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SOLITONS IN N=4 EXTENDED SUPERGRAVITY

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The extended supergravity [1] is believed to be a theory unifying all known interactions and particles. "If the N=8 supergravity theory is to describe Nature, then the list of elementary fields of the theory must have only an indirect relation to the elementary particle spectrum as we perceive it at the very low energies"...: noted by M.Gell-Mann in his "Closing remarks at the Jerusalem Einstein centennial symposium"[2]. Then ..."all or most of the familiar particles would have to correspond to particle-like solutions of the fundamental equations, with a different algebraic behaviour from that of the fundamental fields"... in[3].

The classical equations of supergravity are in fact nonlinear and may possess a great number of possible soliton-like solutions. We discuss here soliton solutions in the N=4 supergravity.

The overall Lagrangian of the SU(4) invariant supergravity was presented in the work of Cremmer, Ferrara and Scherk[4] and contains the following fields:

V_μ^a one graviton,

ψ_μ^i four Majorana spin 3/2 fermions,

A_μ^n three vector fields,

B_μ^n three axial-vector fields,

χ^i four spin 1/2 fermions,

ϕ one scalar field,

B one pseudo-scalar field.

The action of the system was shown in[4] to be invariant under the global SU(4) group. Moreover, equations of motion are in addition SU(1,1) invariant so that the total group of symmetry is SU(4) $\otimes$ SU(1,1).

We shall search for solutions of the classical equations of motion setting the gauge

$$\chi^i = \psi_\mu^i = 0 \quad , \quad A_\mu^n = B_\mu^n = 0 \quad . \tag{1}$$

Then the total system of equations splits into the Einstein gravity equations and the system for ϕ and B fields. The kinetic term in Lagrangian density for ϕ and B is non-polinomial with respect to ϕ field

$$\mathcal{L}_1 = \frac{1}{2} V g^{\mu\nu} \left[\partial_\mu \phi \, \partial_\nu \phi + \exp(4k\phi) \, \partial_\mu B \, \partial_\nu B \right] . \tag{2}$$

Defining a complex function $\mathcal{E} = \exp(-2k\phi) - 2ikB$ one can rewrite (2) in the form:

$$\mathcal{L}_1 = \frac{V}{2} \frac{\partial_\mu \mathcal{E} \, \partial^\mu \bar{\mathcal{E}}}{(\mathcal{E} + \bar{\mathcal{E}})^2} = \frac{V}{2} \frac{\partial_\mu \xi \, \partial^\mu \bar{\xi}}{(1 - \xi \bar{\xi})^2} , \quad \mathcal{E} = \frac{1 + \xi}{1 - \xi} . \tag{3}$$

The Lagrangian is invariant under pseudounitary group SU(1,1)

$$\mathcal{E} \to \frac{\alpha \mathcal{E} + i\beta}{i\gamma \mathcal{E} + \delta} , \quad \alpha\delta + \beta\gamma = 1 . \tag{4}$$

Let us write it in the form of the O(2,1) σ-model. We use for that hyperbolic analog of stereographic projection [5]:

$$\xi = \frac{i n_1 - n_2}{1 + n_0} , \tag{5}$$

where the real vector n_a (a=0,1,2) takes its values on a hyperboloid and is unity normalized

$$\eta^{ab} n_a n_b = 1 , \qquad \eta_{ab} = \text{diag}\,(1,-1,-1).$$

The Lagrangian (3) is then rendered into the nonlinear σ-model one

$$\mathcal{L} = -\frac{V}{8} \eta^{ab} \partial_\mu n_a \partial^\mu n_b . \tag{6}$$

By use of the four dimensional O(2,1) σ-model solutions one can consider their effect on the plane space-time metric and other fields. It is enough for that to perform a global supersymmetric rotation of all fields. Here we would however like to stress more interesting possibility when the system considered is axially symmetric, i.e., the fields depend only on $\rho = \sqrt{x^2 + y^2}$ and z. In this case Lagrangian (3) is simply the Ernst one [6] for stationary vacuum Einstein equations with axial symmetry. The Ernst equations

$$(\mathrm{Re}\,\mathcal{E})\, \nabla^2 \mathcal{E} = \vec{\nabla}\mathcal{E} \cdot \vec{\nabla}\mathcal{E}$$

(with $\vec{\nabla}$ being the three-dimensional differential operator in cylindric coordinates) describe the complete integrable system [7] and the Bäcklund transormations [8], infinite set of conservation laws and N-

soliton solutions [9] (via the inverse transform) have been obtained.

The simplest solutions to the Ernst equations are:

$$\xi = -e^{-i\alpha} \operatorname{cth} \Psi , \tag{7}$$

where real function Ψ satisfies the Laplace equation $\nabla^2 \Psi = 0$. There are the Weyl solution ($\alpha = 0$), and that of Papapetrou ($\alpha = \frac{\pi}{2}$) among them.

The Kerr solution is

$$\xi = p\zeta - iq\eta , \tag{8}$$

where p and q are free parameters obeying the condition $p^2 + q^2 = 1$ and ζ, η are spheroidal coordinates related to Cartesian ones by

$$x = c\sqrt{\zeta^2 - 1}\sqrt{1 - \eta^2} \, \sin\vartheta ,$$
$$y = c\sqrt{\zeta^2 - 1}\sqrt{1 - \eta^2} \, \cos\vartheta ,$$
$$z = c\zeta\eta$$

When p=1, q=0 solution (8) comes to that of Schwarzschild. Infinitezimal local superrotation of these solutions gives:

$$\delta \bar{\chi}^i = i/\sqrt{2}\, \bar{\epsilon}^i (\partial_a \phi + i\gamma_5 \exp(2k\phi) \partial_a B) \gamma^a ,$$
$$\delta \psi^i_a = 1/k\, \bar{\epsilon}^i \overleftarrow{\partial}_a - i/2 \exp(2k\phi) \bar{\epsilon}^i \gamma_5 \partial_a B ,$$
$$\delta \phi = \delta B = \delta A^n_\mu = \delta B^n_\mu = \delta V_{a\mu} = 0 .$$

Thus we get a solution of linearized equations for fermion fields as a function of ϕ_0 and B_0 (fermion zero modes). Using a finite global superrotation one can construct corresponding solution for the full set of fields in terms of $\mathcal{E}$.

Putting $\phi = B = 0$ we get the gravity field equations that are also reduced to the Ernst equations. The analogous technique allows us to calculate corrections to the curved space-time metric (in the case of simple supergravity such a procedure has been performed in [10] using Schwarzschild solution). Detailed results are supposed to be published elsewhere.

The Ernst system considered may also be deduced from self-duality equations for SU(2) gauge potentials

$$F^a_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F^a_{\rho\sigma}$$

in "stationary" state case $\partial/\partial x_4 = 0$. Analogous result may be obtained even when $\partial/\partial x_4 \neq 0$ as well. In this case we have a generalized (four dimensional euclidean) Ernst equations and the axial symmetry condition now reads

$$\bar{\eta}^{3}_{\mu\nu}(\partial_\mu \mathrm{Im}\,\mathcal{E})(\partial_\nu \mathrm{Re}\,\mathcal{E}) = 0,$$

$$(\mathrm{Re}\,\mathcal{E})\partial_\mu^2 \mathcal{E} = (\partial_\mu \mathcal{E})^2 .$$

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Солитоны в N=4 расширенной супергравитации

Рассматривается SU(4) расширенная супергравитация. Показано, что классические уравнения движения для скалярного Φ и псевдоскалярного B полей эквивалентны O(2,1) нелинейной четырехмерной σ-модели. В случае аксиальной симметрии стационарные уравнения движения сводятся к уравнению Эрнста, так же как и для чистой гравитации. Уравнения Эрнста интегрируются методом обратной задачи. Использование суперповорота позволяет получить солитонные решения для полной супергравитации.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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Makhankov V.G., Pashaev O.K. E2-82-506
Solitons in N=4 Extended Supergravity

SU(4) extended supergravity is considered. The classical equations of motion for scalar Φ and pseudoscalar B fields are shown to be equivalent to the O(2,1) nonlinear four-dimensional σ-model. The axially symmetric stationary equations of motion are reduced to the Ernst equations as well as that for pure gravitation. The Ernst equations are integrable via the inverse transform technique. Soliton solutions for full supergravity may be obtained by means of superrotation.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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