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THE STATIC $Q\bar{Q}$ FORCE FROM INSTANTON GAS AND NUMERICAL LATTICE CALCULATIONS

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1. Introduction

During the last two years it became evident, that lattice QCD Monte-Carlo studies are able to produce many nonperturbative, phenomenologically relevant numbers with a reasonable degree of consistency: string tension, gluon condensate, meson masses, etc.[1]. However, our understanding of the underlying dynamical mechanisms, in particular of the vacuum structure, remained still poor. Actually, phase transitions and narrow weak-to-strong coupling crossovers observed in different lattice models in dependence on the bare coupling g_0 (or on the corresponding lattice scale a) could be related to the condensation of topological objects on the scale of a lattice unit (monopoles, vortices)[2]. However, in order to comprehend the interpolation between the perturbative (Coulomb-like) and the confinement region characterized by physical length scales one would like to identify excitations of arbitrary large size (in lattice units) which would survive in the continuum limit.

Such configurations are well-known from the quasiclassical continuum approach: instantons and their "dilute gas configurations"[3]. Their typical scale size - till now introduced either by hand via an infrared cutoff ρ_c in the most naive way or by repulsive hard core interactions[4] - should be viewed as a physical quantity to be extracted from phenomenological information. Callan, Dashen and Gross (CDG) were the first, who tried to relate the quasiclassical approach to the lattice theory proposing the determination of an effective Wilson-type lattice action[5]. In discussing instanton effects they concentrated on the weak-to-strong coupling crossover and studied this in terms of the β function. The strong coupling branch of the latter was taken from the large order high temperature expansion for the string tension $\sigma(a,g_0)$ required to be independent on the lattice spacing a [6]. Viewing the Yang-Mills vacuum as a polarizable medium CDG claimed the instanton β function to be driven by the permeability of the gas of instantons, the maximum scale of which is set by the lattice constant($\rho_c \simeq a$). Really CDG found a departure from the perturbative behaviour at $g_0 \simeq 1$ as indicated also by the strong coupling expansion. However, the instanton curve did not smoothly interpolate between the weak and the strong coupling branch, rather overshooting the latter. Thus, one had to rely on additional mechanisms coming into operation while the instanton gas were yet very dilute (the fraction of space-time occupied by instantons being less than 1%)[4,7].

In a recent paper[8] we could overcome this difficulty calculating the β function directly by renormalizing the triple-gluon-vertex in a momentum subtraction (MOM) scheme and taking the leading instanton gas corrections into account. Here, the inverse scale size ρ_c^{-1} plays the role of a fixed, mass-like parameter. The resulting curve interpolates satisfactorily between the perturbative and strong coupling branches if the space-time packing fraction takes values from 1 to 5%. (This is a region, where dipole-like interactions between instantons and antiinstantons remarkably influence the results). However, the chosen MOM scheme bears its own problems. First the β function becomes gauge dependent. Secondly, we do not know what the strong coupling branch really looks like in this scheme. Therefore a more restrictive estimate of the packing fraction and of ρ_c , resp. has remained impossible.

In this paper we shall do better. We study the "Coulomb-confinement transition" in terms of the interaction force in between of an infinitely heavy quark-antiquark pair. It is related to the expectation values of rectangular Wilson loops $W(T,R)$ extending over Euclidean time T and distance R $(R \ll T)$ between the sources by

$$F(R) = -\frac{1}{\Delta T \cdot \Delta R} \ln \frac{W(T,R)\cdot W(T-\Delta T, R-\Delta R)}{W(T-\Delta T,R)\cdot W(T,R-\Delta R)} . \qquad (1)$$

We will calculate this quantity for SU(2) and SU(3) gauge groups within the dilute gas approximation including corrections due to instanton interactions and confront the results with MC data. In this way we estimate the distances up to which the instanton gas reasonably describes the force and find the typical instanton scale size. The static $Q\bar{Q}$ force presented in a Coulomb-like form with an effective coupling by

$$F(R) = \frac{C_F}{4\pi}\,\frac{g_{eff}^2(R)}{R^2} , \qquad C_F = \frac{N_c^2-1}{2N_c} \qquad (2)$$

enables us to specify another, physically motivated β function which avoids gauge dependence and is given in terms of a unique renormalization scheme at both weak and strong coupling.

On the lattice the force has been calculated recently with good statistics by Lang et al.[9] for the SU(2) case. In the transition region they established the independence of the results with respect to the chosen lattice action. It has been shown by Creutz[10] that the SU(2) data at small distances fit well into dependence (2), if the effective coupling is given by the two-loop renormalization group expression ($N_f = 0$ for the pure Yang-Mills case)

$$g_{eff}^2 \longrightarrow g^2(R) = \left(\beta_0 \ln \frac{1}{(R\Lambda_R)^2} + \frac{\beta_1}{\beta_0} \ln \ln \frac{1}{(R\Lambda_R)^2}\right)^{-1} , \tag{3}$$

$$\beta_0 = \frac{1}{16\pi^2} \frac{1}{3}(11 N_c - 2N_f) , \quad \beta_1 = \frac{1}{(16\pi^2)^2} \frac{1}{3}(34 N_c^2 - (10 N_c + 6 C_F) N_f).$$

Creutz found the ratio Λ_R/Λ_L , where Λ_L is related to the renormalization group behaviour of the bare coupling $g_0(a)$ in the given lattice theory, and a recent theoretical estimate[11] agreed. For SU(3) the potential has been investigated[12] on the basis of the few available data points of Creutz[13]. Thus, we decided to take data of our own measuring Wilson loop expectation values with the help of Pietarinen's heat bath procedure[14]. This allowed to improve the fit of the ratio $\Lambda_L/\sqrt{\sigma}$, too.

In the SU(3) case the static force can be compared with what is known from phenomenological, nonrelativistic quarkonium potentials[15]. Our lattice data deviate significantly from the phenomenological curves for $R \lesssim .4$ fm. It has been argued that this difference is due to the absence of light fermions[12]. Therefore, we discuss our pure Yang-Mills data considering Wilson's action as an effective action with inclusion of the smallest virtual quark-antiquark loops (tantamount to a renormalization of the bare Yang-Mills coupling). We find that this correction does not bring the lattice data nearer to the effective quarkonium potential.

In order to make this paper widely selfconsistent we sketch in section 2 our present understanding of the instanton gas picture. Section 3 describes the calculation of the static $Q\bar{Q}$ force. In section 4 we shall present our numerical results and draw the conclusions.

2. The Instanton Gas Model for the Yang-Mills Vacuum

The instanton gas model for the ground state of SU(N) Yang-Mills theories takes all tunnelings between topological distinct prevacua into account by considering Gaussian fluctuations around superpositions of single (anti) instantons taken in the singular gauge[3]. The vacuum-to-vacuum transition amplitude is then represented as a grand canonical partition function of an interacting instanton-antiinstanton gas

$$Z_{inst}(V) = \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \sum_{\varepsilon_i = \pm} \int \frac{d\varrho_i}{\varrho_i} \int_V d^4 z_i \int [dR_i] \; d_0(\varrho_i) \exp(-V_{int}) , \tag{4}$$

ϱ_i, z_i and R_i are the scale size, the position and the global orientation with respect to SU(N), resp. $d_0(\varrho)$ denotes the single-instanton amplitude[16]

$$d_0(\varrho) = C_N \left(\frac{8\pi^2}{g_0^2}\right)^{2N_c} (\varrho\Lambda)^{\frac{11 N_c}{3}} \varrho^{-4} ,$$

$$C_N = \frac{4.6}{\pi^2} \frac{1}{(N_c-1)!(N_c-2)!} (\Lambda_{P.V.}/\Lambda)^{\frac{11 N_c}{3}} \exp(-1.68 N_c) . \tag{5}$$

The scale parameter Λ refers to a particular regularization scheme which will be chosen here in accordance with the Wilson lattice scheme. The corresponding Λ_L is related to the Pauli-Villars $\Lambda_{P.V.}$ by[17]

$$\Lambda_{P.V.}/\Lambda_L = \begin{cases} 21.55 & \text{for } SU(2) \\ 31.31 & SU(3) . \end{cases} \tag{6}$$

The interaction potential V_{int} in eq. (4) collects all nonfactorizing contributions arising from the classical action, from quantum effects (i.e., from the multiscattering expansion of the fluctuation determinants) as well as from the expansion of the collective coordinate Jacobian. At large distances dominates the classical dipole-dipole interaction[3]

$$V_{int} \simeq V_{dipole} = -8\pi^2 \sum_{i,j} \overset{+}{D}(i)^a_{\mu\nu} \, \overset{-}{D}(j)^a_{\mu\nu'} \left(\delta_{\nu\nu'} - 4 \frac{\Delta_\nu \Delta_{\nu'}}{\Delta^2}\right)/\Delta^4 \tag{7}$$

with $\Delta = \overset{+}{z}_i - \overset{-}{z}_j$, and where $\overset{\varepsilon}{D}{}^a_{\mu\nu} = \frac{\varrho^2}{g_0} R^a_\alpha \eta^{\varepsilon}_{\alpha\mu\nu}$ is called dipole moment (the η symbols were invented by 't Hooft[16]). If one takes only expression (7) into account in calculating the partition function, then the thermodynamical limit does not exist because of the infrared divergent scale size integration. Usually one avoids this problem by an ad hoc cutoff ρ_c. One has then to fix ρ_c or the related packing fraction

$$f_o(\rho_c) = 2 \int_0^{\rho_c} \frac{d\rho}{\rho}\, d_o(\rho)\, \frac{\pi^2}{2}\, \rho^4 \qquad (8)$$

from an additional dynamical information. However, this method makes the instanton gas model inconsistent with some low energy theorems[18]. This disaster could be avoided, if one were able to identify a repulsive force among the contributions to V_{int}. Instead of this we have put in by hand a hard core repulsion[4].

$$V_{h.c.} = \begin{cases} 0 & \\ \infty \end{cases} \text{for} \quad \Delta^4 \gtrless a' \rho_1^2 \rho_2^2 \qquad (9)$$

involving a "large" dimensionless diluteness parameter a'. For a' it is possible to estimate a lower bound from the condition

$$|V_{dipole}| < 2 \cdot \frac{8\pi^2}{g_o^2}$$

for a given instanton-antiinstanton pair of the same group orientation. It turns out $a' > 6$. We have shown[4] that the hard core induces a selfconsistent cut-off of the instanton density at large sizes

$$d(\rho,\bar\rho) = d_o(\rho)\, \exp\left(-a_N \frac{\rho^2}{\bar\rho^2}\right), \qquad a_N = \frac{11 N_c}{6} - 2, \qquad (10)$$

where the r.m.s. radius $\bar\rho$ is completely fixed by the parameter a' or equivalently by the packing fraction

$$f_o(\bar\rho) = \pi^2 \int \frac{d\rho}{\rho}\, d(\rho,\bar\rho)\, \rho^4 = \left(1 - \frac{6}{11 N_c}\right)\frac{2}{a'}. \qquad (11)$$

In this way the pressure (i.e., the vacuum energy density), the gluon condensate and certain two-point functions of gluonic currents at zero momentum get the right, renormalization group dependence on the coupling constant[7], thus satisfying the requirements of the low energy theorems. We mention here an independent approach to the same problem[19], where analogous results are achieved, independent of certain details concerning the region of small distances between instantons.

For a' values up to $O(100)$ we expect that the interactions (7) play an important role. In Ref.[4] they have been dealt with by means of a functional averaging procedure in its Gaussian approximation. One finds an approximate expression for the pressure

$$P \simeq P_{h.c.} - \frac{3}{16\pi^2}(N_c^2-1)\frac{1}{\bar\rho^4}\int_0^\infty dx\, x^3 \ln\left[1 - F^4(x)(\pi^2\chi_o)^2\right], \qquad (12)$$

where
$$F(x) = \frac{4}{x^2}\left(1 - \frac{x^2}{2} K_2(x)\right)$$

is related to the Fourier transform of the instanton field and where the susceptibility χ_o is given by

$$\pi\chi_o = \frac{1}{N_c^2-1}\, x_o(\bar\rho)\, f_o(\bar\rho), \qquad x_o(\bar\rho) \equiv \frac{8\pi^2}{g^2(\bar\rho)} = \frac{11 N_c}{3} \ln \frac{1}{\bar\rho\Lambda}.$$

From eq. (12) one easily deduces the correction to the instanton density due to dipole interactions

$$d_{eff}(\rho) = d_o(\rho,\bar\rho)\left\{1 + \frac{3}{16}\frac{\rho^4}{\bar\rho^4}\, x_o \cdot \pi^2\chi_o \cdot \int_0^\infty dx\, x^3 \frac{F^4(x)}{1 - F^4(x)(\pi^2\chi_o)^2}\right\}. \qquad (13)$$

Our experience with the calculation of the permeability of the interacting instanton gas shows[4] that the corrections due to the enhanced one-instanton distribution (13) are large compared with those coming from two-instanton correlations. Therefore, in calculating the static $Q\bar Q$ force we will not try to treat them and restrict ourselves to an account of expression (13). Then one induces only a small error, if one replaces $\bar\rho$ by the usual cut-off ρ_c, for simplicity. In what follows we will really do so. But we want to keep in mind the relation between f_o and a' acc. to eq. (11), in order to have a better control on the influence of dipole-like interactions.

3. The Static $Q\bar Q$ Force

First let us discuss pure Yang-Mills theory. The static potential for a heavy quark-antiquark system has been studied within the quasi-classical approach first by the Princeton group[3,20]. Instead we prefer here the force as defined by eq. (1), because in the given ratio of

Wilson loop expectation values unpleasant perimeter dependent terms and Z factors depending on the cusp angles of the rectangular contour are cancelling out[21].

The dilute gas derivation for the Wilson loop expectation value $W(T,R)$ with $T \gg R$ assumes the factorization into single instanton contributions along the Euclidean time axis.

$$W(T,R) = Z_{inst}^{-1} \sum_n \frac{1}{n!} \prod_{i=1}^{n} \sum_{\varepsilon_i = \pm} \int_0^{\rho_c} \frac{d\rho_i}{\rho_i} \int d^4 z_i \int [dR_i] \; d_{eff}(\rho_i) \times$$
$$\times \; \mathrm{Tr} \left(\prod_{i=1}^{n} g_i \, \mathcal{U}(\vec{z}_i, \rho_i) \, g_i^{-1} \cdot \prod_{i=n}^{1} g_i \, \mathcal{U}^{-1}(\vec{z}_i - \vec{R}, \rho_i) \, g_i^{-1} \right), \tag{14}$$

where the global group orientations $g_i(R_i)$ written in the fundamental representation have been taken out off the matrices

$$\mathcal{U}(\vec{z},\rho) = \exp\left(i g_0 \int_{-\infty}^{\infty} dx_4 \, A_4^{inst}(x_4 - z_4, \vec{z}, \rho) \right). \tag{15}$$

The trace in expression (14) factorizes after integrating over the group orientations. One gets

$$W(T,R) = \exp \; T \cdot 2 \int_0^{\rho_c} \frac{d\rho}{\rho} \, d_{eff}(\rho) \, \rho^3 \, \omega\left(\frac{R}{\rho}\right), \tag{16}$$

where

$$\omega\left(\frac{R}{\rho}\right) = \int \frac{d\vec{z}}{\rho^3} \frac{1}{N_c} \mathrm{Tr} \left(\mathcal{U}(\vec{z}) \, \mathcal{U}^{-1}(\vec{z} - \vec{R}) - 1 \right)$$

is a numerically known function[3]. Finally, we arrive at the instanton contribution to the $Q\bar{Q}$ force as

$$F_{inst}(R) = -2 \int_0^{\rho_c} d\rho \, \rho^2 \, d_{eff}(\rho) \, \frac{d}{dR} \, \omega\left(\frac{R}{\rho}\right). \tag{17}$$

This result must be brought together with the short-range, perturbative force taking at least one-loop corrections into account. The latter can be reformulated in terms of the two-loop running coupling (3) by employing a renormalization group analysis,

$$F_{pert} = \frac{C_F}{4\pi} \frac{g^2(R)}{R^2}. \tag{18}$$

The corresponding ratio of scale parameters takes the values[11]

$$\Lambda_R / \Lambda_L = \begin{cases} 20.78 & \text{for } SU(2) \\ 30.19 & \text{for } SU(3) \end{cases}, \tag{19}$$

where Λ_L is defined by

$$\Lambda_L a = \left(\beta_0 g_0^2\right)^{-\frac{\beta_1}{2\beta_0^2}} \exp\left(-\frac{1}{2\beta_0 g_0^2}\right) \tag{20}$$

and refers to the Wilson action. Our aim is to confront the total force

$$F(R) = F_{pert}(R) + F_{inst}(R) \tag{21}$$

with the corresponding lattice data. They can be found at a given lattice scale a by

$$\frac{F(R)}{\Lambda_L^2} = \frac{1}{a^2(g_0)\Lambda_L^2} \; \chi(I,J;g_0) \tag{22}$$

with

$$\chi(I,J;g_0) = -\ln \frac{W(Ia,Ja) \cdot W((I-1)a,(J-1)a)}{W((I-1)a,Ja) \, W(Ia,(J-1)a)} \tag{23}$$

and

$$R\Lambda_L \simeq \sqrt{J(J-1)}\, a(g_0)\Lambda_L, \qquad I \gg J.$$

In a real calculation on a , for instance, 8^4 lattice one has to restrict $I \leq 5$, $J \leq 3$ to get reasonable results.

If one wants to translate the data into physical units, one needs the relation to the string tension which itself is usually taken from the string model by $\sigma = 1/2\pi\alpha'$ with $\alpha' \simeq 1$ GeV^{-2}. From MC data one extracts $\Lambda_L/\sqrt{\sigma}$ by considering the limiting curve $I,J \to \infty$ as an envelope for all ratios $\chi(I,J;g_0)$, which is expected to behave as dictated by eq. (20). For SU(2) we use the value of Creutz $\Lambda_L = .013\sqrt{\sigma}$,[10] whereas for SU(3) we will present our own data, which will include also simple ratios of the type

$$\chi_S(I,J;g_0) = \frac{1}{J-I-1} \ln \frac{W(Ia,Ja)}{W((I+1)a,(J-1)a)} \tag{24}$$

exhibiting relatively small errors due to statistical fluctuations of the W's.

It is interesting to see, how the generation and annihilation of virtual pairs of light quarks can influence the static $Q\bar{Q}$ force. For definiteness we assume $c, b, \ldots$ quarks to be infinitely heavy leaving three flavors of light quarks. We will not discuss here the force mediated by instantons but only compare the perturbative force with the lattice one. F_{pert} is given by the same expression (18) with the running coupling (3) modified by $N_f = 3$ and governed by the ratio/11/

$$\Lambda_R / \Lambda^L_{N_f=3} = 49.03 \qquad \text{for } SU(3). \tag{25}$$

On the lattice the simplest, what one can do, is to replace the Euclidean gauge field action

$$S_{Y.M.} \sim \frac{1}{g_0^2} \sum_{n;\mu,\nu} \mathrm{Tr}\, \Box_{n;\mu,\nu} \tag{26}$$

($\Box_{n;\mu,\nu}$ denotes the ordered product of link variables $U_{m,\mu}$ around a plaquette in the μ, ν plane) by an effective action which takes the smallest virtual quark loops into account. We start from the Wilson fermion action/22/

$$S_F = \sum_{f=1}^{3} \sum_{i,j} \bar{\psi}_i^f (-\delta_{ij} + K_f M_{ij}) \psi_j^f , \tag{27}$$

where i, j include the lattice site m, n colour a, b and Lorentz indices μ, ν, respectively and where

$$M_{m,n;a,b;\mu,\nu} = (1+\gamma^\lambda)_{\mu\nu} U^{ab}_{m,\lambda} \delta_{n,m+\hat\lambda} + (1-\gamma^\lambda)_{\mu\nu} U^{+ab}_{m-\hat\lambda,\lambda} \delta_{n,m-\hat\lambda} . \tag{28}$$

The hopping parameters K_f are related to the bare masses of the quarks and have been estimated recently as functions of g_0 /23/. The fermion degrees of freedom can be integrated out yielding an effective action as

$$\begin{aligned} S_{eff} &= S_{Y.M.} + \sum_f \mathrm{Tr}\, \ln(1 - K_f M) \\ &= S_{Y.M.} + \sum_f 2 K_f^4 \sum_{n,\mu,\nu} \mathrm{Tr}\, \Box_{n;\mu,\nu} + \dots \end{aligned} \tag{29}$$

Thus, in the lowest approximation of the hopping parameter expansion we can carry out a pure Yang-Mills calculation with a renormalized coupling

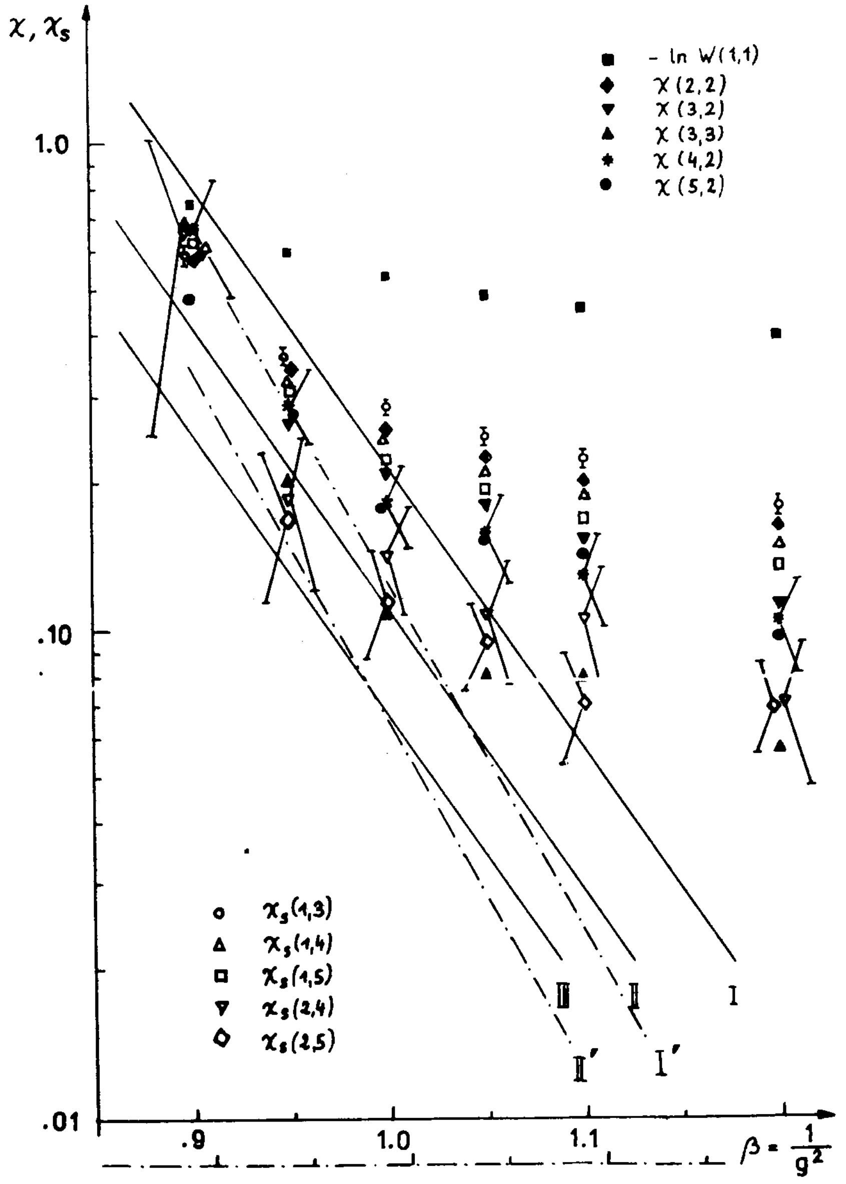

Fig. 1. SU(3) MC data for the ratios χ, χ_s acc. to eqs. (23), (24), resp.

$$\frac{1}{g^2} \longrightarrow \frac{1}{g_0^2} + \Delta\,, \qquad \Delta = 2\sum_{f=1}^{3} K_f^4(g_0)\,. \qquad (30)$$

For $g_0^{-2} \geq .90$ which is the interesting region for us in the SU(3) case there are upper bounds[23] $K_u = K_d \lesssim .194$, $K_s \lesssim .183$ yielding a small shift $\Delta \lesssim .0079$. We will use this constant shift for determining the ratio $\Lambda^L_{N_f=3}/\sqrt{\sigma}$.

4. Results

First let us discuss our numerical Monte-Carlo results for Wilson loop expectation values for the SU(3) gauge group. We applied the heat bath procedure of Pietarinen[14] to a 6^4 lattice with periodic boundary conditions. The data for the ratios (23) and (24) are shown in Fig. 1. Only those data are included for which reasonable statistical errors have been achieved. In order to give an impression of their magnitude we have quoted them at several points. For values $g_0^{-2} \geq 1.0$ we took averages over 20...25 sweeps after reaching equilibrium. For $g_0^{-2} = .95$ and .90 the number was considerably larger (27 and 44 sweeps, resp.). The straight lines I, II and III show the behaviour of the string tension $a^2\sigma$ according to eq. (20) for $N_f = 0$. They correspond to ratios $\Lambda_L/\sqrt{\sigma} = .005$, .007 and .009, respectively. Our data, in particular for the smaller loops known with sufficient accuracy, exclude values $\Lambda_L/\sqrt{\sigma} \lesssim .006$ and favorize a value

$$\Lambda_L = (.007 \pm .001)\sqrt{\sigma}\,, \qquad (31)$$

being in coincidence with the Pietarinen's one[14]. By dashed-dotted lines we marked the small scale shift on the g_0^{-2} axis due to the renormalization (30) in the case of three light quark flavors and the corresponding renormalization group behaviour for the string tension (curve I': $\Lambda^L_{N_f=3}/\sqrt{\sigma} = .0015$, curve II': $\Lambda^L_{N_f=3}/\sqrt{\sigma} = .0020$). We would prefer here

$$\Lambda^L_{N_f=3} = (.0018 \pm .0003)\sqrt{\sigma}\,. \qquad (32)$$

Now let us turn to the discussion of the static quark-antiquark force. On Fig. 2 the corresponding curves are shown for the SU(2) case ($N_f = 0$). We compare here the pure perturbative (Coulomb) behaviour

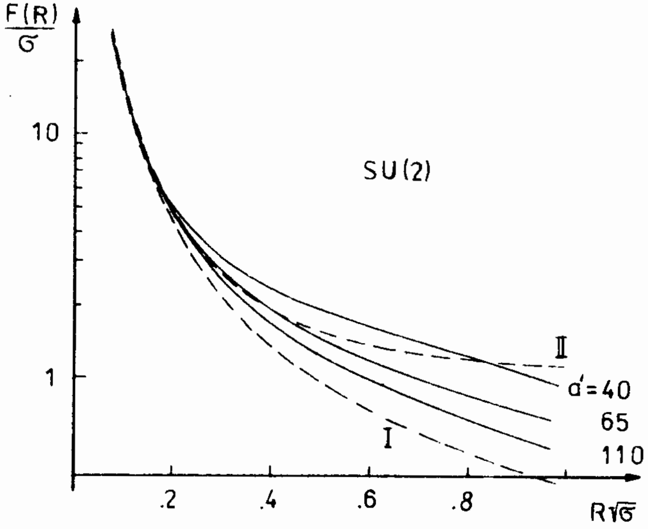

Fig. 2. The instanton mediated static $Q\bar{Q}$ force in the SU(2) ($N_f = 0$) case for different diluteness degrees.

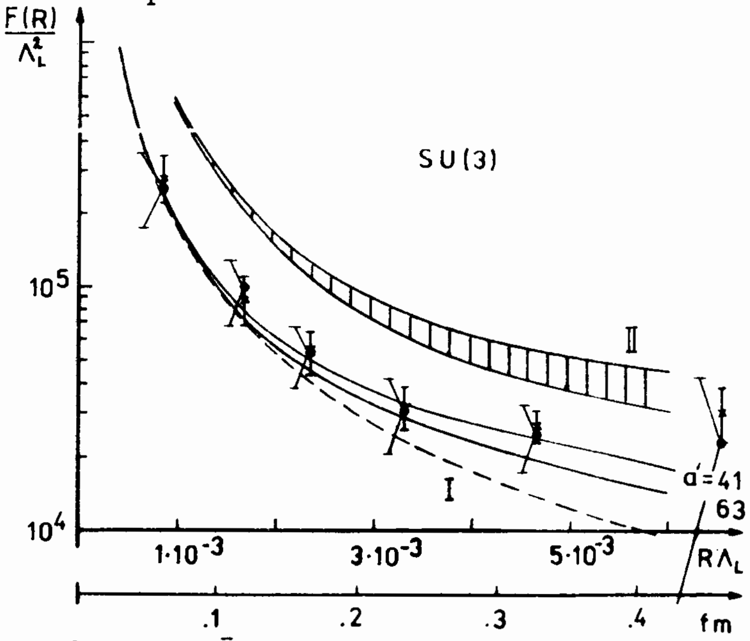

Fig. 3. The static $Q\bar{Q}$ force for SU(3) ($N_f = 0$). The MC data for $\chi(4,2)$, $\chi(5,2)$ are represented by crosses and dots, resp. Curve II shows the phenomenological force acc. to Ref.[25].

acc. to eq. (18) (curve I) and the instanton corrected expression (eq. (21)) with a fit (curve II) to data taken by Lang et al[9] on an 8^4 lattice for the ratios (23) with $I = 4,5$ and $J = 2,3$. There is a clear distinct interval for the diluteness parameter a' , such that the instanton mediated force can be put into agreement with the lattice force up to $R\sqrt{\sigma} \lesssim .45$. Acc. to eqs. (6) and (11) $a' = 65$ corresponds to an instanton scale $\rho_c\sqrt{\sigma} = .50$ being slightly larger than the correctly described distances between the quark-antiquark pair. This "optimal" a' value is related to a packing fraction f_0 of 2%.

A similar picture arises for the SU(3) gauge group. Here the drawn Coulomb force (curve I) and the instanton corrected ones for two diluteness degrees have to be compared with the data points obtained from the ratios $\chi(4.2)$, $\chi(5.2)$ (see Fig. 3). Within their error bars the points show a remarkable independence of the time-like extension of the Wilson loops.
We conclude that the instanton gas is in a good shape with the lattice results for $40 \lesssim a' \lesssim 65$. This is tantamount to $.0047 \gtrsim \rho_c \Lambda_L \gtrsim .0045$ and $4\% \gtrsim f_0 \gtrsim 2.5\%$. At this diluteness degree the dilute gas approximation is intact, if dipole-like interactions are really taken into account. That they yield a non-negligible contribution can be seen from Figs. 4 and 5 where we show the β functions

$$-\frac{\beta}{g_{eff}} \equiv \frac{R}{2} \frac{\partial g^2_{eff}(R)}{\partial R} \frac{1}{g^2_{eff}(R)} , \qquad g^2_{eff}(R) \equiv \frac{4\pi}{C_F} R^2 F(R) \tag{33}$$

in the cases with instanton interactions (IIa) and without interactions (IIb) compared with the perturbative two-loop behaviour (I). On the curves we have marked points corresponding to those distances up to which the instanton mediated forces follow the lattice data ($R\sqrt{\sigma} \simeq .45$ for SU(2), $R\Lambda_L \simeq .004$ for SU(3)). It is interesting to compare the β function defined by eq. (33) with the one we have found considering the triple-gluon vertex with instanton contributions[8]. In both schemes the curves resemble each other in their shape. In the scheme chosen here the departure from the perturbative behaviour is shifted to somewhat larger coupling.

Having determined the diluteness degree of the instanton gas within the above-mentioned limits we would like to ask whether other quantities can consistently be described. The most interesting quantity in this respect is the gluon condensate[24]

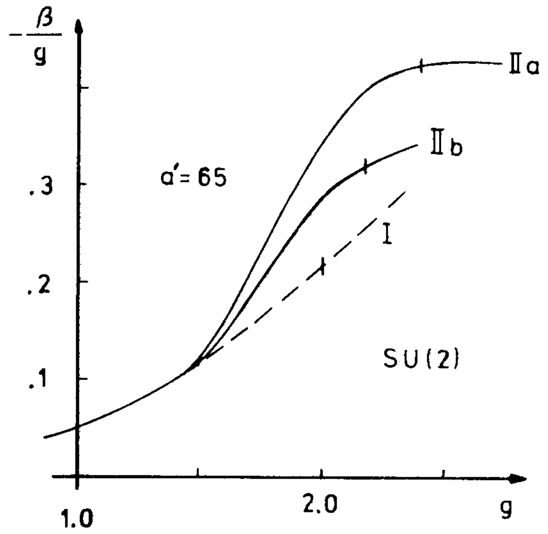

Fig. 4. The instanton driven β function for SU(2) acc. to eq. (33).

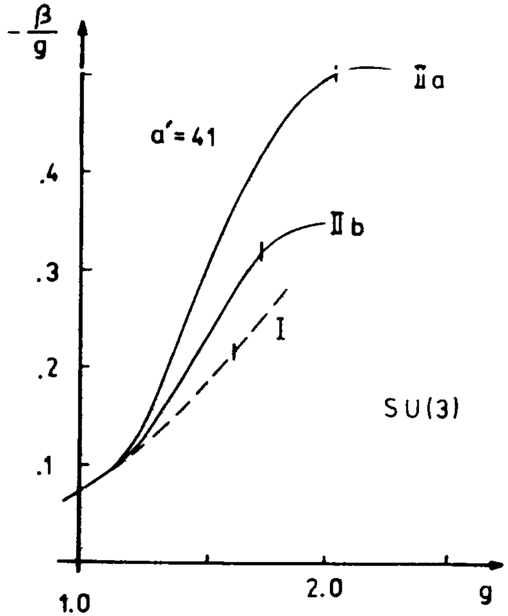

Fig. 5. As Fig. 4 for SU(3).

$$\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} \rangle \simeq 2 \int_0^{\rho_c} \frac{d\rho}{\rho} \int d^4 z \int [dR]\, d_{eff}\, \frac{g^2}{4\pi}\, G^{a\,inst}_{\mu\nu}(z)\, G^{a\,inst}_{\mu\nu}(z) . \qquad (34)$$

For instance for $\alpha' \simeq 63$ we find acc. to eq. (31) for SU(3)

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = (.028 \pm .016)\ GeV^4 \qquad (35)$$

which might be an acceptable estimate within the given error.

For the real case of the colour group SU(3) there exist a lot of competing phenomenological potentials successively describing the charmonium and bottonium spectra[15]. They agree very well at distances 0.1 fm $\lesssim R \lesssim$ 1 fm. For definiteness we have shown in Fig. 3 the potential of the Cornell group[25] (curve II) with the uncertainty due to the error in eq. (31). There is a striking disagreement with the lattice data seen up to $R \simeq$ .4 fm. Can the inclusion of light fermions substantielly improve the situation? At least in the lowest approximation of the hopping expansion as given by eq. (29) we have only a negative answer. Fig. 6 shows this clearly. The lattice data at small distances

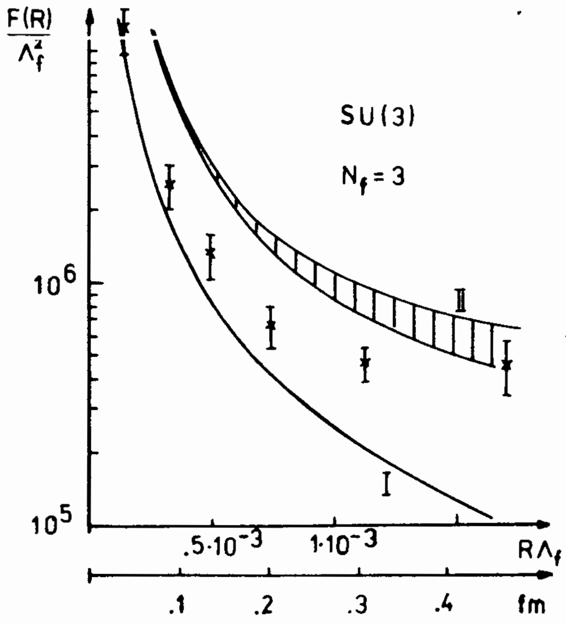

Fig.6. The static $Q\bar{Q}$ force for SU(3) with approximate inclusion of light quarks. Curves I and II show the Coulomb force and the phenomenological force[25], resp. The data points give the ratio $\chi(4,2)$ corrected for charge renormalization (30).

fit well into the perturbative behaviour acc. to eqs. (3,18,25) establishing the correctness of the ratio (25). Of course one could hope for the influence of higher corrections in the hopping expansion. This, however, seems to be in conflict with the recent success in calculating meson masses within the truncated[23] or "quenched" approximations[26]. More likely there is another interpretation, that phenomenological potentials - at least at small distances - are not related to the interaction energy of idealized heavy quarks at all[24]. In accordance with the arguments of Ref.[24] we see that the static $Q\bar{Q}$ force tends to agree with quarkonium potentials for $R \gtrsim$ .5 fm. However, at such distances instantons are not relevant any more.

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Статическая сила между кварками с точки зрения инстантонного газа и численных расчетов на решетке

Предсказания расчетов по методу Монте-Карло на решетке для статической силы между кварками сравниваются с результатами, полученными в рамках модели инстантонного газа, и определяется типичный размер инстантонов. Представлены данные для разных соотношений вильсоновских петель в случае SU(3) для натяжения струны и фиксировано значение $\Lambda_{Latt} = (0{,}007 \pm 0{,}001)\sqrt{\sigma}$. Инстантонные поправки к пертурбационной силе оказываются существенными для достижения согласия с полученными путем расчетов на решетке данными в области малых расстояний до $\simeq .3$ Фм. Приводятся аргументы в пользу того, что отличие данных в этой области от значения силы, известной из феноменологии, связано с понятием бесконечно тяжелых кварков, но не с пренебрежением виртуальными кварковыми петлями.

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The Static $Q\bar{Q}$ Force from Instanton Gas and Numerical Lattice Calculations

From the comparison of lattice Monte-Carlo predictions for the force between infinitely heavy quarks with the instanton gas results in the cases of SU(2) and SU(3) the typical instanton scale sizes are determined. We present our SU(3) Yang-Mills MC data for different ratios of Wilson loops and establish the value $\Lambda_L = (0.007 \pm 0.001)\sqrt{\sigma}$ for the string tension. The instanton mediated force follows the lattice data inside a small-distance region (up to $\simeq 0.3$ fm), where the data obviously disagree with the perturbative as well as with the phenomenologically known $Q\bar{Q}$ force. We argue that the disagreement with the latter is connected with the idealization of quarks at rest rather than with the absence of virtual light quark pairs.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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