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QUARK-GLUON DISTRIBUTION FUNCTIONS OF HADRONS

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INTRODUCTION

The problem of grounding the principles of Quantum Chromodynamics (QCD) in hard processes has recently occupied an important place in elementary particle physics. Remarkable progress has been achieved in studying the problem. Up to now series experimental data have been collected on lepton-nucleon weak and electromagnetic processes, large angle hadron-hadron scattering, inclusive production of hadrons with large transverse momenta, processes of e^+e^--annihilation into hadrons, production of massive lepton pairs. The theoretical study of the above-mentioned problem has been successful also. For instance, calculations of the known physical values, taking into consideration higher orders of the perturbation theory QCD have been done, new QCD effects have been predicted and calculated. All this has allowed a comprehensive comparison of the predictions of the theory with the experimental data. In general a good agreement between QCD and experimental data has been obtained. However, certain difficulties have arisen. Some of them are caused by the fact that QCD predictions are related to the values which cannot be experimentally measured or require additional experimental information to verify them. Q^2-evolution of quark and gluon distribution functions is one of these predictions. As is known, QCD provides, in principle, two equivalent definitions of this prediction: as a system of Lipatov-Altarelli-Parisi (LAP)[1] integro-differential equations of the evolution for distribution functions, or as a system of algebraic equations of the evolution of the momenta of these functions[2]. Though the predictions for the momenta are relatively simple, their comparison with the experimental data is a complicated and not quite correct task. For instance, structural functions can be measured only in a limited region of the interval of changes of the variable $x \in [0,1]$. Therefore, in order to determine the momenta one has to extrapolate the experimental values of structure functions in the whole interval. Thus obtained values of the momenta may have notable uncertainty. That is why it is highly desirable to have QCD predictions immediately for the structure functions $F(x,Q^2)$ or for quark and gluon distribution functions (DF) $f_i(x,Q^2)$ as was often mentioned in papers[3,4].

Values of $f_i(x,Q^2)$ in the discrete set of points for various x and Q^2 can be obtained by solving LAP equations by numerical methods. These equations being very complicated with next-to-leading corrections, their numerical integration involves many difficulties. Besides, the tabular representation of $f_i(x,Q^2)$ is inconvenient for practical applications, and in some important cases (Drell-Yan process, production of hadrons with high p_T, etc.) it is almost useless. Therefore, the problem of the most convenient and rational presentation of QCD predictions for deep inelastic processes has been solving through the search for approximate solutions of the evolution equations in an explicit analytical form. Much has been done for evolution equations (EE) in the leading order (LO)/4/, but quite recently/5/ an explicit analytical form for non-singlet (valence) distribution function has been obtained which approximately satisfies EE of the next-to-leading order. Singlet and gluon distribution functions have not been considered in this work.

We consider this situation to be unsatisfactory. Even rough estimations show that the next-to-leading corrections play an important role in the region of experimentally obtained transfer momenta. Moreover, it turned out that at any Q^2 their contribution grows as the momentum number n or value of Bjorken variable x increases, and n, x being big enough, QCD perturbation theory becomes senseless. These circumstances indicate, that next-to-leading corrections should be taken into consideration when proving the experimental status of QCD.

In this paper we have obtained a new solution of evolution equations in LO and next-to-leading order. We have found in explicit analytical form all distribution functions included in these equations, i.e., non-singlet, singlet and gluon distribution functions. As in our previous papers/6/, the initial conditions for EE and the explicit form of x-dependence of approximated solutions are obtained on the basis of the phenomenological quark-gluon model of the nucleon. The model has been suggested and investigated in out paper/7/. It is based on some assumptions on quark-gluon nucleon structure in agreement with QCD.

We would like to remind of the fact that the initial conditions and, consequently the form of x-dependence for EE solutions, are not the subject of the perturbation theory. This does not allow their thorough calculation within QCD at present time, and one have to replace a strict theory by phenomenological models or the information extracted from experimental data. Almost all approximate solutions found in paper/4,5/ have been obtained through the latter way. We have tried to employ the first possibility as we think that prescribing the initial conditions of EE on the basis of the phenomenological model is more preferable than the use of experimental information which has no physical sense. In fact, the model we base on in this paper allows much

more unity in describing various processes. It enables us of taking account of different bonds between them, inaccessible for the experimental approach.

In recent years the problems of influence of various non-perturbative effects on the behaviour of deep non-elastic processes at moderate Q^2 ($Q^2 \lesssim 10$ GeV^2/c^2) has been widely discussed. Among them twist power corrections for structure functions predetermined by the contribution of Wilson operators of higher twists are best known. Account of these corrections is an open problem and is not considered in this paper. We will turn to another type of non-perturbative effects caused by the large scale structure of the hadron showing itself at moderate Q^2 as three extended constituents - spectroscopic (constituent) quarks. It is interesting to note that unlike twist effects, the just mentioned effects make contributions not only at large x, but also in the whole region of this variable. To take account of these effects we use our version of two-level model of a hadron. Other variants of the model are presented in papers/8,9/. We think they have some drawback from the point of view of the modern development of the hadron theory.

On the basis of our results the calculations have been done for the structure functions of deep inelastic $\nu(\bar{\nu})N$-, $e(\mu)N$-scattering; longitudinal asymmetry of the electron scattering on the polarized proton target A_1, the structure function of π-meson F_π observed in the process of massive lepton pair production in πN collisions. It is shown, that the enumerated theoretical predictions are in good agreement with a large number of experimental data.

1. DISTRIBUTION FUNCTIONS AND THEIR Q^2-EVOLUTION IN QCD

Let us introduce the necessary standard definitions for quarks and gluons distribution functions (DF) in QCD and their connection with the structure functions.

1. In the leading order (LO) DF are introduced by the only way originated from the comparison with the formulae of parton model. The formulae of DF connection with the structure functions in this case are as follows/2/:

$$F_2(x,Q^2) = \frac{5}{18}\, x\Sigma(x,Q^2) + \frac{1}{6} x f_v(x,Q^2), \qquad (1)$$

$$F_3(x,Q^2) = -3 f_v(x,Q^2), \qquad (2)$$

where F_2, F_3 are respectively the structure function of the deep inelastic $e(\mu)N$- and $\nu(\bar{\nu})N$-scattering on the isoscalar target in the channel of charged currents. The DF moments of valence quarks $\langle f_v \rangle_n$, gluons $\langle f_g \rangle_n$ and singlet combination

$$\Sigma(x,Q^2) = 3f_v(x,Q^2) + 8f_s(x,Q^2), \tag{3}$$

where f_s is DF of sea quarks, are identified in the normalization point $Q_0^2 = \mu^2$ with matrix elements of non-singlet A_n^{NS}, gluon A_n^G and singlet A_n^ψ Wilson operators of twist 2. The introduced moments of DF have the following QCD Q^2-evolution:

$$\langle f_v(s)\rangle_n = \int_0^1 dx\, x^{n-1} f_v(x,Q^2) = \langle f_v(0)\rangle_n\, e^{-d_{NS}^n \cdot s}, \tag{4}$$

$$\begin{aligned}\langle \Sigma(s)\rangle_n &= \{(1-\alpha_n)\langle \Sigma(0)\rangle_n - \tilde{\alpha}_n \langle f_g(0)\rangle_n\} e^{-d_+^n \cdot s}\\ &+ \{\alpha_n \langle \Sigma(0)\rangle_n + \tilde{\alpha}_n \langle f_g(0)\rangle_n\} e^{-d_-^n \cdot s}\end{aligned} \tag{5}$$

$$\begin{aligned}\langle f_g(s)\rangle_n &= \{\alpha_n \langle f_g(0)\rangle_n - \epsilon_n \langle \Sigma(0)\rangle_n\} e^{-d_+^n \cdot s}\\ &+ \{(1-\alpha_n)\langle f_g(0)\rangle_n + \epsilon_n \langle \Sigma(0)\rangle_n\} e^{-d_-^n \cdot s}\end{aligned} \tag{6}$$

The standard evolution variable

$$s = -\ln \frac{\bar{g}^2(Q^2)}{\bar{g}^2(Q_0^2)} \tag{7}$$

is used here. In the approximation under the consideration

$$\bar{g}^2_{LO}(Q^2) = \frac{16\pi^2}{\beta_0 \ln Q^2/\Lambda^2}, \quad \beta_0 = 11 - \frac{2}{3} n_f . \tag{8}$$

Other parameters of EE are given in Appendix B.

As is known[10,11], taking account of corrections for LO leads to ambiguity in the way of determining quark and gluon distribution functions of nucleon. Already in next-to-leading order the parton model connections for the structure functions of deep inelastic processes are violated. This fact makes the parton interpretation of the corresponding QCD formulae rather difficult and leads to the mentioned violation.

When choosing one or another method of DF determination one usually takes into consideration convenience or desire to prescribe additional conditions for DF (e.g., renormalization independence[11]). Below we shall consider two most wide-spread modes of definition denoting them a)[10] and b)[11]. The mode dependent expressions shall be labelled by the corresponding index.

2. In method a)[10] the matrix elements of local Wilson operators are identified in the normalization point $Q^2 = Q_0^2$ with moments of the corresponding DF. In this case Q^2-evolution of DF is controlled by renormalization group functions β and γ_n only,

which are calculated in double-loop approximation. Coefficient functions $C_n^{(i)}(1,\bar{g}^2(Q^2))$ of Wilson operators are factorized and considered as the moments of elementary cross sections of the certain current scattering onto quarks or gluons. An advantage of such a determination of DF consists in most explicit parton interpretation of QCD formulae.

In method a) quark and gluon distribution functions depend upon the method of theory renormalization. This is not a problem, however, as the renormalization dependence mutually reduces in the product of DF and coefficient functions $C_n^{(i)}(1,\bar{g}^2(Q^2))$, which constitute the observed values. Below we shall use the $\overline{MS}$ scheme of renormalization.

The considered way of DF determination is characterized in the moments representation by the following connection between the structure functions and quark and gluon distribution functions:

$$\begin{aligned}M_2(n,Q^2) &= \int_0^1 dx\, x^{n-2} F_2(x,Q^2) = \frac{1}{6}\langle f_v^{(a)}(Q^2)\rangle_n \Big(1 + \\ &+ \frac{\bar{g}^2(Q^2)}{16\pi^2}\bar{B}_{2,n}^{NS}\Big) + \frac{5}{18}\Big\{\langle \Sigma^{(a)}(Q^2)\rangle_n \Big(1 + \frac{\bar{g}^2(Q^2)}{16\pi^2}\bar{B}_{2,n}^{\psi}\Big) + \frac{\bar{g}^2(Q^2)}{16\pi^2}\bar{B}_{2,n}^{G}\langle f_g^{(a)}(Q^2)\rangle_n\Big\},\end{aligned} \tag{9}$$

$$\begin{aligned}M_3(n,Q^2) &= \int_0^1 dx\, x^{n-2}(xF_3(x,Q^2)) = \\ &= -3\langle f_v^{(a)}(Q^2)\rangle_n \Big(1 + \frac{\bar{g}^2(Q^2)}{16\pi^2}\bar{B}_{3,n}^{NS}\Big).\end{aligned}$$

$\bar{B}_{k,n}$ are coefficients of expansion of Wilson coefficient functions $C_n^{(i)}(1,\bar{g}^2(Q^2))$ in effective couplimg constant

$$\frac{\bar{g}^2(Q^2)}{16\pi^2} = \frac{1}{\beta_0 \ln Q^2/\Lambda^2}\Big(1 - \frac{\beta_1}{\beta_0^2}\frac{\ln\ln Q^2/\Lambda^2}{\ln Q^2/\Lambda^2}\Big),$$

where $\beta_1 = 102 - \frac{38}{3} n_f$. Explicit expressions of $\bar{B}_{k,n}$ are given in Appendix A. In this Appendix one finds also the connection of F_2 and F_3 with DF in representation of Bjorken variable x, obtained through Mellin transformation of formulae (9).

DF moments in this case obey the following evolution equations:

$$\langle f_v^{(a)}(s)\rangle_n = \langle f_v^{(a)}(0)\rangle_n H_{NS}^n(Q^2, Q_0^2) \exp\{-d_{NS}^n \cdot s\}$$

$$
\begin{aligned}
\langle\Sigma^{(a)}(s)\rangle_n &= \{(1-\alpha_n)\langle\Sigma^{(a)}(0)\rangle_n - \tilde{\alpha}_n\langle f_g^{(a)}(0)\rangle_n\} H^n_{+\psi}(Q^2,Q_0^2)\times \\
&\times e^{-d_+^n s} + \{\alpha_n\langle\Sigma^{(a)}(0)\rangle_n + \tilde{\alpha}_n\langle f_g^{(a)}(0)\rangle_n\} H^n_{-\psi}(Q^2,Q_0^2)\, e^{-d_-^n s}, \\
\langle f_g^{(a)}(s)\rangle_n &= \{\alpha_n\langle f_g^{(a)}(0)\rangle_n - \epsilon_n\langle\Sigma^{(a)}(0)\rangle_n\}\times \\
&\times H^n_{+g}(Q^2,Q_0^2)\, e^{-d_+^n s} + \{(1-\alpha_n)\langle f_g^{(a)}(0)\rangle_n + \\
&+ \epsilon_n\langle\Sigma^{(a)}(0)\rangle_n\} H^n_{-g}(Q^2,Q_0^2)\, e^{-d_-^n s}.
\end{aligned} \tag{10}
$$

d_i^n, α_n, $\tilde{\alpha}_n$, ϵ_n and $H^n_{\pm i}$ parameters of evolution equations determined by group renormalization functions γ_n and β are given in Appendix B.

3. Method b) of DF determination has been suggested in paper/11/. The connection of structure functions moments with DF is as follows:

$$
M_2(n,Q^2) = \frac{5}{18}\langle\Sigma^{(b)}(Q^2)\rangle_n + \frac{1}{6}\langle f_v^{(b)}(Q^2)\rangle_n , \tag{11}
$$

$$
M_3(n,Q^2) = -3\langle f_v^{(b)}(Q^2)\rangle_n \left(1 + \frac{\bar{g}^2(Q^2)}{16\pi^2}(\bar{B}^{NS}_{3,n} - \bar{B}^{NS}_{2,n})\right). \tag{12}
$$

As is seen, a characteristic feature of this determination is a parton formula for F_2 and, as it turns out to be (see Appendix A), a simpler formula for F_3 as compared with mode a). This makes it more suitable for practical applications. Renormalization invariance of DF is not the less remarkable feature of mode b). It follows obviously from/11/ due to renormalization invariance of $M_2(n,Q^2)$.

The evolution of moments has now the following form:

$$
\begin{aligned}
\langle f_v^{(b)}(s)\rangle_n &= \langle f_v^{(a)}(s)\rangle_n \left(1 + \frac{\bar{g}^2(Q^2)}{16\pi^2}\bar{B}^{NS}_{2,n}\right), \\
\langle\Sigma^{(b)}(s)\rangle_n &= \langle\Sigma^{(a)}(s)\rangle_n \left(1 + \frac{\bar{g}^2(Q^2)}{16\pi^2}\bar{B}^{\psi}_{2,n}\right) + \langle f_g^{(a)}(s)\rangle_n \frac{\bar{g}^2(Q^2)}{16\pi^2}\bar{B}^{G}_{2,n} \\
\langle f_g^{(b)}(s)\rangle_n &= \langle f_g^{(a)}(s)\rangle_n \left(1 - \frac{\bar{g}^2(Q^2)}{16\pi^2}\bar{B}^{G}_{2,n}\right) - \langle\Sigma^{(a)}(s)\rangle_n \frac{\bar{g}^2(Q^2)}{16\pi^2}\bar{B}^{\psi}_{2,n}.
\end{aligned} \tag{13}
$$

To simplify the writing we have expressed the formulae of DF connection with the structural functions through DF - $f_i^{(a)}$, which evolutionize according to equations (10).

4. The above-given formulae of DF connection with the structure functions as well as the formula of the moments evolution may change if non-perturbative effects are included.

Now we shall consider the effects, mentioned in the introduction and connected with the presence of large-scale structural formations, i.e., spectroscopic (constituent) quarks, in hadrons. The latter are the conglamerates consisting of one valence quark surrounded by the sea of quark-antiquark pairs and gluons.

In the framework of the proposed two-level model the existence of three spectroscopic quarks in a nucleon is taken into consideration. Their potential interaction ensured confinement (first structural level). Standard methods of QCD perturbations theory are considered to be acceptable for the description of the parton structure of the constituent quarks (second structural level). That means that DF of current valence $f_{v/q}$, sea $f_{s/q}$ quarks and gluons $f_{g/q}$ in the constituents quark q are considered as solutions of QCD EE (4)-(6) in LO and (10), (13) in NO.

In this model the connection of the mentioned DF with the observable structure functions is as follows:

$$
\begin{aligned}
F_2(x,Q^2) &= x\int_x^1 \frac{dy}{y}\phi(y)\{f_{v/q}(x/y,Q^2) + \frac{20}{3}f_{s/q}(\frac{x}{y},Q^2)\} \\
F_3(x,Q^2) &= -3\int_x^1 \frac{dy}{y}\phi(y) f_{v/q}(\frac{x}{y},Q^2).
\end{aligned} \tag{14}
$$

Here $\phi(y)$ is a function of the spectroscopic quark distribution in y part of the longitudinal momentum $P_\parallel$ of the nucleon, defined in the infinite momentum frame ($P_\parallel \to \infty$) IMF by the expression

$$
\phi(y) = \lim_{P\to\infty} \frac{\int d\Gamma\, \delta(y - \frac{p_{\parallel 3}}{P_\parallel})|\Phi_3(p_1,p_2,p_3)|^2}{\int d\Gamma\, |\Phi_3(p_1,p_2,p_3)|^2}, \tag{15}
$$

where

$$
d\Gamma = \delta^4(P - \sum_{i=1}^{3} p_i)\prod_{i=1}^{3} dp_i^4\, \theta(\frac{p_{\parallel i}}{P_\parallel}). \tag{16}
$$

The wave function of 3-quark system $\Phi_3(p_1,p_2,p_3)$ is a solution of the Schrödinger relativistic equation with the harmonic oscillation potential. The relativistically covariant condition of Takabayashi/12/ is prescribed for it to get rid of the growing non-physical time-like modes. The final expression of $\phi(y)$ is as follows:

$$
\phi(y) = \frac{\exp\{-\gamma(1-3y)^2\}\,\mathrm{erf}\{\sqrt{3\gamma}(1-y)\}}{\int_0^1 \exp\{-\gamma(1-3y)^2\}\,\mathrm{erf}\{\sqrt{3\gamma}(1-y)\}dy}, \tag{17}
$$

where γ is the oscillator parameter. Let us use now relations (14), (17) for the quantitative phenomenological estimations of the contribution of non-perturbative effects caused by the presence of three spectroscopic quarks in the nucleon.

2. INITIAL CONDITIONS FOR THE QCD EVOLUTION EQUATIONS

Let us prescribe non-perturbation initial conditions for QCD EE on the basis of our earlier developed [7] statistical parton model with Regge symptotic.

One of the starting points of the model has become the assumption on the possible reconstruction of the distribution functions of valence f_v, sea f_s quarks and gluons f_g in the hadron H according to their behaviour in the vicinity of $x \sim 0$ with the help of a certain system of operators $A_H^{(i)}$

$$\begin{aligned} f_v(x,Q_0^2) &= \hat{A}_H^{(v)}(\bar{f}_v,\bar{f}_s,\bar{f}_g \mid x), \\ f_s(x,Q_0^2) &= \hat{A}_H^{(s)}(\bar{f}_v,\bar{f}_s,\bar{f}_g \mid x), \\ f_g(x,Q_0^2) &= \hat{A}_H^{(s)}(\bar{f}_v,\bar{f}_s,\bar{f}_g \mid x), \\ x &\in [0,1], \end{aligned} \tag{18}$$

where

$$f_i(x,Q_0^2) \underset{x\to 0}{\longrightarrow} \bar{f}_i(x,Q_0^2). \tag{19}$$

The region of small values of x is characterized by high multiplicity of protons and absence of kinematic correlations between them. The limit $x \to 0$ corresponds to the well-known Regge limit $\nu \to \infty$, Q^2-fixed. This allows a number of physically plansible assumptions on the behaviour of distribution functions in this region and we can try to determine the form of the limiting functions $\bar{f}_i$.

The Regge analysis of the amplitude of the virtual Compton effect leads to the following expressions:

$$\begin{aligned} &\bar{f}_v(x,Q_0^2) = a_v x^{-a(0)}, \quad \bar{f}_s(x,Q_0^2) = a_s x^{-1}, \\ &\bar{f}_g(x,Q_0^2) = g x^{-1} e^{-\beta x}. \end{aligned} \tag{20}$$

Here the valence quark distributions are connected with the change of the non-singlet A_2-meson Regge trajectory (the intercept $a(0) = \frac{1}{2}$) the distribution of sea quarks and gluons - with the exchange of the vacuum trajectory (pomeron). We have introduced a preasymptotic factor $e^{-\beta x}$, additional to the Regge behaviour, in the gluon distribution $\bar{f}_g$. It is interpreted as a statistical Boltzman exponent.

Extrapolating operators $\hat{A}_H^{(i)}$ have been constructed through the systematic taking into consideration of permissible quark-gluon configurations in the nucleon and kinematic correlation factors. This consideration is based on the statistical parton ideas of Bjorken, Paschos [22] and Kuti, Weisskopf [23]. It should be stressed, that all the assumptions used to obtain $\hat{A}_H^{(i)}$ do not contradict the qualitative picture of the intrahadronic picture put forward by the quantum chromodynamics in recent years. To make the picture full we give the explicit form of extrapolating operators:

$$\hat{A}_H^{(i)}(\bar{f}_v,\bar{f}_s,\bar{f}_g \mid x) = \frac{\delta \ln Z_k^{-1}\{\bar{f}\}}{\delta \ln \bar{f}_i(x)} \tag{21}$$

the producing functional $Z_k\{\bar{f}\}$ is defined by the expression

$$\begin{aligned} Z_k\{\bar{f}\} = \frac{1}{2\pi K!} \int_{-\infty}^{+\infty} d\xi\, e^{i\xi} \left(\int_0^{\infty} dx\, e^{-i(\xi-i0)x} \bar{f}_v(x)\right)^k \\ \times \exp\left\{\int_0^{\infty} dx\, e^{-i(\xi-i0)x} (8\bar{f}_s(x) + \bar{f}_g(x))\right\}. \end{aligned} \tag{22}$$

k is the number of valence quarks and antiquarks in hadron H. Despite logarithmic divergences ($\bar{f}_{s,g} \sim 1/x$)*, expressions (19), (21) are finite for DF. This is predetermined by the fact, that the above-mentioned divergences can be factorized and disappear during variational differentiation. Let us give the predictions of the model for the case of the nucleon, π-meson and spectroscopic quark - a conglomerate of a valence quark and the sea of quark-antiquark pairs and gluons around it.

Together with the distribution functions f_i we shall write down the expressions of their moments necessary for further considerations

$$\langle f(Q^2)\rangle_n = \int_0^1 dx\, x^{n-1} f(x,Q^2). \tag{23}$$

---

* The unfixed number of degrees of freedom for the system of sea quark-antiquark pairs and gluons in the hadron is the reason for these divergences.

1. Nuçleon distributions:

$$f_{v/N}(x,Q_0^2) = x^{-1/2}\,\frac{(1-x)^{r}}{B(1/2, r+1)}\,\frac{D^{(2)}_{q,1}(1-x)}{D^{(3)}_{q,1}(1)}, \tag{24}$$

$$f_{s/N}(x,Q_0^2) = \frac{r-g}{8x}(1-x)^{r+1/2}\,\frac{D^{(3)}_{q,1}(1-x)}{D^{(3)}_{q,1}(1)}, \tag{25}$$

$$f_{g/N}(x,Q_0^2) = \frac{g}{x}e^{-\beta_g x}(1-x)^{r+1/2}\,\frac{D^{(3)}_{g,1}(1-x)}{D^{(3)}_{g,1}(1)}, \tag{26}$$

$$\langle f_{v/N}(Q_0^2)\rangle_n = \frac{B(r+1, n-1/2)}{B(r+1, 1/2)}\,\frac{D^{(2n+1)}_{q,1}(1)}{D^{(3)}_{q,1}(1)}, \tag{27}$$

$$\langle f_{s/N}(Q_0^2)\rangle_n = \frac{r-g}{8}B(r+3/2, n-1)\,\frac{D^{(2n+1)}_{q,1}(1)}{D^{(3)}_{q,1}(1)}, \tag{28}$$

$$\langle f_{g/N}(Q_0^2)\rangle_n = gB(r+3/2, n-1)\,\frac{D^{(2n+1)}_{g,n}(1)}{D^{(3)}_{g,1}(1)}. \tag{29}$$

2. π-meson distributions:

$$f_{v/\pi}(x,Q_0^2) = x^{-1/2}\,\frac{(1-x)^{r-1/2}}{B(1/2, r+1/2)}\,\frac{D^{(1)}_{q,1}(1-x)}{D^{(2)}_{q,1}(1)}, \tag{30}$$

$$f_{s/\pi}(x,Q_0^2) = \frac{r-g}{8x}(1-x)^{r}\,\frac{D^{(2)}_{q,1}(1-x)}{D^{(2)}_{q,1}(1)}; \tag{31}$$

$$f_{g/\pi}(x,Q_0^2) = \frac{g}{x}e^{-\beta_g x}(1-x)^{r}\,\frac{D^{(2)}_{g,1}(1-x)}{D^{(2)}_{g,1}(1)}, \tag{32}$$

$$\langle f_{v/\pi}(Q_0^2)\rangle_n = \frac{B(r+1/2, n-1/2)}{B(r+1/2, 1/2)}\,\frac{D^{(2n)}_{q,1}(1)}{D^{(2)}_{q,1}(1)}, \tag{33}$$

$$\langle f_{s/\pi}(Q_0^2)\rangle_n = \frac{r-g}{8}B(r+1, n-1)\,\frac{D^{(2n)}_{q,1}(1)}{D^{(2)}_{q,1}(1)}, \tag{34}$$

$$\langle f_{g/\pi}(Q_0^2)\rangle_n = gB(r+1, n-1)\,\frac{D^{(2n)}_{g,n}(1)}{D^{(2)}_{g,1}(1)}. \tag{35}$$

3. Distributions in spectroscopic quark q:

$$f_{v/q}(x,Q_0^2) = x^{-1/2}\,\frac{(1-x)^{r-1}}{B(r, 1/2)}\,\frac{D^{(0)}_{q,1}(1-x)}{D^{(1)}_{q,1}(1)}, \tag{36}$$

$$f_{s/q}(x,Q_0^2) = \frac{r-g}{8x}(1-x)^{r-1/2}\,\frac{D^{(1)}_{q,1}(1-x)}{D^{(1)}_{q,1}(1)}, \tag{37}$$

$$f_{g/q}(x,Q_0^2) = \frac{g}{x}e^{-\beta_g x}(1-x)^{r-1/2}\,\frac{D^{(1)}_{g,1}(1-x)}{D^{(1)}_{g,1}(1)}, \tag{38}$$

$$\langle f_{v/q}(Q_0^2)\rangle_n = \frac{B(r, n-1/2)}{B(r, 1/2)}\,\frac{D^{(2n-1)}_{q,1}(1)}{D^{(n)}_{q,1}(1)}, \tag{39}$$

$$\langle f_{s/q}(Q_0^2)\rangle_n = \frac{r-g}{8}B(r+1/2, n-1)\,\frac{D^{(2n-1)}_{q,1}(1)}{D^{(1)}_{q,1}(1)}, \tag{40}$$

$$\langle f_{g/q}(Q_0^2)\rangle_n = gB(r+1/2, n-1)\,\frac{D^{(2n-1)}_{g,n}(1)}{D^{(1)}_{g,1}(1)}. \tag{41}$$

Here

$$D^{k}_{j,m}(y) = \Phi(g+m-1,\ r+\frac{k}{2};\ -\beta_j y) \tag{42}$$

$\Phi(\alpha,\beta;z)$ is a degenerated hypergeometric function.

3. APPROXIMATE SOLUTION OF QCD EVOLUTION EQUATIONS

Knowing the initial conditions for the evolution equations in point $Q^2 = Q_0^2$ for the nucleon (24)-(29), π-meson (30)-(35) and spectroscopic quark (36)-(41), we shall find the approximate solutions corresponding to the three cases. Let us use a well-known variational method of test functions. The essence of the method consists in the following:

a) We shall find the solutions of EE in the same form, as the initial conditions are prescribed, introducing the dependence of parameters r, g, β_q, β_g upon Q^2. We shall limit ourselves by the linear approximation in the evolution variable s and write down

$$g(Q^2) = g_{(0)} + g_{(1)}\cdot s,$$

$$r(Q^2) = r_{(0)} + r_{(1)}\cdot s, \tag{43}$$

$$\beta_{q,g}(Q^2) = \beta_{(0)} + \beta_{(1)q,g}\cdot s.$$

In the next order (NO) of QCD perturbation theory the initial conditions, parameters $r_{(0)}$, $g_{(0)}$, $\beta_{(0)}$ and evolution parameters $g_{(1)}$, $\beta_{(1)q}$, $\beta_{(1)g}$ depend upon the method of DF determination (cf. Sect.1) and upon the value of QCD-parameter Λ. In LO-approximation there is no dependence of such kind.

Thus the problem of finding the approximate solution is reduced to the searched for parameters $r_{(0)}$, $g_{(0)}$, $\beta_{(0)}$, $r_{(1)}$, $g_{(1)}$, $\beta_{(1)q}$, $\beta_{(1)g}$ of the test functions.

b) Parameter $r_{(1)}$ can be determined from the QCD-predictable threshold $x \to 1$ behaviour of DF [13]. It has the following value:

$$r_{(1)} = \frac{4C_F}{\beta_0} = \frac{16}{25}, \qquad (44)$$

where $C_F = \frac{4}{3}$, $\beta_0 = 11 - \frac{2}{3}n_f$, $n_f = 4$ is a number of quark flavors (for details see ref. [6]).

c) The initial conditions parameters $r_{(0)}$, $g_{(0)}$, $\beta_{(0)}$ can be determined through the comparison with the experimental data on deep inelastic lepton-nucleon scattering at fixed $Q^2 = Q_0^2$.

d) Evolution parameters $g_{(1)}$, $\beta_{(1)q,g}$ (see ref. [6,7]) can be determined through the minimization of the following square functional:

$$\bar{\chi}^2[f] = \sum_{i=v,\Sigma,g} \sum_{n}^{\infty} \int_0^1 ds\, g(s,n) \{ \langle f_i(s|g_{(1)}, \beta_{(1)q,g}) \rangle_n - \langle f_i^{QCD}(s) \rangle_n \}^2, \qquad (45)$$

where $\langle f_i(s|g_{(1)}, \beta_{(1)q,g}) \rangle_n$ are the DF moments calculated through formulae (27)-(29) for the nucleon, (33)-(35) for π-meson (39)-(41) for spectroscopic quark, where the required parameters are present; momenta $\langle f_i^{QCD}(s) \rangle_n$ are calculated using right parts of EE (4)-(6), (10), (13) with regard to a considered approximation and mode of DF determination. The initial conditions of EE are determined by the equation $\langle f_i^{QCD}(0) \rangle_n = \langle f_i(0) \rangle_n$ and parameters $r_{(0)}$, $g_{(0)}$, $\beta_{(0)}$ found earlier; $g(s,n)$ is a weight function controlling the accuracy distribution in the region of averaging. Naturally we shall consider only the finite region and choose the function in the simplest form $g(s,n) = \theta(20-n)\theta(1-s)$ (interval $0 \le s \le 1$ at $Q^2 = 9$ GeV^2 and $\Lambda = 0.5$ GeV corresponds to $9 \le Q^2 \le 18 \cdot 10^3$ GeV^2).

The conditions of minimum for functional $\bar{\chi}^2$(45) are a system of three equations in relation to unknown parameters $g_{(1)}$, $\beta_{(1)q}$, $\beta_{(1)g}$. Being very complicated, the system has been numerically solved in the computer.

On the basis of this method we have found in the explicit form the approximate solutions of EE for the nucleon, π-meson and spectroscopic quarks in LO- and NO-approximations of the perturbation theory. The obtained values of the parameter for the nucleon are listed in Table 1; for the π-meson, in Table 2; for the spectroscopic quark, in Table 3. The errors of the approximate solutions do not exceed 5-6% up to $Q^2 \sim 20 \cdot 10^3$ GeV^2, decreasing sharply as Q^2 decreases.

4. CALCULATION OF OBSERVED VALUES AND COMPARISON WITH EXPERIMENTAL DATA

1. Up to now we have been considering the logarithmic QCD effects only. However, for the calculation of the observed structur

Table 1

Parameters / Method of determination	Λ (GeV/c)	$\tau_{(0)}$	$g_{(0)}$	$\beta_{(0)}$	$g_{(1)}$	$\beta_{(1)q}$	$\beta_{(1)g}$	Q_0^2 (GeV^2)
$NO_{(a)}$	0.5	2.31	1.14	-2.15	0.621	-1.38	5.953	
	0.3	2.27	1.15	-2.06	0.623	-1.35	5.97	
	0.1	2.20	1.13	-1.99	0.618	-1.299	5.96	
$NO_{(b)}$	0.5	1.97	0.992	-2.32	0.050	-2.89	6.0	9
	0.3	1.97	0.992	-2.32	0.061	-2.78	6.5	
	0.1	1.97	0.992	-2.32	0.066	-2.59	6.4	
LO	does not depend upon	1.97	0.992	-2.32	0.619	-0.995	5.951	

Table 2

Parameters / Approximation	$g_{(0)}$	$\tau_{(0)}$	$\beta_{(0)}$	$g_{(1)}$	$\tau_{(1)}$	$\beta_{(1)q}$	$\beta_{(1)g}$	Q_0^2
LO	1.06	1.5	-1.50	0.99	0.64	-0.29	6.25	16

Table 3

Parameters / Approximation	Λ	$g_{(0)}$	$\tau_{(0)}$	$\beta_{(0)}$	$g_{(1)}$	$\tau_{(1)}$	$\beta_{(1)q}$	$\beta_{(1)g}$	γ	Q_0^2
LO		1.04	1.27	1.11	0.33	16/25	-0.23	3.04	0.02	1
$NO_{(b)}$	0.17	1.06	1.30	0.44	0.25	16/25	-0.97	-	0.02	

functions to be compared with the experimental data power mass effects m^2/Q^2 (m is mass of nucleon-target) should be taken into consideration, because their contribution to the structure functions at moderate Q^2 is comparable with the contribution of earlier considered $\bar{g}^2(Q^2)$ corrections. Like $\bar{g}^2(Q^2)$ corrections, they are especially high at large and small Q^2. Mass effects are of pure kinematic nature and can be taken into consideration within the limits of ξ-scaling formalism /14/. We would like to note, however, that ξ-scaling enables us of taking into correct consideration the mass effects in the region of x ($x \leq 0.8$) only. This fact should be taken into account when using the formulae of structure functions calculation obtained on its basis (the formulae are given in Appendix B).

Taking into consideration everything mentioned above we calculate structure functions F_2^{ep} and $F_3^{\nu(\bar{\nu})N}$ taking account of next-to-leading corrections and power mass effects. For this purpose it is necessary to substitute structure functions F_2^{NO} and F_3^{NO} corresponding to the case with zero mass of the target (see the formulae of Sect.1) in the formulae of Appendix B.

2. We have compared thus obtained numerical values of structure functions F_2^{ep} and $F_3^{\nu(\bar{\nu})N}$ with the experimental data from SLAC /15/, FNAL /16/ ($e(\mu) + p \to e(\mu) + X$) and CDHS /17/ ($\nu(\bar{\nu}) + N$ (isoscalar target) $\to \mu^-(\mu^+) + X$). It turned out, that curves calculated at $\Lambda = 0.5$ GeV are in good agreement with the experimental points. These curves and the mentioned experimental data are given in Fig.1, as well as curves for F_2^{LO} and F_3^{LO} calculated in LO /7/ with the same value of $\Lambda = 0.5$ GeV. These curves are given just for comparison. As is seen, the experimental situation of the problem allows one to speak about the fact that the new trends in the behaviour of the structure functions introduced by the corrections (e.g., a stronger growth at low x) correspond better to the trend which shows itself in the experiment.

It should be notices, that the value of $\Lambda = 0.5$ GeV, we have chosen earlier, is not the result of an all-sided optimization*. Extraction of QCD parameter Λ from the experimental data is quite a delicate problem and must be considered specially what is beyond the purpose of the present paper.

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*Among those not taken into consideration there are, e.g., "twist" corrections of power character $(M_0^2/Q^2)^n$, M_0 is the hadron scale. Presently, it is impossible to take into account within the limits of QCD.

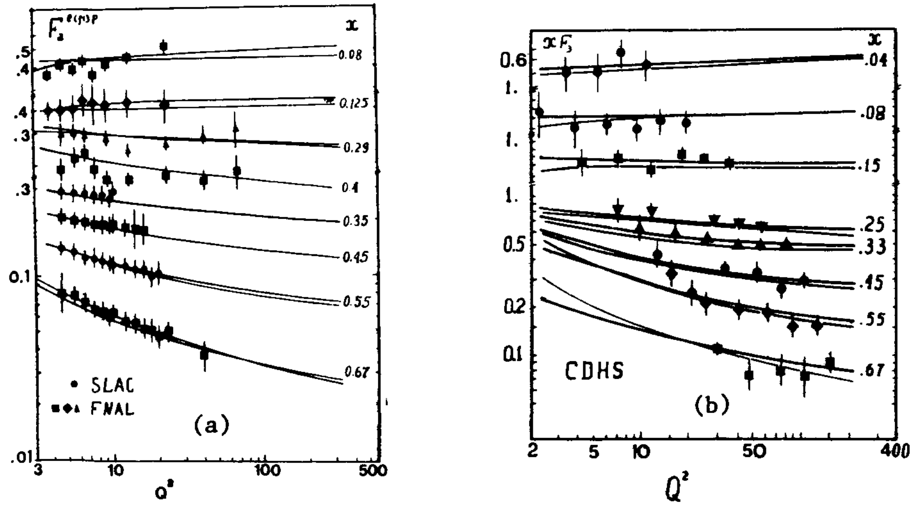


Fig.1. Continuous-line curves are: (a) F_2^{LO} , (b) xF_3^{LO}, dash-line curves - (a) F_2^{NO} , (b) xF_3^{NO} ($\Lambda = 0.5$ GeV). Dots are the experimental data of SLAC /15/, FNAL /16/ ($e(\mu)+N \to e(\mu)+X$), CDHS /17/ ($\nu(\bar{\nu})+N \to \mu^-(\mu^+)+X$).

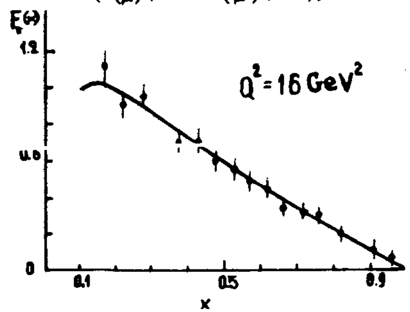


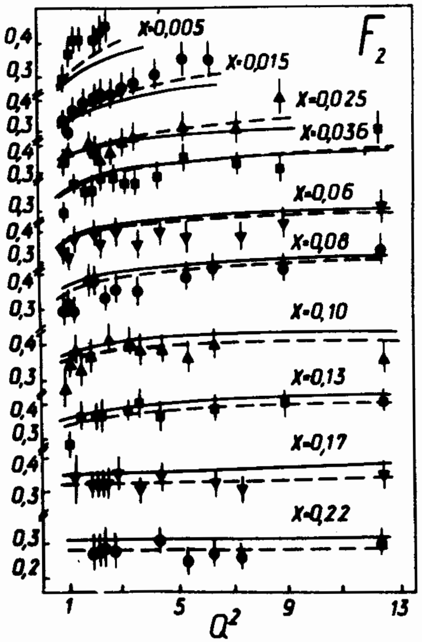
Fig.2. Structure function of π-meson $F_\pi = 2.4x(f_{v/\pi} + 3.7f_{s/\pi})$. The curve is a theoretical prediction. Dots are the experimental data /18/.

3. We have calculated the structural function of π-meson

$$F_\pi(x,Q) = 2.4x\{f_{v/\pi}(x,Q^2) + 3.7f_{s/\pi}(x,Q^2)\}$$

which determines the cross section of production of massive lepton pair in πN-interactions at high energies. Substituting obtained expressions (30), (31) as distribution functions of valence $f_{v/\pi}$ and seq $f_{s/\pi}$ quarks in π-meson, we compare F_π with the experimental data. The results of the comparison are given in Fig.2. The available experimental material being not enough /18/, it is impossible to do a more detailed analysis of the obtained predictions for F_π .

4. Let us calculate, at last, the nucleon structural functions $F_2^{e(\mu)N}$, $F_3^{\nu(\bar{\nu})N}$ in the two-level model at moderate Q^2 ($Q^2 \leq 10$ (GeV/c)^2) where we expect the most essential mani-



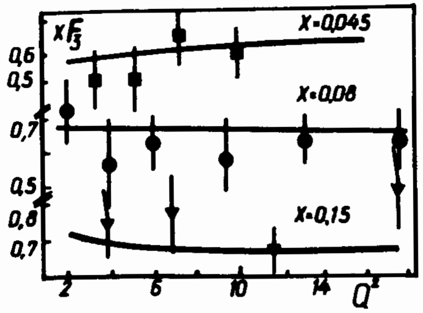
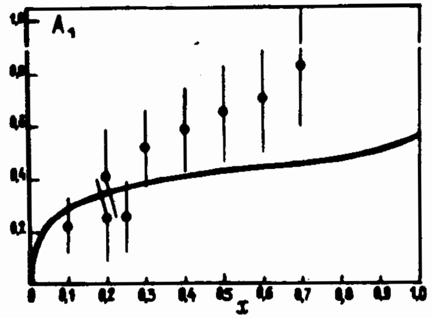


Fig.3. Structure functions (a) $F_2^{e(\mu)p}$ (b) $xF_3^{\nu(\bar\nu)N}$ in the region of small x and Q^2. Continuous-line curves are (a) F_2^{LO}, (b) xF_3^{LO}. Dash-line curves are (a) F_2^{NO}. The large scale spectoscopic structure of the nucleon is taken into consideration. Dots represent the experimental data [16,17].

Fig.4. The coefficient of the longitudinal asymmetry A_1 in the deep inelastic polarized ep - scattering. The theoretical curve is calculated with the large-scale spectroscopic structure of the proton taken into consideration. Dots represent the experimental data [19].



festation of the spectroscopic quark structure of hadrons in deep inelastic scattering. Let us use now the formulae (14) and the obtained solving QCD EE functions of distribution of quark and gluons in the spectroscopic quark to determine the value of the structure functions for various x and Q^2. The results of the comparison with the experimental data [16,17] are shown in Fig.3. Additionally a satisfactory agreement with the experimental data [19] has been obtained for measurements of coefficient of the longitudinal asymmetry A_1 in scattering of electrons on the polarized proton target (see Fig.4).

CONCLUSION

In this paper we have obtained explicit expressions for the distribution functions of quarks and gluons inside the nucleon and π-meson which possess Q^2-dependence predicted by QCD in leading and next-to-leading orders of the perturbation theory. In other words, these distribution functions are new approximate solutions of QCD evolution equations. We have found the explicit forms of singlet and gluon functions of distribution, which possess this property in the next-to-leading order of the perturbation theory.

In the considered approximation the way of determining the distribution functions and corresponding evolution equation is ambiguous. In this paper we have obtained approximate solutions for two well-known definition [10,11].

We have specified the list of formulae of DF connection with the structure functions in $\overline{MS}$ scheme and added new formulae $(\bar B_k^G(z))$ which are necessary for practical calculations.

We would like to indicate some characteristic features of the obtained solutions.

a) They have a correct threshold behaviour $x \to 1$ from the point of view of perturbation theory of QCD.

b) The predictions of the phenomenological quark-gluon model of nucleon [6] have been used as the initial conditions of the evolution equations, but not the empiric parametrization of the experimental data at $Q^2 = Q_0^2$.

c) x-dependence of the approximate solutions at all Q^2 has also been chosen on the basis of this model.

Our approach to the approximate solving of the evolution equations based on the phenomenological model has wider possibilities as compared with the approaches based on empiric parametrizarions [4], e.g., it is possible to calculate theoretically from the unique point of view distribution functions in any hadron (π, K, etc.) to obtain multiparticle and correlation functions. Calculation of these functions has become quite an actual problem.

Using the mentioned results we have calculated the structure functions $F_2^{e(\mu)N}$ and $F_3^{\nu(\bar\nu)N}$ of deep inelastic $e(\mu)N$- and $\nu(\bar\nu)N$ scattering, and the structure function F_π of π-meson which characterizes the cross section of production of massive muon pairs in πN-interactions at high energies. It is shown that the obtained predictions are in good agreement with the experimental data. Besides the logarithmic perturbative QCD corrections for the structure functions, we have taken into account the power mass corrections. For this purpose we have used the standard formalism of ξ-scaling. Besides, we have studied within the framework of the two-level model of hadrons the influence of non-perturbation effects caused by the presence of large-scale structures, like spectroscopic quarks, in the nucleon on the processes of deep inelastic scattering. We have also calculated the asymmetry coefficient A_1 in the scattering of polarized electrons on polarized protons.

Thus, the model described from the unique point of view a large number of phenomena in physics of elementary particles.

APPENDIX A

We give the formulae of structure functions F_i^{NO} connection with the distribution functions in the next-to-leading order. For the considered methods a)[10] and b)[11] of determining the distribution functions they have the following form:

$$\text{a)}\quad F_2^{NO}(x,Q^2) = F_2^{(a)PM}(x,Q^2) + \frac{\bar g^2(Q^2)}{16\pi^2}\int_x^1 \frac{dy}{y}\{\bar B_2^{\psi}(\frac{x}{y})\times$$

$$\times F_2^{(a)PM}(y,Q^2) + \frac{5}{18} y f_g^{(a)}(y)\bar B_2^G(\frac{x}{y})\}, \tag{A1}$$

$$xF_3^{NO}(x,Q^2) = xF_3^{(a)PM}(x,Q^2) + \frac{\bar g^2(Q^2)}{16\pi^2}\int_x^1 \frac{dy}{y} F_3^{(a)PM}(y,Q^2)\, y\bar B_3^{\psi}(\frac{x}{y}),$$

$$\text{b)}\quad F_2^{NO}(x,Q^2) = F_2^{(b)PM}(x,Q^2),$$

$$xF_3^{NO}(x,Q^2) = xF_3^{(b)PM}(x,Q^2) + \frac{\bar g^2(Q^2)}{16\pi^2}\times \tag{A2}$$

$$\times\int_x^1 \frac{dy}{y}(yF_3^{(b)PM}(y,Q^2))(\bar B_3^{\psi}(\frac{x}{y}) - \bar B_2^{\psi}(\frac{x}{y})),$$

where $F_m^{(n)PM}$ are calculated through the formulae of the parton model.

$$F_2^{(n)PM}(x,Q^2) = \frac{5}{18}\Sigma^{(n)}(x,Q^2) + \frac{1}{6} f_v^{(n)}(x,Q^2) \tag{A3}$$

$$F_3^{(n)PM}(x,Q^2) = -3f_v^{(n)}(x,Q^2).$$

Convolution kernels $\bar B_k^{\psi}(x)$, $\bar B_k^{G}(x)$ are melline transformations of coefficients $\bar B^{\psi}_{k,n}$, $\bar B^{G}_{k,n}$ appeared in (9),(12),(13). Let us put down these values in the renormalization scheme $\overline{MS}$.

$$\bar B^{\psi}_{2,n} = \bar B^{NS}_{2,n} = \frac{4}{3}\{3\sum_{k=1}^{n}\frac{1}{k} - 4\sum_{k=1}^{n}\frac{1}{k^2} - \frac{2}{n(n+1)}\sum_{k=1}^{n}\frac{1}{k} + \tag{A4}$$

$$+4\sum_{s=1}^{n}\frac{1}{s}\sum_{k=1}^{s}\frac{1}{k} + \frac{3}{n} + \frac{3}{n+1} + \frac{2}{n^2} - 9\},$$

$$\bar B^{\psi}_{3,n} = \bar B^{NS}_{3,n} = \bar B^{NS}_{2,n} - \frac{4}{3}\,\frac{4n+2}{n(n+1)},$$

$$\bar B^{G}_{2,n} = 2n_f\{\frac{4}{n+1} - \frac{4}{n+2} + \frac{1}{n^2} - \frac{n^2+n+2}{n(n+1)(n+2)}(1+\sum_{k=1}^{n}\frac{1}{k}),$$

$$\bar B_2^{\psi}(x) = \frac{4}{3}x\{2(1-x)\ln\frac{1-x}{x} + 4x[\frac{\ln(1-x)}{1-x}]_+ - \tag{A5}$$

$$-\frac{3x}{(1-x)_+} + 3 + 4x - 4x\frac{\ln x}{1-x} - (\frac{2}{3}\pi^2 + 9)\delta(1-x)\},$$

$$\bar B_3^{\psi}(x) = \bar B_3^{NS}(x) = \bar B_2^{NS}(x) - \frac{4}{3}2x(1+x),$$

$$\bar B_2^{G}(x) = 2n_f\, x\{((1-x)^2 + x^2)\ln\frac{1-x}{x} + 8x(1-x) - 1\}.$$

Function $\bar B_2^G(x)$ did not appear in any previous paper. Here $n_f = 4$ is a number of quark flavors. Index "+" in singular expressions indicates the following rules of integration

$$\int_0^1 \frac{h(x)dx}{(1-x)_+} \equiv \int_0^1 dx\,\frac{h(x)-h(1)}{1-x},$$

$$\int_0^1 dx\, h(x)[\frac{\ln(1-x)}{1-x}]_+ \equiv \int_0^1 dx\,\frac{\ln(1-x)}{1-x}[h(x)-h(1)],$$

$h(x)$ is a function, regular in boundary points.

APPENDIX B

We give the list of formulae for the calculation of EE parameters (scheme $\overline{MS}$):

$$H^n_{NS}(Q^2, Q_0^2) = 1 + \frac{\bar g^2(Q^2) - \bar g^2(Q_0^2)}{16\pi^2} Z^n_{NS}, \tag{B1}$$

$$H^n_{\pm i}(Q^2,Q_0^2) = 1 + \frac{\bar g^2(Q^2)-\bar g^2(Q_0^2)}{16\pi^2} Z^n_\pm + K^{in}_\pm \times \left\{ \frac{\bar g^2(Q_0^2)}{16\pi^2}\left[\frac{\bar g^2(Q^2)}{\bar g^2(Q_0^2)}\right]^{d^n_\pm - d^n_\pm} - \frac{\bar g^2(Q^2)}{16\pi^2}\right\}, \tag{B2}$$

$$Z^n_{NS} = \frac{\gamma^{(1),n}_{NS}}{2\beta_0} - \frac{\gamma^{(0),n}_{NS}}{2\beta_0^2}\beta_1 , \qquad Z^n_\pm = \frac{\gamma^{(1),n}_{\pm\pm}}{2\beta_0} - \frac{\lambda^n_\pm}{2\beta_0^2}\beta_1 , \tag{B3}$$

$$K^{\psi n}_\pm = \frac{\gamma^{(1),n}_{\pm\mp}}{2\beta_0+\lambda^n_\pm-\lambda^n_\mp}, \qquad K^{Gn}_\pm = \frac{\gamma^{(1),n}_{\pm\mp}}{2\beta_0+\lambda^n_\pm-\lambda^n_\mp}\cdot\frac{\gamma^{(0),n}_{\psi\psi}-\lambda^n_\mp}{\gamma^{(0),n}_{\psi\psi}-\lambda^n_\pm}, \tag{B4}$$

$$d^n_{NS} = \frac{\gamma^{(0),n}_{NS}}{2\beta_0}, \quad d^n_\pm = \frac{\lambda^n_\pm}{2\beta_0}, \quad \alpha_n = \frac{\gamma^{(0),n}_{NS}-\lambda^n_+}{\lambda^n_- - \lambda^n_+}, \quad \tilde\alpha_n = \frac{\gamma^{(0),n}_{\psi G}}{\lambda^n_- -\lambda^n_+}, \quad \epsilon_n = \frac{\alpha_n(1-\alpha_n)}{\alpha_n}. \tag{B5}$$

Table 4

n	$\gamma^{(1),n}_{NS}$	$\gamma^{(1),n}_{\psi\psi}$	$\gamma^{(1),n}_{\psi G}$	$\gamma^{(1),n}_{G\psi}$	$\gamma^{(1),n}_{GG}$
2	71.37	55.56	-60.34	-55.56	60.34
4	120.14	119.28	10.34	-27.40	151.61
6	147.00	146.82	22.08	-18.28	201.94
8	166.39	166.34	25.96	-14.08	238.16
10	181.78	181.74	27.25	-11.67	267.48
12	194.63	194.58	27.51	-10.11	297.44
14	205.7	205.7	27.29	- 9.00	314.2
16	215.4	215.4	26.82	- 8.17	333.7
18	224.1	224.1	26.25	- 7.52	351.2
20	231.9	231.9	25.63	- 7.00	367.3

The explicit expressions for $\gamma^{(1),n}$ are extremely cumbersome. They can be found in papers /20/ and, in simpler form, in ref. /21/. In Table 4 we give their numerical values used in our paper

$$\lambda^n_\pm = \frac12\left[\gamma^{(0),n}_{\psi\psi}+\gamma^{(0),n}_{GG} \pm \sqrt{(\gamma^{(0),n}_{\psi\psi}-\gamma^{(0),n}_{GG})^2 + 4\gamma^{(0),n}_{\psi G}\gamma^{(0),n}_{G\psi}}\right] \tag{B6}$$

$$\begin{aligned}
\gamma^{(1),n}_{--} &= [D^n_1(\gamma^{(0),n}_{\psi\psi}-\lambda^n_+)+D^n_2\gamma^{(0),n}_{\psi G}]T_n ,\\
\gamma^{(1),n}_{-+} &= [D^n_1(\gamma^{(0),n}_{\psi\psi}-\lambda^n_-)+D^n_2\gamma^{(0),n}_{\psi G}]T_n ,\\
\gamma^{(1),n}_{+-} &= [D^n_3(\gamma^{(0),n}_{\psi\psi}-\lambda^n_+)+D^n_3\gamma^{(0),n}_{\psi G}]T_n ,\\
\gamma^{(1),n}_{++} &= [D^n_3(\gamma^{(0),n}_{\psi\psi}-\lambda^n_-)+D^n_3\gamma^{(0),n}_{\psi G}]T_n .
\end{aligned} \tag{B7}$$

$$\begin{aligned}
D^n_{\binom{1}{3}} &= \pm[\gamma^{(0),n}_{\psi G}\gamma^{(1),n}_{\psi\psi} + (\lambda^n_\mp - \gamma^{(0),n}_{\psi\psi})\gamma^{(1),n}_{\psi G}],\\
D^n_{\binom{2}{4}} &= \pm[\gamma^{(0),n}_{\psi G}\gamma^{(1),n}_{G\psi} + (\lambda^n_\mp - \gamma^{(0),n}_{\psi\psi})\gamma^{(1),n}_{GG}],\\
T_n &= [\gamma^{(0),n}_{\psi G}(\lambda^n_- - \lambda^n_+)]^{-1}.
\end{aligned} \tag{B8}$$

According to the definition

$$\gamma^n_{ij}(g^2) = \gamma^{(0),n}_{ij}\frac{\bar g^2}{16\pi^2} + \gamma^{(1),n}_{ij}\left(\frac{\bar g^2}{16\pi^2}\right)^2 \tag{B9}$$

γ^n_{ij} is a matrix of anomaly dimensions of Wilson operators. Here

$$\gamma^{(0),n}_{G\psi} = -16\,\frac{n^2+n+2}{3n(n^2-1)},$$

$$\gamma^{(0),n}_{NS} = \gamma^{(0),n}_{\psi\psi} = \frac{8}{3}\left[1-\frac{2}{n(n+1)}+4\sum_{k=1}^{n}\frac{1}{k}\right],$$

$$\gamma^{(0),n}_{\psi G} = -4n_f\,\frac{n^2+n+2}{n(n+1)(n+2)}$$

$$\gamma^{(0),n}_{GG} = \frac{4}{3}n_f + 6\left[\frac13 - \frac{4}{n(n+1)} - \frac{4}{(n+1)(n+2)} + 4\sum_{k=2}^{n}\frac{1}{k}\right].$$

APPENDIX C

Kinematic mass effects due to non-zero mass of the nucleon-target m can be taken into account on the basis of ξ-scaling formalism [14]. In this case the structure functions can be calculated through formulae:

$$F_2(x,Q^2) = \left(\frac{x}{\xi}\right)^2 K^3(x,Q^2)\bar{F}_2(\xi,Q^2) + \frac{6m^2x^3}{Q^2}K^4(x,Q^2) \times \int_\xi^1 \frac{dz}{z}\bar{F}_2(z,Q^2) + \frac{12m^4}{Q^4}x^4K^5(x,Q^2) \times \int_\xi^1 dz \int_z^1 \frac{dy}{y^2}\bar{F}_2(y,Q^2), \quad (C1)$$

$$F_3(x,Q^2) = \frac{x}{\xi}K^2(x,Q^2)\bar{F}_3(\xi,Q^2) + \frac{2m^2x^2}{Q^2}K^3(x,Q^2)\int_\xi^1 \frac{dz}{z}\bar{F}_3(z,Q^2), \quad (C2)$$

where

$$K(x,Q^2) = \left(1+4x^2\frac{m^2}{Q^2}\right)^{-1/2}, \quad (C3)$$

$$\xi = \frac{2x}{1+\sqrt{1+4x^2\, m^2/Q^2}}, \quad (C4)$$

$\bar{F}_i$ are the structure functions when $m=0$.

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Бедняков В.А. и др. E2-82-467

Кварк-глюонные функции распределения в адронах

В явном аналитическом виде приближенно решены уравнения КХД, описывающие Q^2-эволюцию функций распределения кварков и глюонов в нуклонах и π-мезонах во втором приближении по эффективной константе сильного взаимодействия $\bar{g}^2(Q^2)$. Начальные условия этих эволюционных уравнений рассчитаны на основе феноменологической кварк-глюонной модели нуклона. В области умеренных Q^2 (1 ГэВ $^2/c^2 \leq Q^2 \leq 10$ ГэВ $^2/c^2$) рассмотрены эффекты кваркового конфайнмента в рамках модели релятивистского осциллятора. Исходя из найденных функций распределения рассчитаны структурные функции глубоконеупругого $\nu(\bar{\nu})N$-, $e(\mu)N$ -рассеяния, коэффициент асимметрии A_1 в рассеянии электронов на поляризованной нуклонной мишени и сечения рождения массивных лептонных пар в πN-взаимодействии. Получено хорошее согласие с экспериментальными данными.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1982

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Quark-Gluon Distribution Functions of Hadrons

The QCD equations with next-to-leading corrections which describe Q^2-evolution of nucleon (π-meson) quark and gluon distribution functions are solved approximately on the basis of a phenomenological quark-gluon model in an explicit analytical form. Using the relativistic oscillator model quark confinement effects are considered in moderate Q^2 region $(1\ (\mathrm{GeV}/c)^2 \leq Q^2 \leq 10\ (\mathrm{GeV}/c)^2)$. On the basis of the found distribution functions the structure functions of deep inelastic $\nu(\bar{\nu})N$-, $e(\mu)N$ -scattering, asymmetry coefficient A_1 in the scattering of polarized electrons on the polarized nucleon target and cross sections of production of massive lepton pairs in πN interaction are calculated. A good agreement with the experimental data is obtained.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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