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PARASTATISTICS AND GAUGE SYMMETRIES

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In recent years the hadron physics has made a considerable progress due to the discovery of the quark colour degree of freedom[1-6] and the gauge theory of quark-gluon interaction developed on its basis[7,8], QCD, which is expected to substitute the "old-fashioned" theory of hadron strong interactions. The colour symmetry is supposed to be perfect[7] though this perfection and reasons for it are not yet established. Therefore, for a better understanding of the nature of the colour symmetry it is expedient to apply again to its original formulation.

As is known the main reason for assuming quarks to have an additional internal degree of freedom with three values came from the problem of quark statistics raised by developing the hadron spectroscopy: quarks when constituting baryons turned out to be in a state symmetric in all variables (the so-called "symmetric model") in contradiction with the Pauli principle. This problem was solved in paper by Bogolubov, Struminsky, and Tavkhelidze[1] who proposed to introduce additional quantum numbers which antisymmetrize the total wave function. The idea was further proposed independently in a number of works[2-6]. Further the new degree of freedom was called "the quark colour", and on its basis the gauge colour $SU(3)_c$ symmetry and the corresponding (massless) vector fields, "gluon" octet, were introduced[5-7].

However still another solution existed to the problem of quark statistics proposed by Greenberg[9], which did not assume quarks to have a new degree of freedom and was based on the application of the para-Fermi-statistics of order (or rank) 3 to quarks. The statistics admits to place three identical (spinor) particles in the same quantum-mechanical state. The field theory corresponding to generalized statistics of this type was formulated by Green[10] and Volkov[11] and then studied by many authors[12-16]. It is now called the "parafield theory".

Unfortunately, in recent years these two approaches have been confused, and in the literature, especially in reviews, they are erroneously identified with each other, through even

Greenberg and Messiah[16] have stressed the difference between parastatistics and the degeneracy in some internal coordinate. The state space in the first case is narrower than that in the second case. However, the exact degeneracy makes it possible to establish a certain connection between these state spaces, if states degenerated in the internal coordinate are considered as a single state. It is only that sense in which a restricted equivalence of the parastatistics to the internal-coordinate degeneracy is to be understood.

It has turned out that the use for the quark description of one para-Fermi field of the third order, instead of three colour Fermi-fields, results in the theory equivalent to SO(3) rather than to SU(3)colour symmetry [17,18]. Moreover, it has appeared that in the para-field theory it is impossible at all to formulate the nonlinear Lagrangian equivalent to the Yang-Mills Lagrangian of SU(3) - symmetry.

In our opinion, this result represents a certain principal difficulty for understanding the perfection of colour symmetry. If the exact degeneracy in colour holds for quark states, the colour becomes then nonobservable in principle. Quarks in this case should certainly be considered as identical particles obeying the para-Fermi-statistics of order 3 and should be described by one para-Fermi-field admitting the hidden SO(3) rather then SU(3) gauge symmetry.

Consider the derivation of this result in more detail.

We make use of the so-called Green ansatz[10,16] by which the para-Fermi-field of order 3 is represented as a sum of three usual Fermi-fields

$$\psi(x) = \sum_{A=1}^{3} \psi_A(x) \qquad (1)$$

commuting, however , with each other. Such commutation relations between Fermi-fields are called anomalous in cantrast to the normal ones when all Fermi-fields anticommute with each other (all Bose-fields commute with each other and with Fermi-fields). The para-field (1) itself obeys the Green trilinear relations which we need not to consider.

The free-Dirac-para-field theory contains two independent

conserved currents

$$j_\mu(x) = \frac{1}{2}\left[\bar\psi(x), \gamma_\mu \psi(x)\right] = \sum_{A=1}^{3} \bar\psi_A(x)\gamma_\mu\psi_A(x), \qquad (2)$$

$$j'_\mu(x) = \frac{1}{2}\left\{\bar\psi(x), \gamma_\mu \psi(x)\right\}$$
$$= \frac{1}{2}\left(\bar\psi_1\gamma_\mu\psi_2 + \bar\psi_1\gamma_\mu\psi_3 + \bar\psi_2\gamma_\mu\psi_3 + h.c.\right), \qquad (3)$$

where $\bar\psi$ is the Dirac-conjugated field.
In (2) and (3) and in what follows brackets denote the commutator; while braces,the anticommutator. Basides, for currents and analogous quantities the normal form is implied in the sense of subtraction of vacuum expectation values of the type

$$j_\mu(x) \to j_\mu(x) - \langle j_\mu(x)\rangle_0 = \sum_{A=1}^{3} :\bar\psi_A(x)\gamma_\mu\psi_A(x): \qquad (4)$$

Obviously, the current (2) is local: two such currents at points separated by a space-like interval commute with each other. For that reason it can form a local interaction only with the usual Bose-field $\mathcal{A}_\mu(x)$, which we identify with the electromagnetic field

$$\mathcal{H}_{e.m.}(x) = e' j_\mu(x)\mathcal{A}_\mu(x). \qquad (5)$$

Note that in such an interpretation the effective electromagnetic charge equals e' times three[11,15], and in order that the field charge be simply the elementary charge e , one should put

$$e' = e/3. \qquad (6)$$

For instance, for u- ,d- , and s -quarks described by the para-field (instead of colour quarks) the charges should be as follows

$$Q_u = 2e/3, \quad Q_d = Q_s = -e/3. \qquad (7)$$

In this sense the fractional charges of quarks could be related with para-statistics of the latter.

The current (3) is nonlocal: its three components in the r.h.s. taken at points separated by a space-like interval anticommute with each other. In this sense it behaves like a paraboson of order 3.Just this order 3 is distinguished with respect to all others. The anticommutator currents (3) for the second order is local, for the fourth order it is also the third-order para-boson but its order does not coincide with the order 4 of the initial para-Fermi-field. For the fifth and higher order it possesses no para-locality properties. Therefore, to construct the local interaction with that current we should introduce a vector para-Bose-field of order 3.(Such a field may be introduced also when the initial para-Fermi-field is of the fourth order, however, in this case Green mutual para-commutation relations take no place, and we go beyond the scope of the para-field theory. In any case the para-Bose-field is of order 3). It is represented by the sum of three usual Bose-fields <u>anticommuting with</u> each other:

$$\mathcal{B}_\mu(x) = \sum_{A=1}^{3} \mathcal{B}_\mu^A(x). \tag{8}$$

If we also require the components $\psi_A(x)$ and $\mathcal{B}_\mu^B(x)$ to commute with each other at $A=B$ and to <u>anticommute</u> for $A \neq B$, then from the fields (1) and (8) we can construct the local interaction

$$\mathcal{H}'(x) = \frac{1}{2}\{ j_\mu'(x), \mathcal{B}_\mu(x)\} = \sum_{A=1}^{3} j_\mu^A(x)\mathcal{B}_\mu^A(x). \tag{9}$$

where

$$\begin{aligned} j_\mu^1(x) &= \bar\psi_2(x)\gamma_\mu\psi_3(x) + h.c.,\\ j_\mu^2(x) &= \bar\psi_1(x)\gamma_\mu\psi_3(x) + h.c.,\\ j_\mu^3(x) &= \bar\psi_1(x)\gamma_\mu\psi_2(x) + h.c. \end{aligned} \tag{10}$$

We can also construct the tensor, nonlinear in field $\mathcal{B}_\mu$, which is a para-boson:

$$\begin{aligned} \mathcal{F}_{\mu\nu}(x) &= \partial_\mu\mathcal{B}_\nu(x) - \partial_\nu\mathcal{B}_\mu(x) + \frac{ig}{2}[\mathcal{B}_\mu(x), \mathcal{B}_\nu(x)]\\ &= \sum_{A=1}^{3}\mathcal{F}_{\mu\nu}^A(x), \end{aligned} \tag{11}$$

where

$$\mathcal{F}_{\mu\nu}^A = \partial_\mu\mathcal{B}_\nu^A - \partial_\nu\mathcal{B}_\mu^A + ig\sum_{A\neq B\neq C\neq A}\mathcal{B}_\mu^B\mathcal{B}_\nu^C \tag{12}$$

and g is an interaction constant.

Now we are able to build up some "copy" of the Yang-Mills Lagrangian in the para-field theory

$$\begin{aligned} \mathcal{L}(x) &= -\frac{1}{4}\mathcal{F}_{\mu\nu}^2(x) + \frac{1}{2}[\bar\psi(x), (i\gamma_\mu\partial_\mu - m)\psi(x)] +\\ &\quad + g'\{j_\mu'(x), \mathcal{B}_\mu(x)\} \end{aligned} \tag{13}$$

$$\begin{aligned} &= -\frac{1}{4}\sum_{A=1}^{3}(\mathcal{F}_{\mu\nu}^A)^2 + \sum_{A=1}^{3}\bar\psi_A(x)(i\gamma_\mu\partial_\mu - m)\psi_A(x) +\\ &\quad + g'\sum_{A\neq B\neq C\neq A}\bar\psi^A(x)\gamma_\mu\psi^B(x)\mathcal{B}_\mu^C(x). \end{aligned} \tag{14}$$

With the commutation relations for components $\bar\psi^A$, ψ^B , $\mathcal{B}_\mu^C$ the locality of the interaction (14) can be easily verified.

The Lagrangian (14) resembles in form the Yang-Mills Lagrangian with respect to the group of rotations in a three dimensional space with axes denoted by indices 1,2,3. However, such rotations are not admitted by anomalous commutatuon relations between different components. To get rid of this obstacle, transition should be made to fields with normal commutation relations. The way is to define the Klein operator $\mathcal{K}$ commuting with components ψ^1 and $\mathcal{B}_\mu^1$ and anticommuting with $\psi^2, \psi^3, \mathcal{B}_\mu^2, \mathcal{B}_\mu^3$. For free fields it is expressed via operators N_2 and N_3 of numbers of particles (both bosons and fermions) with indices 2 and 3

$$\mathcal{K} = \exp\{i\pi(N_2 + N_3)\}. \tag{15}$$

That such an operator can be introduced for interacting fields was shown in refs./17,18/. It has the properties:

$$\mathcal{K}^+ = \mathcal{K}^{-1} = \mathcal{K}, \quad \mathcal{K}^2 = 1. \tag{16}$$

Now Green components can be replaced by fields

$$\Psi_1 = \psi_1 \mathcal{K}\,, \quad \Psi_2 = -i\,\psi_2 \mathcal{K}\,, \quad \Psi_3 = \psi_3\,. \tag{17}$$
$$B^1_\mu = \mathcal{B}^1_\mu \mathcal{K}\,, \quad B^2_\mu = -i\,\mathcal{B}^2_\mu \mathcal{K}\,, \quad B^3_\mu = \mathcal{B}^3_\mu\,.$$

The fields Ψ_A and B^A_μ obey normal commutation relations.

With the Klein transformation (17) the Lagrangian (14) becomes:

$$\mathcal{L}(x) = -\frac{1}{4}\vec{F}^{\,2}_{\mu\nu}(x) + \overline{\vec{\Psi}}(x)(i\gamma_\mu\partial_\mu - m)\vec{\Psi}(x) - -ig'\vec{B}_\mu(x)\cdot[\overline{\vec{\Psi}}(x)\times\gamma_\mu\vec{\Psi}(x)]\,, \tag{18}$$

where $\vec{\Psi}$ and $\vec{B}_\mu$ are vectors

$$\vec{\Psi} = (\Psi_1, \Psi_2, \Psi_3)\,, \quad \vec{B}_\mu = (B^1_\mu, B^2_\mu, B^3_\mu) \tag{19}$$

and

$$\vec{F}_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + g\,\vec{B}_\mu \times \vec{B}_\nu\,. \tag{20}$$

The Hamiltonian (18) turns completely into the Yang-Mills Hamiltonian of the gauge $SO(3)$-symmetry if we equate the interaction constants g and g' in (18) and (20). Below this assumption will be justified when we shall formulate the gauge principle for para-fields in the quaternion representation.

Thus, in the Greenberg para-quark theory it is possible to formulate two local interactions of the para-quark field with other fields: the electromagnetic interaction with a Bose-field $\mathcal{A}_\mu(x)$ and quark-gluon interaction with a "para-gluon" field $\mathcal{B}_\mu(x)$. The latter appears to be equivalent of the gauge field of $SO(3)$ forming a vector of this group (and not a gluon octet, as it occurs for the $SU(3)$ colour symmetry).

One may ask: Why would be bad the $SO(3)$ symmetry as a colour quark gauge symmetry? There are two objections (see, e.g.[19]). The first: For such a symmetry the condition of asymptotic freedom limits the number of different sorts ("flavours") of quarks by the maximal number 2. Consistency of this condition could be achieved with the experimental discovery of at

least five sorts of quarks if they are placed in generations

$$(u,d)\,,\ (c,s)\,,\ (b,t)$$

and if each generation is assumed to have its own triplet of gluons. But in this case the gluon universality, and as a result the relation between quarks of different generations, would disappear.

The second objection comes from the real character of $SO(3)$-symmetry that makes no distinction between the quark and antiquark. For that reason its singlets cover not only usual mesons, $q\bar{q}$, and baryons, qqq, but also diquarks qq and analogs of baryons in which one quark is replaced by an antiquark, $qq\bar{q}$. Because gluons are vectors, the $SO(3)$ singlet would also be a bound state of a quark with one gluon (gluon-quark)

$$\frac{1}{2}\{q, G\} = \sum_{A=1}^{3} q_A G_A\,.$$

All these unusual objects would have fractional charges and could exist in a free state without causing the infrared instability. No such states have been observed yet. If, however, the observation of fractional residual charges $(\pm 1/3)e$ will be verified in Millikan-type experiments[20], this will be on the contrary an indication of the existence of such objects, and consequently, a strong argument in favour of the colour $SO(3)$ symmetry.

It could be expected that the $SO(3)$ symmetry is an exact subgroup of the broken colour $SU(3)$-symmetry restoring only asymptoticaly at short distances. Then the first objection related with the asymptotic freedom is removed, while the second (the existence of fractional charged objects) remains still valid. (A possibility to explain experiments on the observation of fractional charges on this basis has recently been proposed also in ref.[21]). The para-field theory, admits, in principle, a possibility to formulate the broken $SU(3)$ symmetry[22], but in this case we should go beyond the scope of the theory of one parafield and introduce two additional para-fields constructed as two other linear independent combinations of the type (1).

So, the parastatistics, on the one hand, picks out the third order only for which some Yang-Mills Lagrangian can be constructed in the para-field theory, and thus seems to justify the "three-colourness" of quarks, but on the other hand it gives an unambiguous indication that only the real $SO(3)$ exact gauge symmetry can be related to this hidden "three-colour".

Our last remark concerns the possibility of straightforward formulation of the gauge principle for para-fields. Apart from the Green ansatz (1) for a para-field of the third order another ansatz may be put forward on the basis of quaternions. Such a para-field can be represented as the sum

$$\psi(x) = \sum_{A=1}^{3} \Psi_A(x) e_A \,, \tag{21a}$$

$$\bar{\psi}(x) = -\sum_{A=1}^{3} \bar{\Psi}_A(x) e_A \,, \tag{21b}$$

where $\Psi_A(x)$ are Fermi-fields with normal mutual commutation relations; and e_A, quaternion units defined by the relations

$$e_A e_B = -\delta_{AB} + \epsilon_{ABC} e_C \,, \quad \bar{e}_A = -e_A \,. \tag{22}$$

The free Lagrangian

$$\mathcal{L}(x) = \frac{1}{2}[\bar{\psi}(x), (i\gamma_\mu \partial_\mu - m)\psi(x)] - \sum_{A=1}^{3} \bar{\Psi}_A(x)(i\gamma_\mu \partial_\mu - m)\Psi_A(x). \tag{23}$$

is obviously invariant under the global phase transformations

$$\psi'(x) = e^{\varphi}\psi(x)e^{-\varphi}, \quad \bar{\psi}'(x) = e^{\varphi}\bar{\psi}(x)e^{-\varphi} \tag{24}$$

with arbitrary vector quaternion φ :

$$\varphi = \sum_{A=1}^{3} \varphi^A e_A \,. \tag{25}$$

Next, we require the invariance of the theory not only under global but also under local transformations (24), when phases φ are functions of time and coordinates, x. For this purpose it is necessary to introduce the covariant derivative

$$D_\mu(x) = \partial_\mu + \frac{g}{2}\mathcal{B}_\mu(x) \tag{26}$$

which contains the para-Bose-field $\mathcal{B}_\mu(x)$ constructed by analogy with (28) out of the Bose-fields with normal mutual

relations

$$\mathcal{B}_\mu(x) = \sum_{A=1}^{3} B_\mu^A(x) e_A \,. \tag{27}$$

Note that the covariant derivative (26) should always enter into the commutator with the field which it operates on. For this commutator we indeed have the covariance

$$[D'_\mu(x), \psi'(x)] = e^{\varphi(x)}[D_\mu(x), \psi(x)]e^{-\varphi(x)}, \tag{28}$$

if the field $\mathcal{B}_\mu(x)$ is transformed by the law

$$\mathcal{B}'_\mu(x) = e^{\varphi(x)}\mathcal{B}_\mu(x)e^{-\varphi(x)} + \frac{2}{g} e^{\varphi(x)}\partial_\mu e^{-\varphi(x)} \,. \tag{29}$$

Then we arrive at the gauge Lagrangian

$$\mathcal{L}(x) = -\frac{1}{4}\mathcal{F}^2_{\mu\nu}(x) + \frac{1}{2}[\bar{\psi}(x), (i\gamma_\mu\partial_\mu - m)\psi(x)] + \frac{ig}{4}[\bar{\psi}(x), [\mathcal{B}_\mu(x), \gamma_\mu \psi(x)]], \tag{30}$$

where $-i\mathcal{F}_{\mu\nu}(x) = \partial_\mu \mathcal{B}_\nu(x) - \partial_\nu \mathcal{B}_\mu(x) + \frac{g}{2}[\mathcal{B}_\mu(x), \mathcal{B}_\nu(x)]$.

With the use of representations (21) and (27) the Lagrangian (30) can be rewritten in a form of the Yang-Mills Lagrangian (18) (one also should employ well known identities for mixed scalar and vector products of three vectors in order to convert the field interaction into the form it enters into (18)).

So, the "three-colourness" of quarks could be associated with the possible quaternion representations of para-fields. However, in this case, as should be expected, the group of internal symmetry is $SO(3)$. We note also that the gauge principle has naturally led to the proposed earlier equality of constants g and g' in (18) and (20).

A natural generalisation of the quaternion representation of para-fields would be their octanion representation /23/. However, the open question is how to formulate the gauge principle for such fields in view of the nonassociative property of octanions.

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Говорков А.Б. Е2-82-470

Парастатистика и калибровочные симметрии

Анализируется возможность формулировки калибровочных симметрий в теории параполя Грина и отмечается выделенность калибровочной $SO(3)$ -симметрии. Гипотеза паракварков Гринберга оказывается неэквивалентной гипотезе цветовой $SU(3)_c$ -симметрии кварков. Рассматриваются особенности калибровочной $SO(3)$ -симметрии и возможность схемы, в которой она выступает как точная подгруппа нарушенной $SU(3)_c$ -симметрии. Обсуждается также формулировка калибровочного принципа для параполей, представляемых с помощью кватернионов.

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Parastatistics and Gauge Symmetries

A possible formulation of gauge symmetries in the Green parafield theory is analysed and the $SO(3)$ gauge symmetry is shown to be on a distinct status. The Greenberg paraquark hypothesis turns out to be not equivalent to the hypothesis of quark colour $SU(3)_c$ symmetry. Specific features of the gauge $SO(3)$ symmetry are discussed, and a possible scheme where it is an exact subgroup of the broken $SU(3)_c$ symmetry is proposed. The direct formulation of the gauge principle for the parafield represented by quaternions is also discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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