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WILSON LINE DISTRIBUTIONS IN HOT SU(2) GLUODYNAMICS: MONTE-CARLO RESULTS AND INSTANTON GAS ESTIMATES

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1. A remarkable breakthrough has been achieved in the last two years in the numerical evaluation of nonperturbative properties of gluodynamics by putting this theory on a lattice [1]. Numerical values have been obtained for the string tension [2], the gluon condensate [3], the lightest glueball [4] and some other meson masses [5], the topological susceptibility [6], etc., with a surprising degree of consistency. Still, one has to admit that the understanding of the structure of the vacuum is poor as yet. The activity of monopoles and vortices [7] defined in terms of the center of the group, appears intrinsically related to the weak-to-strong coupling crossover [8], but bears no direct relevance to the continuum limit. Exploiting the tool of Monte-Carlo simulation in asking good questions should finally help to develop or improve analytical methods for nonperturbative phenomena without recourse to the lattice.

Essentially the only nonperturbative continuum approach practized today, is the quasiclassical approximation to the functional integral, in particular the dilute instanton gas approximation (DGA) [9]. Unfortunately, no instanton amplitudes are available with higher loop corrections under control, while the one loop amplitude explodes badly at large sizes. Dealing with more general configurations in the partition function meets severe difficulties. There are arguments, however, that the correct treatment of collective coordinates for not infinitely dilute gases gives rise to a hard core, stabilizing the instanton size scale [10]. Moreover, in this version the instanton gas does not contradict anymore the low energy theorems [11]. The resulting model is able to describe the static quark-antiquark force at distances below the confinement scale [12]. For finite temperature studies it has become customary to take the naive instanton gas approximation for granted [13]. It is argued that high enough temperature sets the scale for the coupling constant and acts as an external cut-off for the instanton sizes, such that there is nothing to bother about. In this spirit the high temperature instanton gas has been used to discuss the confining transition and to estimate the corresponding temperature T_c [14]. From our above-mentioned point of view (concerning T=0) it seems more natural to define a saturation temperature T_{sat}, at which the zero temperature characteristics of the instanton gas are reached, and to wonder about its relation to be deconfinement and/or other phase transitions.

For this purpose, we have undertaken a study of the SU(2) lattice gauge model at finite temperature, collecting some more information than gathered by previous workers [15]. Monte-Carlo simulation works on a finite $N_s^3 \times N_t$ lattice, usually with periodic boundary conditions in all directions. Temperature $T \neq 0$ is imposed by taking $N_t = \beta/a = 1/aT \ll N_s$. A particular gauge invariant Wilson loop operator, closed by periodicity, is the Wilson line

$$\Omega(\vec{x}) = \frac{1}{N} \mathrm{tr} \prod_{\tau=1}^{N_t} U_4(\vec{x}, \tau) = \cos\Phi(\vec{x}) . \qquad (1)$$

Its expectation value serves as order parameter of the deconfining transition and is understood as

$$L = \langle \Omega(\vec{x}) \rangle = \exp[-\beta F_Q(\beta)] \qquad (2)$$

in terms of the change of the system's free energy due to the static source. The observation of $L \to 0$ as T approaches T_c from above has been evidence for onset of confinement [15], usually quoted at $T_c = (0.35 \pm 0.05)\sqrt{\sigma(0)}$ ($\sigma(0)$ zero temperature string tension). In the confining phase $L = 0$, i.e., the inherent global Z_2 symmetry is dynamically realized, while it is spontaneously broken in the high temperature phase. More information than just the average Wilson line L, concerning the mechanisms trying to restore the symmetry in the hot phase, is simply obtained by recording the distribution of Ω values over volume and MC time. In particular, we intend to isolate in this way the instanton effect.

2. The order parameter $\Omega(\vec{x})$ defines a projection of the full four-dimensional theory onto three-space. The image of a finite temperature periodic instanton [16]

$$A_\mu^a = \frac{1}{g} \bar{\eta}^a_{\mu\nu} \partial_\nu \ln\phi , \quad \phi = 1 + \sum_{n=-\infty}^{\infty} \frac{\rho^2}{(x - n\beta \hat{e}_4)^2} \qquad (3)$$

under this mapping is simple: the Wilson line in the field of the instanton appears as an island of negative Ω in a positive background or vice versa. We denote it $\Omega_1(r, \rho, T) = \cos I(r, \rho, T)$ ($r = |\vec{x} - \vec{x}_{inst}|$ is the distance from the center of the instanton; ρ, the instanton size). The naive instanton gas expression for the average Wilson line is

$$L = \exp[2 \int \frac{d\rho}{\rho^5} D(\rho, T) d^3 x_{inst} (\cos I - 1)] , \qquad (4)$$

and the Wilson line phase distribution is

$$W(\Phi) = 1 + 2 \sum_{m=1}^{\infty} \cos(m\Phi) \times \exp[2 \int \frac{d\rho}{\rho^5} D(\rho, T) d^3 x_{inst} (\cos(mI) - 1)] . \qquad (5)$$

A characteristic feature of the instanton picture is the very rapid onset of effects with falling temperature until saturation takes over. The dampling of larger instantons due to the $T \neq 0$ amplitude [13]

$$D(\rho, T) = D(\rho, 0)\exp\left(-\frac{2N}{3}(\pi\rho T)^2 + \dots\right) \qquad (6)$$

is gradually lifted, and the critical space-time fraction $f \approx 3\%$, known from the comparison with the lattice $Q\bar{Q}$ force, is reached around $T/\Lambda_{PV} \approx 1$. Since roughly $L \approx 1-4f$, the contribution of instantons to the discordering of the Wilson line cannot be important. We are more interested to identify the saturation temperature T_{sat} with respect to the deconfinement temperature.

3. Our Monte-Carlo data were taken running the SU(2) heat bath iteration program of Ardill and Moriarty [17] at the EC 1060 computer of JINR. The lattice sizes were 8^3x2, 8^3x3 and 8^3x4. (We have checked that no essential effects of the final volume were present). Typically, sweeps through the lattice have been iterated 100 ÷ 200 times for each value of coupling $B = 4/g_0^2$. Because we were interested in a relatively fine-binned Wilson line distribution we could not work with a finite subgroup program. In fig.1 we show the order parameter as function of temperature, where the two-loop relationship

$$T/\Lambda_L = 1/N_t a\Lambda_L = \frac{1}{N_t}(\gamma_0 g_0^2)^{\gamma_1/2\gamma_0^2}\exp\frac{1}{2\gamma_0 g_0^2}. \qquad (7)$$

has been used throughout, although not fully justified for $N_t = 2$ near the transition. It is not very safe to determine T_c from the vanishing of L. In the transition region our 8^3x3 data are averages over $|L|$ excluding tunnelings. For comparison, the instanton prediction is shown. The temperature

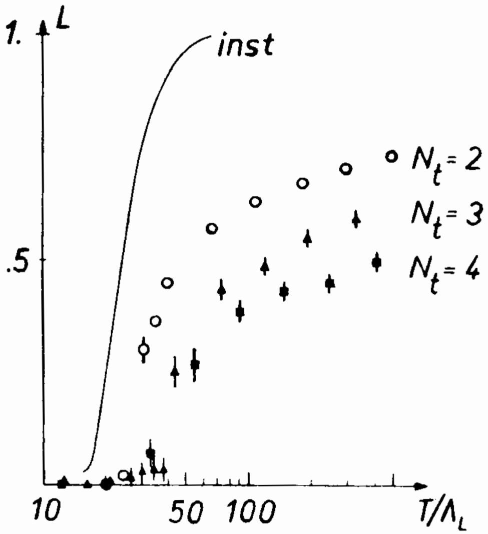


Fig.1. Average Wilson line as function of temperature, and the instanton gas prediction (eq. (4)).

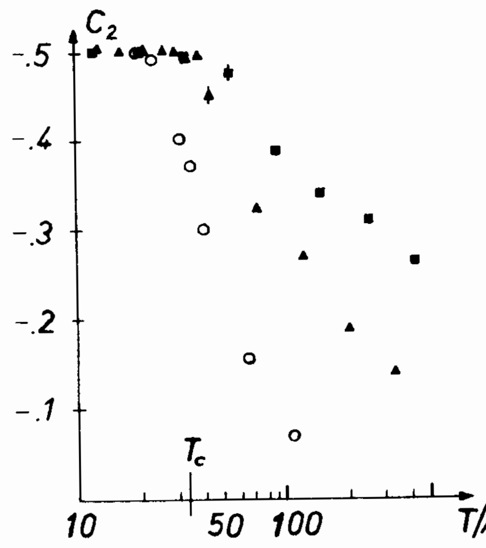


Fig.2. $C_2 = \langle\cos 2\Phi\rangle$ as function of temperature. Data point symbols as Fig.1.

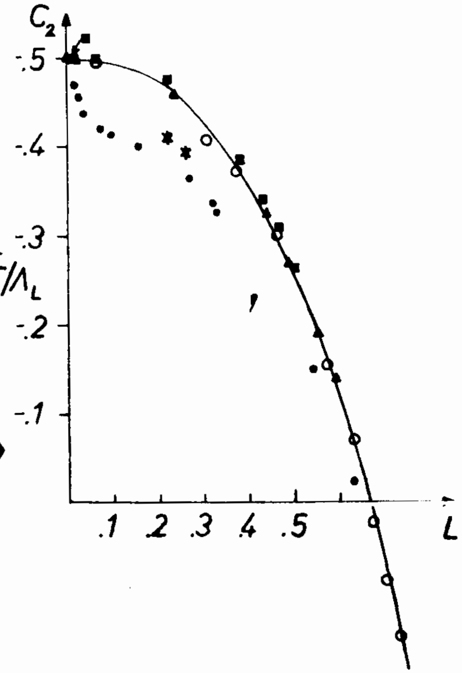


Fig.3. C_2 versus L for all temperatures and lattice sizes. Data point symbols as Fig.1. Full circles ● refer to the effective model of Ref. [19]; stars *.to an improved model with additional off-axis next neighbour coupling.

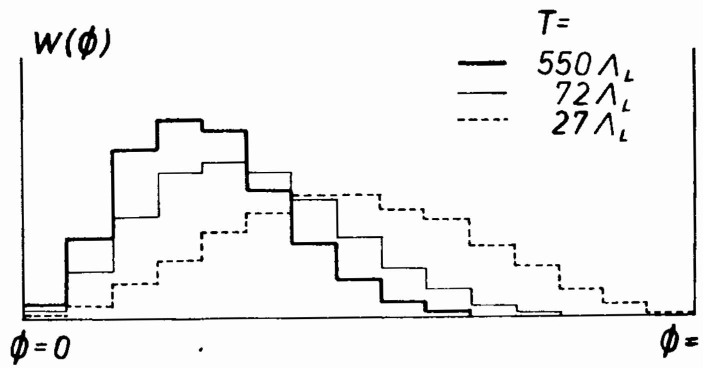


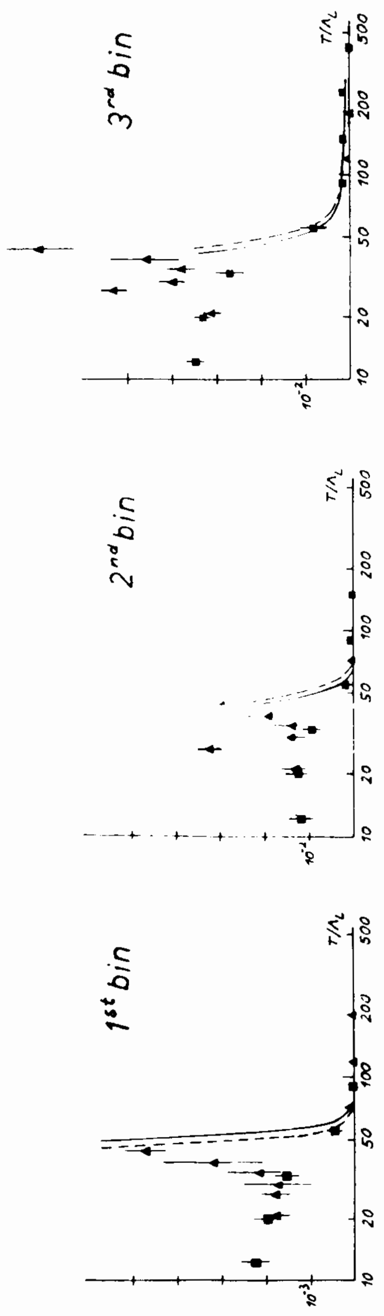
Fig.4. Distribution of eigenvalues, $W(\Phi)$, at three temperatures for the SU(2) model studied on a 8^3x3 lattice.

characterizing the sudden onset of instanton bubbles is fixed here taking the standard value $\Lambda_{PV}/\Lambda_L = 21.55$ [18] into account. From this figure it should also be clear that the order parameter L or its logarithm $\beta F_Q(\beta)$ shows no well-defined continuum limit at given temperature. Plotting $\beta F_Q(\beta)$ versus β, however, one can see a minimal slope between $T=80\,\Lambda_L$ and $200\,\Lambda_L$, corresponding to an internal energy of $U_Q=(25\pm5)\Lambda_L$, in coincidence for $N_t=2\div4$.

A clearer signal for the onest of the new, confining phase can be obtained by measuring the Fourier coefficient of the phase distribution $C_2=\langle\cos(2\Phi)\rangle$ depending on the temperature. At T_c the value $C_2=-0.5$ is reached, which does not change anymore at lower temperature. Disregarding again the $N_t=2$ points, we find $T_c=(34\pm2.5)\Lambda_L$ (see Fig.2). Remarkably enough, data for all our N_t happen to fall onto a universal curve shown in Fig.3. This outstanding feature served to discriminate the behaviour of the order parameter and its fluctuations from that of the effective theory obtained in Ref. [19] within the strong coupling approximation.

We show in Fig.4 the distribution of phase for some temperature values, measured on the $8^3\times3$ lattice. The peak moves slowly over a huge temperature interval until finally a stable, fully symmetric distribution of constant width is reached. This distribution does not directly allow to infer the effective potential in [23] terms of the order parameter, since the symmetry is explicitly broken by the Monte-Carlo procedure starting from ordered configurations and because it is influenced by the "kinetic" part of the effective action. This unknown coupling proved essential in Fig.3.

In contrast to the average Wilson line, the population of the deepest bins of the phase distribution shows a dramatic temperature effect in the same range of $T\approx50\,\Lambda_L$. In Fig.5 we show the content of the five bins in the region $120°<\Phi<180°$, as measured on the $8^3\times3$ and $8^3\times4$ lattices, in dependence on the temperature. If we assume that the distribution for phases in this region reflects that part of three-space influenced by the innermost core of the instantons (with $I(r,\rho,T)>2\pi/3$), we are able to understand the sudden rise. This amounts, of course, to an "experimental" determination of the ratio of the respective Λ parameters figuring up in our (one-loop Pauli-Villars) instanton density and in our Monte-Carlo measurement of the deconfinement temperature. Notice, however, the weakly temperature dependent background already present in the 4th and 5th bins. The instanton thresholds are drawn into these plots in correspondence with an effective ratio $\Lambda_{PV}/\Lambda_L=42\pm3$. This value differs from the value quoted above, which has been determined perturbatively at one-loop level, by a factor of two. This should not be considered too embarrasing since the instan-



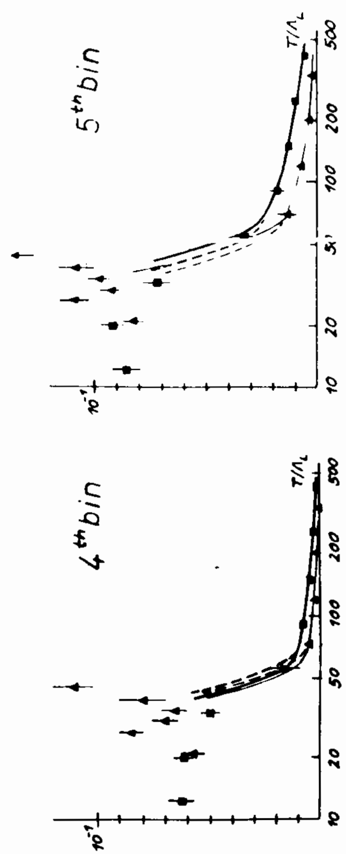


Fig.5. Content of five bins in the region $180°>\Phi>120°$, and the instanton gas prediction for $R=\Lambda_{PV}/\Lambda_L=42$ (full line). The broken line corresponds to $\delta R/R = 8\%$. Data point symbols as Fig.1.

ton amplitude does not account for higher loop effects, and in view of the fact that the extraction of nonperturbative effects out of Monte-Carlo data often meets some problems in recovering "theoretical" values [20].

The extraction of the string tension, its temperature dependence, and of the interquark force above T_c, which follows the line of arguments as explained on case of the internal energy associated with a single static quark, is in progress and will be discussed elsewhere.

4. We will now discuss some implications in connection with the instanton gas model and with the physical interpretation it offers for the threshold temperature detected in the Wilson line distribution. The DGA does not allow to estimate or even describe the onset of confinement. This is in accord with the study of the static $Q\bar{Q}$ force at $T=0$ [12], measured on the lattice, which can be described by the instanton gas at intermediate distances (up to 0.3 fermi) while the gas gives an almost realistic value for the gluon condensate. If we fix for the high temperature instanton gas T_{sat} and Λ_{PV} such that the SU(2) gas has just the maximum packing fraction $f=2.5\%$ [12] and accounts for the gluon condensate $\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} \rangle = (2/3)\cdot 0.012\ \mathrm{GeV}^4$, we get a Paulo-Villars $\Lambda_{PV}=190$ MeV while $T_{sat}=1.2\Lambda_{PV}=228$ MeV. These values are correct within some 20% because of the uncertainty in the SU(2) condensate itself and the degree of saturation by instantons. The study of the Wilson line distribution has provided two characteristic temperatures, T_c and T_{sat} with a well defined ratio $T_{sat}/T_c = 1.47 \pm 0.23$. Accepting the above numbers in physical units, we get $\Lambda_L=(4.6 \pm 1.2)$ MeV and $T_c = (155 \pm 55)$ MeV. Apart from these somewhat academic values, the ratio of the two transition temperatures agrees well with that found by Kogut et al. [21] in a Monte-Carlo study of the quark condensate. This can be understood as a confirmation of Shuryak's scenario [22] of two hadronic phase transitions, and the role instantons play in breaking chiral symmetry. It is interesting to notice, that a signal indicating the chiral transition could be found studying pure gluodynamics. What kind of change in the vacuum structure is responsible for the onset of confinement, but does not change essentially its local properties like the gluon condensate? The answer remains unknown, so far.

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Ильгенфриц Э.-М., Крипфганц Й. Распределения по вильсоновской струне в "горячей" SU(2) глюодинамике: результаты по методу Монте-Карло и инстантонные оценки E2-82-481

Обсуждаются результаты, полученные по методу Монте-Карло, касающиеся параметра порядка деконфайнмента и его флюктуации, в рамках приближения разреженного инстантонного газа. Найден пороговый эффект в распределении вильсоновской струны, объясняемый активностью инстантонов. На основе этого фиксировано отношение между температурами насыщения инстантонного газа и восстановления Z_2 симметрии. Приведены также и другие результаты численного счета.

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Ilgenfritz E.-M., Kripfganz J. Wilson Line Distributions in Hot SU(2) Gluodynamics: Monte-Carlo Results and Instanton Gas Estimates E2-82-481

We discuss Monte-Carlo results concerning the order parameter related to confinement and its distribution from the point of view of the dilute instanton gas approximation. We identify a threshold effect in the Wilson line distribution with the onset of instantons and establish the relation between the temperature of instanton saturation and the confinement temperature. Some other details of our Monte-Carlo results are reported as well.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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