of $\mathfrak{L}$ such that for any $\gamma_1 \in \mathfrak{N}$ and $\gamma_2 \in \mathfrak{N}$ either $\gamma_1 \subset \gamma_2$ or $\gamma_2 \subset \gamma_1$ holds. With this the following lemma is true.

<u>Lemma 1</u>. The coefficient function $F_\Gamma(x,\underline{k})$ of a graph Γ contributing to $Q^a(x)$ has the representation

$$F_\Gamma(x,\underline{k}) = C\int_0^\infty da_1 \dots \int_0^\infty da_2 \{\prod_{\gamma\in\mathfrak{L}} I_{\sigma_\gamma} + \sum_{\mathfrak{N}}\prod_{\gamma\in\mathfrak{N}}(-M_{\sigma_\gamma}^{a_\gamma+\omega_\gamma+s_\gamma})\prod_{\gamma\bar{\in}\mathfrak{N}} I_{\sigma_\gamma}\}\times$$

$$\times\prod_{\gamma\in\mathfrak{L}}\sigma^{s_\gamma+4\mathfrak{K}_\gamma}\prod_{\ell=1}^{L} P_\ell(\frac{1}{2i}\partial_\xi, a)\,\Gamma_\beta(x_\pi,\underline{\eta},\underline{k})\,\mathcal{E}(\underline{a})|_{\xi=0}, \quad (A.7)$$

* ω depends on $i_1\dots i_\ell$, that means on the number and sorts of fields ϕ_i only. It is the same for all graphs contributing to the coefficient function $F_{i_1\dots i_\ell}$.

where

$$\mathcal{E}(a) = \prod_{\ell=1}^{L}\exp[ia_\ell(-m_\ell^2+i\epsilon)],$$

$$a_\gamma = \begin{cases} a & \text{for } \gamma\in\mathfrak{L}_c \\ 0 & \text{for } \gamma\bar{\in}\mathfrak{L}_c,\end{cases}$$

s_γ are sufficiently large numbers. L denotes the number of lines in Γ and

$$I_\gamma f(\sigma_\gamma) = f(1).$$

The function

$$\Gamma_\beta(x_\pi,\underline{\eta},\underline{k}) \equiv \Gamma_{\beta_1\dots\beta_2}(x_\pi,\eta_1\dots\eta_2, k_1\dots k_n)$$

contains the combinatorical functions and will be specified later. The parameters β_ℓ are given by

$$\beta_\ell = a_\ell\pi_\ell^2 \quad (A.8)$$

with $\Pi_\ell = \prod_{\gamma\in\ell}\sigma_\gamma$ and the η_ℓ by

$$\eta_\ell = \Pi_\ell\xi_\ell, \quad (A.9)$$

x_π is given by (2.7) and

$$\Pi = \prod_{\gamma\in\mathfrak{L}_c}\sigma_\gamma. \quad (A.10)$$

To show (A.7) we start with the corresponding unrenormalized coefficient function

$$F_\Gamma^{un}(x,k) = C\int_r^\infty da_1\dots\int_r^\infty da_2\prod_{\ell=1}^{L}P(\frac{1}{2i}\partial, a)\times \quad (A.11)$$

$$\times\Gamma_a(x,\xi,k)\,\mathcal{E}(a)|_{\xi=0},$$

where r is the UV-regularization parameter. One gets this representation by a partial Fourier transform out of the standard a-representation /7/. The function $\Gamma_{\underline{a}}(x,\underline{\xi},\underline{k})$ is given by

$$\Gamma_{\underline{a}}(x,\underline{\xi},\underline{k}) =$$

$$=(A(\underline{a}))^{-2}\exp[-\frac{i}{4}\frac{D(\underline{a})}{A(\underline{a})}x^2 - \frac{i}{2}\sum_i xk_i\frac{A_i(\underline{a})}{A(\underline{a})} - \frac{i}{4}\frac{(\Sigma k_i A_i(\underline{a}))^2}{D(\underline{a})A(\underline{a})} +$$

$$+ i\frac{\sum k_i k_j A_{ij}(\underline{a})}{D(\underline{a})} - i\frac{k(\underline{a},\underline{\xi})}{D(\underline{a})} - \frac{i}{4}\frac{(\sum \xi_\ell \bar{B}_\ell(\underline{a}))^2}{D(\underline{a})\,A(\underline{a})} - \qquad (A.12)$$
$$- \frac{i}{2}\frac{\sum x\xi_\ell \bar{B}_\ell(\underline{a})}{A(\underline{a})} - \frac{i}{2}\frac{\sum k_i A_i(\underline{a})\sum\xi_\ell \bar{B}_\ell(\underline{a})}{D(\underline{a})\,A(\underline{a})} - 2i\frac{\sum \xi_\ell \tilde{B}_{\ell i}(\underline{a})k_i}{D(\underline{a})}].$$

An n-tree of the graph Γ is a set of lines of Γ with no loops and n components of connectedness. So T_1 is a 1-tree. T_2' is a 2-tree containing the vertices V_x and V_y in different components of connectivity. T_{2i}', resp., T_{2ij}' are 2-trees containing the vertices V_x and V_i, resp. V_x, V_i, and V_j in one component and V_y in the other. With this we have

$$D(\underline{a}) = \sum_{T_1}\prod_{\ell\in T_1} a_\ell\,, \qquad A_i(\underline{a}) = \sum_{T_{2i}'}\prod_{\ell\in T_{2i}'} a_\ell\,, \qquad (A.13)$$
$$A(\underline{a}) = \sum_{T_2'}\prod_{\ell\in T_2'} a_\ell\,, \qquad A_{ij}(\underline{a}) = \sum_{T_{2ij}'}\prod_{\ell\in T_{2ij}'} a_\ell\,.$$

Now we assume that the lines are oriented. T_1^ℓ is a 1-tree containing the line ℓ and $T_1^{\ell,x}$ is a 1-tree in which the line ℓ is directed to the vertex V_x. The sum Σ^x goes over all vertices of $T_1^{\ell,x}$ between V_x and the line ℓ. With this we have

$$\bar{B}_\ell(\underline{a}) = \sum_{T_1^\ell}\prod_{\ell'\bar{\in} T_1^\ell} a_{\ell'}\,, \qquad (A.14)$$
$$\sum_i \tilde{B}_{\ell,i}(\underline{a})k_i = \sum_{T_1^{\ell,x}}\prod_{\ell'\bar{\in} T_1^{\ell,x}} a_\ell\,(\Sigma^x k)\,.$$

Now let T be a 1-tree with loop. Introduce an orientation in the loop and let $\sum \pm\xi_\ell$ be the sum over the lines of the loop where the sign shows the orientation of the line ℓ to the loop. Then

$$k(\underline{a},\underline{\xi}) = \sum_{T_i}\prod_{\ell\bar{\in} T_i} a_\ell\Big(\sum \pm\xi_\ell\Big)^2. \qquad (A.15)$$

Now let's consider the action of the subtraction operator $\mathfrak{M}^a$ on an x-prop subgraph $\gamma\in\mathfrak{L}_c$. With (A.5) we have

$$\mathfrak{M}^a_\gamma F_\Gamma^{un}(\underline{x},\underline{k}) = M_{\sigma_\gamma}^{a_\gamma+\omega_\gamma+s_\gamma}\sigma_\gamma^{s_\gamma}\int\int_r^\infty da_1\ldots\int_r^\infty da_{L_\gamma}\times \qquad (A.16)$$
$$\times\prod_{\ell\in\gamma}P_\ell\Big(\frac{1}{2i}\partial_\xi, a\Big)\Gamma_a^\gamma\Big(\frac{x_\sigma}{\sigma},\xi,\sigma\underline{p}\Big)\mathfrak{E}_\gamma(\underline{a})\Big|_{\xi=0}\times F_{\Gamma/\gamma}^{un}(\underline{k},\underline{p})d\underline{p}\,.$$

Here the first factor is the a-representation of γ and the second factor is the coefficient function of the graph Γ/γ. $\underline{p}$ are the external momenta of γ. From (A.13), (A.14), and (A.15) it follows

$$\Gamma_a^\gamma\Big(\frac{x_{\sigma_\gamma}}{\sigma_\gamma},\frac{\xi}{\gamma},\sigma_\gamma\underline{p}\Big) = \sigma_\gamma^{4\mathcal{R}_\gamma}\Gamma_{\underline{\sigma_\gamma^2 a}}\big(x_{\sigma_\gamma};\underline{\sigma_\gamma\xi},\underline{p}\big)\,.$$

Now the first factor in the r.h.s. of (A.16) has again the form of the a-representation, and we can integrate over $\underline{p}$ to get

$$\mathfrak{M}_\gamma^a F_\Gamma^{un}(x,k) = M_{\sigma_\gamma}^{a_\gamma+\omega_\gamma+s_\gamma}\sigma_\gamma^{4\mathcal{R}_\gamma+s_\gamma}\times \qquad (A.17)$$
$$\times\int_r^\infty da_1\ldots\int_r^\infty da_L\prod_{\ell=1}^{L}P_\ell\Big(\frac{1}{2i}\partial_\xi, a\Big)\Gamma_\beta(x_\pi,\underline{\eta},\underline{k})\,\mathfrak{E}(\underline{a})\Big|$$

with β, π, η given by (A.8), (A.9), (A.10). In the same way we get for a 1PI subgraph γ

$$M_\gamma F_\Gamma^{un}(x,\underline{k}) = M_{\sigma_\gamma}^{a_\gamma+\omega_\gamma+s_\gamma}\sigma_\gamma^{4\mathcal{R}_\gamma+s_\gamma}\times \qquad (A.18)$$
$$\times\int_r^\infty da_1\ldots\int_r^\infty da_L\prod_{\ell=1}^{L}P_\ell\Big(\frac{1}{2i}\partial_\xi, a\Big)\Gamma_{\underline{\beta}}(x,\underline{\eta},\underline{k})\,\mathfrak{E}(\underline{a})\Big|_{\xi=0}$$

with $a_\gamma=0$ ($\gamma\bar{\in}\mathfrak{L}_c$). For latter convenience we have introduced the number s_γ which does not change anything. By repeating this procedure we get a contribution to (A.7). Now let $\mathfrak{F}$ be a forest of x-prop resp. 1PI subgraphs. Then the structure of the R-operation is given by the braces in (A.7) where the sum goes over all forests. Taking into account (A.17) and (A.18) we have shown (A.7) with summation over forests. In ref. /1/ it was shown that this sum is equivalent to the sum over the nests. The proof given there is valid for our case too. So the representation (A.7) is shown.

To analyse the structure of the integral (A.7) we divide the integration domain into Hepp sectors $0\le a_{i_1}\le\ldots\le a_{i_L}$ and consider the contribution from one sector, say $0\le\underline{a}_1\le\underline{a}_2\le\ldots\le\underline{a}_L$. In this sector we introduce new variables

$$t_\ell = a_\ell/a_{\ell+1} \qquad (\ell = 1,2,\ldots,L-1) \qquad (A.19)$$
$$t_L = a_L$$

with the integration domain $0 \le t_\ell \le 1$ $(\ell = 1,2,\dots,L-1)$ and $0 \le t_L$. The Jacobian is $\partial(a_1 \dots a_L)/\partial(t_1 \dots t_L) = \prod_{\ell=1}^{L} t_\ell^{\ell-1}$. The inverse transformation looks like

$$a_\ell = \prod_{k=\ell}^{L} t_k . \tag{A.20}$$

Further we divide the set of nests into classes. Let $P(1),\dots,P(L)$ be a permuttation P of the numbers $1,2,\dots,L$ and let γ_ℓ denote the subgraphs $\gamma_\ell = \{P(1),P(2),\dots,P(\ell)\}$. A subgraph γ_ℓ is called increasing if $P(\ell)>P(\ell+1)$and is called decreasing if $P(\ell)>P(\ell+1)$. We put formally $P(L+1)=L+1$ so that γ_L is increasing for all permutations P. Let $\mathfrak{N}^P$ be the nest $\mathfrak{N}^P=\{\gamma_1,\gamma_2,\dots,\gamma_L\}$ and $\mathfrak{N}^P_{inc}$ resp. $\mathfrak{N}^P_{dec}$ the subnests of $\mathfrak{N}^P$ consisting of increasing, resp., decreasing γ_ℓ only. In ref. /1/ the relation

$$\prod_{\gamma\in\mathfrak{L}} I_{\sigma_\gamma} + \sum_{\mathfrak{N}} \prod_{\gamma\in\mathfrak{N}} (-M_{\sigma_\gamma}) \prod_{\gamma\bar\in\mathfrak{N}} I_{\sigma_\gamma} = \sum_{P} \prod_{\gamma\in\mathfrak{N}^P_{dec}} (-M_{\sigma_\gamma}) \prod_{\gamma\in\mathfrak{N}^P_{inc}} (I_{\sigma_\gamma} - M_{\sigma_\gamma}) \prod_{\gamma\bar\in\mathfrak{N}^P} I_{\sigma_\gamma} \tag{A.21}$$

was shown.

So the contribution from one Hepp sector and one permutation P to (A.7) looks like

$$F_\Gamma^{H,P}(x,k) = C \int_0^1 dt_1 \dots \int_0^1 dt_{L-1} \int_0^\infty dt_L \prod_{\ell=1}^{L} t_\ell^{\ell-1} \times$$

$$\times \prod_{\ell:\gamma_\ell\in\mathfrak{N}^P_{dec}} \sum_{n_\ell=0}^{\omega_\ell+a_\ell+s_\ell} \frac{1}{n_\ell!} \partial^{n_\ell}_{\sigma_\ell} \prod_{\ell:\gamma_\ell\in\mathfrak{N}^P_{inc}} \frac{1}{(\omega_\ell+a_\ell+s_\ell+1)!} \partial^{\omega_\ell+a_\ell+s_\ell+1}_{\sigma_\ell} \times \tag{A.22}$$

$$\times \prod_{\ell=1}^{L} \sigma^{4\mathcal{R}_\ell+s_\ell} P_\ell\left(\frac{1}{2i}\partial_\xi, a\right) \Gamma_{\underline{\beta}}(x_\pi, \underline{\eta}, \underline{k})\, \mathcal{E}(\underline{t}) \Big|_{\substack{\xi=0 \\ \sigma_\ell=0\ (\gamma_\ell\in\mathfrak{N}^P_{dec}) \\ \sigma_\ell=\theta_\ell\ (\gamma\ \in\mathfrak{N}^P_{inc})}} ,$$

where we have taken $\sigma_\gamma = 1$ for $\gamma\bar\in\mathfrak{N}^P$ and denoted $\sigma_{\gamma_\ell} = \sigma_\ell$, $\omega_{\gamma_\ell} = \omega_\ell$, $s_{\gamma_\ell} = s_\ell$ for $\gamma\in\mathfrak{N}^P$. The mass exponential has the form

$$\mathcal{E}(\underline{t}) = \prod_{\ell=1}^{L} \exp[i\, t_L \sum_{k=\ell}^{L-1} t_k (-m^2 + i\epsilon)]$$

and we have employed (2.9) and

$$(I_\sigma - M_\sigma^b) f(\sigma) = \frac{1}{(b+1)!} \partial_\sigma^{b+1} f(\sigma)\big|_{\sigma=\theta} , \quad \theta\in[0,1] .$$

As the next we will show that (A.22) is a sum of terms

$$F_\Gamma^{H,P}(x,k) = C \int_0^1 dt_1 \dots \int_0^1 dt_{L-1} \int_0^\infty dt_L \left(\frac{x^2}{t_{\ell_1} \dots t_L}\right)^a \times$$

$$\times \prod_{\ell=1}^{L} t_\ell^{N_\ell} \exp\left[-\frac{i}{4}\frac{x^2}{t_{\ell_1},\dots,t_L} V_1(\underline{t})\right] u_1(x^2,\underline{t}) \tag{A.23}$$

with $N_\ell \ge 0$ for $\ell = 1,2,\dots,\ell_1-1$ and $N_\ell \ge [\frac{a}{2}]$ for $\ell = \ell_1,\dots,L$ and $a \ge 0$ u_1 is an analytic function in t_ℓ and exponential decreasing for $t_L \to \infty$. V_1 is an analytic function in t_ℓ and does not depend on t_L.

To show (A.23) we introduce auxiliary variables

$$\tilde t_\ell = a_{P(\ell)} / a_{P(\ell+1)} \quad (\ell = 1,2,\dots,L-1)$$

$$\tilde t_L = a_{P(L)} . \tag{A.24}$$

The inverse is

$$a_{P(\ell)} = \prod_{k=\ell}^{L} \tilde t_k . \tag{A.25}$$

Because of

$$\tilde t_\ell = t_{P(\ell)} \dots t_{P(\ell+1)-1} \quad \text{for} \quad \gamma_\ell\in\mathfrak{N}^P_{inc},$$

$$\tilde t_\ell = [t_{P(\ell+1)} \dots t_{P(\ell)-1}]^{-1} \quad \text{for} \quad \gamma_\ell\in\mathfrak{N}^P_{dec} \tag{A.26}$$

$\tilde t_\ell$ is analytic in t_ℓ for $\gamma_\ell\in\mathfrak{N}^P_{inc}$ and only $\tilde t_L$ depends on t_L so that t_ℓ with $\ell < L$ and $\gamma_\ell\in\mathfrak{N}^P_{inc}$ are bounded in the integration domain.

Introduce

$$r_\ell = \tilde t_\ell\, \sigma_\ell^2 , \tag{A.27}$$

it is clear that due to

$$r_\ell = \beta_{P(\ell)} / \beta_{P(\ell+1)} , \quad \beta_{P(\ell)} = \prod_{k=\ell}^{L} r_k \tag{A.28}$$

the combinatorical functions (A.13), (A.14), and (A.15) are polynomials in r_ℓ .

Now we claim that

$$\prod_{\ell=1}^{L} P_\ell\left(\frac{1}{2i}\partial_\xi, \alpha\right)\Gamma_\beta(x_\pi, \eta, k)\Big|_{\xi=0}$$

is a sum of terms

$$\prod_{\ell=1}^{L} \pi_\ell^{\bar n_\ell}\, \alpha_\ell^{\bar n'_\ell}\, r_\ell^{s'_\ell}\, u(x_\pi, r) \exp\left[-\frac{i}{4}\frac{x^2}{\tilde t_{\ell_0}, \dots, \tilde t_L} V_1(r)\right], \qquad (A.29)$$

where u and V_1 are analytic in x^2 and r_ℓ (for nonnegative r_ℓ). ℓ_0 is the minimal number such that γ_{ℓ_0} containing the vertices V_x and V_y in one component of connectivity. $\bar n_\ell$, $\bar n'_\ell$ and s' are numbers with $0 \le \bar n_\ell \le n_\ell$ and $\bar n'_\ell \ge n'_\ell$. To show (A.29), we refer to /1/, where this relation was shown for $\Gamma_\beta(x_\pi, 0, k)$ (The contribution corresponding to the scalar theory). Especially there was shown that V_1 does not depend on r_L. So we consider the contribution from the propagator polynomial $\prod_{\ell=1}^{1} P_\ell(\frac{1}{2i}\partial_\xi, \alpha)$. By definition every term in it contains at least $\bar n'_\ell \ge n'_\ell$ factors α_ℓ and not more than $\bar n_\ell \le n_\ell$ factors ∂_ξ. Every derivative ∂_{ξ_ℓ} produces a factor π_ℓ owing to $\partial_{\xi_\ell} = \pi_\ell \partial_{\eta_\ell}$. The derivatives ∂_{η_ℓ} give factors before the ex-potential containing the combinatorical functions $A(r)$, $A_1(r)$ and so on, which are polynomials in r_ℓ and contribute to the function u and the factors $r_\ell^{s'_\ell}$ *. So we can write (A.22) as a sum of terms

$$F_\Gamma^{H,P}(x,k) = C\int_0^1 dt_1 \dots \int_0^1 dt_{L-1}\int_0^a dt_L \prod_{\ell=1}^{L} t_\ell^{\ell-1} \times$$

$$\times \prod_{\ell:\gamma_\ell\in\mathfrak{N}^P_{dec}} \sum_{n_\ell=0}^{\omega_\ell+a_\ell+s_\ell} \frac{1}{n_\ell!}\partial_{\sigma_\ell}^{n_\ell} \times$$

$$\times \prod_{\ell:\gamma_\ell\in\mathfrak{N}^P_{inc}} \frac{1}{(\omega_\ell + a_\ell + s_\ell + 1)!}\partial_{\sigma_\ell}^{\omega_\ell+a_\ell+s_\ell+1} \times \qquad (A.30)$$

$$\times \prod_{\ell=1}^{L} \sigma_\ell^{4\mathcal{R}_\ell+s_\ell}\, \pi_\ell^{\bar n_\ell}\, \alpha_\ell^{\bar n'_\ell}\, r_\ell^{s'_\ell}\, u(x_\pi, r) \times$$

$$\times \exp\left[-\frac{i}{4}\frac{x^2}{\tilde t_{\ell_0}\dots\tilde t_L}V_1(r)\right]\mathcal{E}(t)\Big| \begin{array}{l}\sigma_\ell = 0 \quad \text{for } \gamma_\ell \in \mathfrak{N}^P_{dec} \\ \sigma_\ell = \theta_\ell \quad \text{for } \gamma_\ell \in \mathfrak{N}^P_{inc}\end{array}$$

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* The numbers s'_ℓ can be negative.

Substituting $\sigma_\ell = r_\ell^{1/2}\,\tilde t_\ell^{-1/2}$ and taking into account

$$\prod_{\ell=1}^{L} \pi_\ell^{\bar n_\ell} = \prod_{k=1}^{L} \sigma_k^{\sum_{\ell=1}^{k} \bar n_{P(\ell)}},$$

$$\prod_{\ell=1}^{L} \alpha_\ell^{\bar n'_\ell} = \prod_{\ell=1}^{L} \tilde t_\ell^{\sum_{k=1}^{\ell} \bar n'_{P(k)}},$$

$$\prod_{\ell=1}^{L} t_\ell^{\ell-1} = \prod_{\ell=1}^{L} \tilde t_\ell^{\ell-1},$$

we have

$$\prod_{\ell=1}^{L} t_\ell^{\ell-1}\sigma_\ell^{4\mathcal{R}_\ell+s_\ell}\pi_\ell^{\bar n_\ell}\alpha_\ell^{\bar n'_\ell} r_\ell^{s'_\ell} =$$

$$= \prod_{\ell=1}^{L} \tilde t_\ell^{-2\mathcal{R}_\ell+\ell-1-\frac{1}{2}s_\ell-\frac{1}{2}\sum_{k=1}^{\ell}(\bar n_{P(K)}-2\bar n'_{P(k)})} \times \qquad (A.31)$$

$$\times r_\ell^{2\mathcal{R}_\ell+\frac{1}{2}s_\ell+s'_\ell+\sum_{k=1}^{\ell}\bar n_{P(k)}}.$$

Remembering that s_ℓ is not bounded from above we require $s_\ell \ge -2s'_\ell$. Further $x_\sigma = f(x^2, \sigma^2)$ (2.10) is analytic in $x^2\sigma^2$ and $\pi = \prod_{k=\ell_0}\sigma_k$ so that

$$x_\pi = f\left(\frac{x^2}{\tilde t_{\ell_0}\dots\tilde t_L}\cdot r_{\ell_0}\dots r_L\right).$$

With this we can rewrite (A.30) as

$$F_\Gamma^{H,P}(x,k) = \int_0^1 dt_1 \dots \int_0^1 dt_{L-1}\int_0^\infty dt_L \prod_{\ell:\gamma_\ell\in\mathfrak{N}^P_{dec}} \sum_{n_\ell=0}^{\omega_\ell+a_\ell+s_\ell} \frac{1}{n_\ell!}\partial_\sigma^{n_\ell} \times$$

$$\times \prod_{\ell:\gamma_\ell\in\mathfrak{N}^P_{inc}} \frac{1}{(\omega_\ell+a_\ell+s_\ell+1)!}\partial_\sigma^{\omega_\ell+a_\ell+s_\ell+1} \times \qquad (A.32)$$

$$\times \prod_{\ell=1}^{L} \tilde t^{-2\mathcal{R}_\ell+\ell-1-\frac{1}{2}s_\ell-\frac{1}{2}\sum_{k=1}^{\ell}(\bar n_{P(k)}-2\bar n'_{P(k)})} \times$$

$$\times u_2\left(\frac{x^2}{\tilde t_{\ell_0}\dots\tilde t_L}, r\right) e^{-\frac{i}{4}\frac{x^2}{\tilde t_{\ell_0}\dots\tilde t_L}V_1(r)}$$

where $u(\frac{x^2}{\tilde{t}_{\ell_0}\dots\tilde{t}_L}, \underline{r})$ is analytic in all its arguments (we have included here the mass exponential) and exponentially decreasing for $t_L \to \infty$.

Every derivative with respect to σ_ℓ does not change this properties and gives a factor $\tilde{t}_\ell^{1/2}$ and possibly a factor $\frac{x}{\tilde{t}_{\ell_0}\dots\tilde{t}_L}$. Taking the derivatives with respect to σ_ℓ we get a sum of terms

$$\int_0^1 dt_1 \dots \int_0^1 dt_{L-1} \int_0^\infty dt_L \prod_{\ell=1}^{L} \tilde{t}_\ell^{\,-1+\frac{a_\ell}{2}+\frac{1}{2}\sum_{k=1} [n_{P(k)}-\bar{n}_{P(k)}-2(n'_{P(k)}-\bar{n}'_{P(k)})]+B_\ell}$$

$$\times \left(\frac{x^2}{\tilde{t}_{\ell_0}\dots\tilde{t}_L}\right)^a u(x^2,\underline{t}) \exp\left[-\frac{i}{4}\frac{x^2}{\tilde{t}_{\ell_0}\dots\tilde{t}_L} V_1(\underline{t})\right] \qquad \text{(A.33)}$$

with $B_\ell = \frac{1}{2}$ for $\gamma_\ell \in \mathfrak{N}^p_{inc}$ and $B_\ell \le 0$ for $\gamma_\ell \in \mathfrak{N}^p_{dec}$ and $a \ge 0$.

One gets the function $V_1(\underline{t})$ from $V_1(\underline{r})$ putting $r_\ell = 0$ for $\gamma_\ell \in \mathfrak{N}^P_{dec}$ and $r_\ell = \theta_\ell$ for $\gamma_\ell \in \mathfrak{N}^P_{inc}$. So it depends on $\tilde{t}_\ell$ for $\gamma_\ell \in \mathfrak{N}^P_{inc}$ only and due to (A.26) it is analytic in t_ℓ. The function u contains all factors occurring from the differentiations and is analytic in x^2 and t_ℓ and exponentially decreasing for $t_L \to \infty$. With

$$\prod_{\ell=1}^{L} \tilde{t}_\ell^{\sum_{k=1}^{\ell} m_{P(k)}} = \prod_{\ell=1}^{L} t_\ell^{\sum_{k=1}^{\ell} m_k}, \quad n_\ell \ge \bar{n}_\ell, \quad n'_\ell \le \bar{n}'_\ell$$

the factor

$$\prod_{\ell=1}^{L} \tilde{t}_L^{\,\frac{1}{2}\sum_{k=1}^{\ell} [n_{P(n)}-n_{P(n)}-2(n'_{P(n)}-n'_{P(n)})]}$$

gives positive powers of t_ℓ only. Further

$$\prod_{\ell=1}^{L} \tilde{t}_\ell^{\,-\frac{1}{2}+B_\ell} = \prod_{\ell:\gamma_\ell \in \mathfrak{N}^P_{dec}} \tilde{t}_\ell^{\,B_\ell}$$

gives positive powers of t_ℓ only. With $\prod_{\ell=1}^{L} t_\ell^{-\frac{1}{2}+\frac{1}{2}a_\ell} = \prod_{\ell=1}^{L} t_\ell^{-\frac{1}{2}} \prod_{\ell=\ell_1}^{L} t_\ell^{\frac{1}{2}a}$ $(\ell_1 = P(\ell_0))$ and $\tilde{t}_{\ell_0}\dots\tilde{t}_L = t_{\ell_1}\dots t_L$ we get (A.23) with

$$N_\ell \ge -\frac{1}{2} \qquad \ell = 1,2,\dots,\ell_1-1$$

$$N_\ell \ge \frac{a}{2} \qquad \ell = \ell_1,\dots,L.$$

Now we argue that only integer powers of t_ℓ enter into the integrand of (A.33) as well as (A.23). Therefore if some N_ℓ is not integer, that would mean that u contains the factor $t_\ell^{1/2}$. We obtain finally

$$N_\ell \ge 0 \quad \text{for} \quad \ell = 1,2,\dots,\ell_1-1$$

$$N_\ell \ge [\tfrac{a}{2}] \quad \text{for} \quad \ell = \ell_1,\dots,L$$

so that (A.23) is proved.

With this the proof of (A.6) is nearly finished, because we have obtained with (A.23) the same representation for the coefficient function $F_\Gamma(x,k)$ as in the scalar theory. Even in ref. /1/ it was shown that (A.23) has for $x^2 \to 0$ the desired behaviour (A.6).

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Received by Publishing Department
on January 22 1982.

Бордаг М., Кашлун Л. E2-82-52

О конусном разложении в калибровочных теориях /КЭД/

Дается обобщение развитого в скалярной теории формализма для вывода разложения на световом конусе в нескалярной, в частности, в калибровочной теории. При этом малость остаточного члена доказывается для произвольной ренормируемой теории с условием наличия инфракрасной регуляризации. Исследуется вид разложения и возникающие в нем конусные операторы. Оказывается, что разложение в теориях с векторными полями /калибровочные теории/ содержит все степени оператора векторного поля. При этом коэффициентные функции не являются независимыми вследствие калибровочной инвариантности. Показано, что использование известных решений тождеств Уорда позволяет вывести разложение на световом конусе с конечным числом операторов и с независимыми коэффициентными функциями. При этом разложение оказывается калибровочно инвариантным.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1982

Bordag M., Kaschluhn L. E2-82-52

On the Light-Cone Expansion in Gauge Field Theories /QED/

The techniques developed for derivation of the light-cone expansion in scalar theories for its application to nonscalar, especially gauge theories are generalized. For this reason the smallness of the remainder is proved in an arbitrary renormalizable theory, provided an infrared regularization is present. The structure of the expansion and the arising light-cone operators are derived. It turns out, that the light-cone expansion in theories involving vector-fields (gauge theories) contains all powers of the vector-field operator. In this case the coefficient functions are not independent due to the gauge invariance. It is shown that with the help of the known solutions of the Ward-identities one can get a light-cone expansion with a finite number of operators and with independent coefficient functions. This expansion is gauge invariant.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1982