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SUPERSYMMETRIC QUASIPOTENTIAL EQUATIONS

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1. INTRODUCTION

Usually, when we are dealing with the relativistic two-particle problem in the framework of QFT, it is convenient to use the Bethe-Salpeter equation. However, in this case there arise some difficulties because of the nondefinite sign of the norm of the two-particle amplitude. The origin of these difficulties is the existence in the theory of one nonphysical parameter, relative-time or its conjugate, relative-energy. An extremely useful procedure for removing these difficulties has been suggested by Logunov and Tavkhelidze [1]. The main idea of Logunov-Tavkhelidze is the equality of times of both particles in the center-of-mass system, i.e., the relative time is put to be zero [1]. The theory developed on the base of this idea as well as its manifestly covariant modifications [2-6] constitute a powerful method for studying the two-particle problem in QFT [7,8].

On the other hand, in the last years supersymmetric quantum field theories are intensively developed. An essential characteristic of such theories is the unification of the bosonic and fermionic fields in one multiplet. For this reason some of divergences from the bosonic sector are cancelled with the ones from the fermionic sector. On the whole supersymmetric QFR's have less divergences than the ordinary theories. There is a promise that in some case of extended supersymmetric theories the divergences do not exist. As an example we can point out the supersymmetric SU(4) Yang-Mills theory, where there is no charge renormalization in the three-loop approximation [9,10]. There is a hope that these renormalizations do not exist in any order of perturbation theory as well. In that case the supersymmetric SU(4) Yang-Mills theory is a good candidate for the theory which is able to describe the quark confinement phenomen.

In the present report a possibility to construct supersymmetric three-dimensional two-particle equation of the quasipotential type is discussed. Here is considered only the Logunov-Tavkhelidze approach, however, the same can be made also in the case of light-cone variables [11] and for the approach in which the Markov-Yukawa condition is used [2,5,6]. In all the cases, for simplicity, we restrict ourselves only to simple scalar supermultiplets, i.e., superfields describing one scalar, one pseudo-scalar, and one spinor particles.

With the help of these equations the bound-states problem in various supersymmetric theories including the supergravity, supersymmetric electro-weak and Grand Unification theories can be investigated. The quasipotential can be determined in a perturbative way from quantum field theory. In the case,when the Lagrangian contains supersymmetry breaking terms they are included in the quasipotential.

2. SUPERSYMMETRIC BETHE-SALPETER EQUATION

Consider the supersymmetric four-point Green function

$$G(z_1,\dots,z_4)=\langle 0|T(\Phi(z_1)\Phi(z_2)\Phi^+(z_3)\Phi^+(z_4))|0\rangle, \qquad (2.1)$$

where $z=(x_\mu,\theta_\alpha)$, θ is, in general, the four-component anti-commuting Majorana spinor variable and Φ are superfields. It is supposed that G is invariant with respect to the super-Poincare transformations. For the Green function G the following supersymmetric Bethe-Salpeter equation[12] can be written

$$G(z_1,z_2;w_1,w_2)=G_0(z_1,z_2;w_1,w_2)+\int d^8u_1 d^8u_2 d^8v_1 d^8v_2 \times$$
$$\times D_0(z_1,u_1)D_0(z_2,u_2)K(u_1,u_2;v_1,v_2)G(v_1,v_2;w_1,w_2), \qquad (2.2)$$

where $D_0(z_1,z_2)$ is the supersymmetric free-particle propagator

$$D_0(z_1,z_2)=\langle 0|T(\Phi(z_1)\Phi(z_2))|0\rangle \qquad (2.3)$$

and K is the invariant Bethe-Salpeter kernel.

As in the usual case, we can introduce a complete system of intermediate states. Then G can be represented in the following form

$$G=\sum_n \langle 0|T(\Phi(z_1)\Phi(z_2))|n\rangle\langle n|T(\Phi^+(w_1)\Phi^+(w_2))|0\rangle$$
$$=\sum_n \Psi_n(z_1,z_2)\Psi_n^+(w_1,w_2), \qquad (2.4)$$

where by

$$\Psi_n(z_1,z_2)=\langle 0|T(\Phi(z_1)\Phi(z_2))|n\rangle \qquad (2.5)$$

the Bethe-Salpeter amplitude is denoted and $x_0^a > y_0^b$ $(a,b=1,2)$ is assumed. Then, substituting (2.4) into (2.2) we obtain the corresponding homogeneous supersymmetric Bethe-Salpeter equation for the two-particle amplitude

$$\int d^8u_1 d^8u_2 G_0^{-1}(z_1,z_2;u_1,u_2)\Psi_n(u_1,u_2)=\int d^8u_1 d^8u_2 d^8v_1 d^8v_2$$
$$\times D_0(z_1,u_1)D_0(z_2,u_2)K(u_1,u_2;v_1,v_2)\Psi_n(v_1,v_2). \qquad (2.6)$$

In eqs. (2.2) and (2.6) it is convenient to introduce the collective coordinates. In the equal-mass case, we restrict ourselves to

$$Z=\frac{1}{2}(z_1+z_2)=\frac{1}{2}(x_\mu^1+x_\mu^2,\ \theta_\alpha^1+\theta_\alpha^2) \qquad (2.7)$$

and

$$z=z_1-z_2=(x_\mu^1-x_\mu^2,\ \theta_\alpha^1-\theta_\alpha^2) \qquad (2.8)$$

are the super-center-of-mass coordinate and the super-relative coordinates, respectively. It is easy to see that with respect to the supertransformations the center-of-mass coordinate (2.7) is transformed as a coordinate in the superspace but the transformation law of the relative coordinate (2.8) is

$$z\to z'=\{x_\mu^1-x_\mu^2+\frac{1}{2}\epsilon\gamma_\mu(\theta^1-\theta^2),\ \theta^1-\theta^2\}, \qquad (2.9)$$

where ϵ is the anticommuting spinor parameter of the super-transformations.

Transition to the momentum space, with respect to x, is performed as in the ordinary case. Then the Bethe-Salpeter eq. (2.2) can be written symbolically in the following way:

$$G=G_0+G_0 \underset{\vee}{K} \underset{\vee}{G}, \qquad (2.10)$$

where G_0 is the two-particle supersymmetric disconnected Green function, and the integration over intermediate spinor variables is denoted by $\vee$, the integration over intermediate momentum variables also being taken into account. The solution of eq. (2.10) can be found by iteration, i.e.,

$$G=G_0+G_0 \underset{\vee}{K} \underset{\vee}{G_0}+G_0 \underset{\vee}{K} \underset{\vee}{G_0} \underset{\vee}{K} \underset{\vee}{G_0}+\dots \qquad (2.11)$$

In the supersymmetric case in (2.11) there are, in general, less singular terms than in the ordinary case. However, there also exist unphysical parameters: the relative coordinate or ots conjugate relative energy.

3. SUPERSYMMETRIC TWO-TIME GREEN FUNCTION

To make the theory free from the difficulties caused by the relative time (energy), we following Logunov-Tavkhelidze /1/ put the relative time in (2.1) and (2.5) to be zero in the c.m.s., i.e.,

$$x_0^1 - x_0^2 = 0. \tag{3.1}$$

However, from (2.8) it follows that this operation is not invariant with respect to the supertransformations. As is well known, the equal-time operation also is not invariant with respect to the Lorentz transformations; the operation (3.1) can be made meaningful in a fixed reference frame, e.g., in the c.m.s. In the supersymmetric case the operation (3.1) also has sense in a fixed reference frame in the superspace. Such "supercenter-of-mass" system is introduced by the conditions

$$\mathbf{p} = 0, \quad \epsilon = 0. \tag{3.2}$$

In an arbitrary reference frame the equal-time condition (3.1) can be written in the following invariant form

$$(L_p)_0^{\nu}\,[x_\nu^1 - x_\nu^2 + \frac{1}{2}\bar{\epsilon}\gamma_\nu(\theta_1 - \theta_2)] = 0, \tag{3.3}$$

where $(L_p)_\mu^{\nu}$ are matrix elements of the boost operator, for which

$$(L_p)_0^{\nu} = n^{\nu} = \frac{p^{\nu}}{\sqrt{p^2}}, \qquad n^2 = 1.$$

Here p is the total momentum of the two-particle system. Note that the momentum p is invariant with respect to the supertransformations.

Transition from the four-time Green functions to the two-time ones and from the two-time B-S amplitudes to the one-time wave function can be made in a covariant manner according to the formulas

$$\tilde{G}(z_1, z_2; w_1, w_2) = \int dx_0^2 dy_0^2\, \delta\{n^{\mu}[x_\mu^1 - x_\mu^2 + \frac{1}{2}\bar{\epsilon}\gamma_\mu(\theta_1 - \theta_2)]\} \tag{3.4}$$
$$\times\, G(z_1, z_2; w_1, w_2)\,\delta\{n^{\mu}[y_\mu^1 - y_\mu^2 + \frac{1}{2}\bar{\epsilon}\gamma_\mu(\theta_1 - \theta_2)]\}$$

and

$$\tilde{\Psi}_n(z_1, z_2) = \int dx_0^2\, \delta\{n^{\mu}[x_\mu^1 - x_\mu^2 + \frac{1}{2}\bar{\epsilon}\gamma_\mu(\theta_1 - \theta_2)]\}\Psi_n(z_1, z_2). \tag{3.5}$$

Going to the momentum space from (3.4) and (3.5) in the "supercenter-of-mass" system we have

$$\tilde{G}(E, \vec{q}, \vec{q}', \theta_1, \theta_2, \theta_1, \theta_2) = \int dq_0 dq_0' G(E, q, q', \theta_1, \theta_2, \theta_1, \theta_2) \tag{3.6}$$

and

$$\tilde{\Psi}_E(\vec{q}, \theta_1, \theta_2) = \int dq_0 \Psi(E, q, \theta_1, \theta_2). \tag{3.7}$$

Consequently in the momentum space the "equal-time" operation (3.1) is replaced by the integration over the relative energies, as in the ordinary case /1/.

For the two-time Green function (3.4) or (3.6) we have the following equation

$$\tilde{G} = \tilde{G}_0 + G_0 K G, \tag{3.8}$$

which can be found from the B-S eq. (2.10) by the "equal-time" operation (3.1). Then, as in the ordinary case, in the supersymmetrical case the quasipotential is determined from the equation

$$[\tilde{G}]^{-1} = [\tilde{G}_0]^{-1} - \frac{1}{2\pi i} V. \tag{3.9}$$

Here the inverse operator is determined by the following condition

$$\int d^3q'' d^4\theta_1'' d^4\theta_2''\, \tilde{G}(E, \vec{q}, \vec{q}''; \theta_1, \theta_2; \theta_1'', \theta_2'')$$
$$\times\, \tilde{G}^{-1}(E, q'', q', \theta_1'', \theta_2'', \theta_1', \theta_2') = \delta^{(3)}(q - q')\,\delta^{\Gamma}(\theta_1 - \theta_1')\,\delta^{\Gamma}(\theta_2 - \theta_2'), \tag{3.10}$$

where $\delta^{\Gamma}(\theta)$ is the Grassmannian δ-function /13/.

4. QUASIPOTENTIAL EQUATION FOR SCALAR CHIRAL SUPERMULTIPLETS

In this section we restrict our consideration to scalar chiral superfields (see Appendix A). The four-particle Green function for these fields can be represented in the following form

$$G = \begin{bmatrix} G^{++,++} & G^{++,-+} & G^{++,+-} & G^{++,--} \\ G^{-+,++} & G^{-+,-+} & G^{-+,+-} & G^{-+,--} \\ G^{+-,++} & G^{+-,-+} & G^{+-,+-} & G^{+-,--} \\ G^{--,++} & G^{--,-+} & G^{--,+-} & G^{--,--} \end{bmatrix}, \qquad (4.1)$$

where

$$G^{\alpha,\beta,\gamma,\delta} = \langle 0|T(\Phi^{\alpha}(x_1,\theta_1)\Phi^{\beta}(x_2,\theta_2)\Phi^{-\gamma}(x_3,\theta_3)\Phi^{\delta}(x_4,\theta_4))|0\rangle \qquad (4.2)$$

are the four-point Green functions (4.2) for the chiral scalar superfields. Here the following notation is used:

$$\Phi^{+}(x,\theta) = \Phi(x,\theta) \qquad \text{and} \qquad \bar{\Phi}^{-}(x,\theta) = \Phi(x,\bar{\theta}),$$

where $-$ means the complex conjugation. For the two-particle wave function we have

$$\Psi(x_1,x_2;\theta_1,\theta_2) = \begin{bmatrix} \Psi^{++}(x_1,x_2;\theta_1,\theta_2) \\ \Psi^{-+}(x_1,x_2;\bar{\theta}_1,\theta_2) \\ \Psi^{+-}(x_1,x_2;\theta_1,\bar{\theta}_2) \\ \Psi^{--}(x_1,x_2;\bar{\theta}_1,\bar{\theta}_2) \end{bmatrix}, \qquad (4.3)$$

where

$$\Psi_p^{\alpha,\beta} = \langle 0|T(\Phi^{\alpha}(x_1,\theta_1)\Phi^{\beta}(x_2,\theta_2))|p,j,j_3\rangle. \qquad (4.4)$$

Superfields Φ contain components with spin 0 and 1/2 consequently the states $|p,j,j_3\rangle$ have the spin

$$j = \ell, \quad \ell \pm 1/2, \quad \ell \pm 1, \qquad (4.5)$$

where ℓ is the orbital momentum with respect to the center-of-mass system. The transformation law of the states $|p,j,j_3\rangle$ with respect to the supertransformations are not discussed here.

From (3.9) it follows that the determination of the quasipotential requires the inverse Green function $\tilde{G}_0^{-1}$ to be found. The corresponding supersymmetric four-particle two-time Green function according to (3.6) is given by

$$\tilde{G}_0(E,\vec{q},\vec{q}\,',\theta_1,\dots,\theta_4) = \int_{-\infty}^{\infty} dq_0 dq_0' G(E,q,q',\theta_1\dots\theta_4). \qquad (4.6)$$

Here G_0 has a matrix form (4.1) with matrix elements in the free case

$$G_0^{\alpha\beta,\gamma\delta} = D_0^{\alpha\gamma}(E,q,\theta_1,\theta_3) D_0^{\beta\delta}(E,q',\theta_2,\theta_4)\delta^{(4)}(q-q'), \qquad (4.7)$$

where D_0 are free supersymmetric propagators given in Appendix A. In the integral (4.6) there are divergent terms. To avoid this difficulty, the Pauli-Villars regularization can be used for the propagators (A.3). Then inserting (4.7) into (4.6), after integration over q_0 and q_0' and removing the regularization we have (in the c.m.s. frame)

$$\tilde{G}_0(E,\vec{q},\vec{q}\,',\theta_1\dots\theta_4) = \frac{}{E\,w}\Big\{ \frac{\mathcal{G}_0(\mathcal{P}_1,\theta_1,\theta_3) \otimes \mathcal{G}_0(Q_3,\theta_2,\theta_4)}{E-2w} + \frac{\mathcal{G}_0(\mathcal{P}_3,\theta_1,\theta_3) \otimes \mathcal{G}_0(Q_3,\theta_2,\theta_4)}{E+2w}\Big\}\delta^{(3)}(\vec{q}-\vec{q}\,'), \qquad (4.8)$$

where

$$\mathcal{G}_0 = \begin{pmatrix} m\,\delta^{\Gamma}(\theta-\theta) & \exp 2\ \theta\mathcal{P}\bar{\theta} \\ \exp 2\ \bar{\theta}\tilde{\mathcal{P}}\theta & m\,\delta^{\Gamma}(\bar{\theta}-\bar{\theta}) \end{pmatrix} \qquad (4.9)$$

and

$$\mathcal{P}_1 = (w,\vec{q}), \quad \mathcal{P}_3 = (E+w,\vec{q}),$$

$$Q_1 = (E-w,-\vec{q}), \quad Q_3 = (-w,-\vec{q}), \quad w = \sqrt{q^2+m^2}$$

and E is the center-of-mass energy.

It can be verified that the two-fermion component of $\mathcal{G}_0$ coincides with the corresponding two-time Green function for the free spin 1/2 particles /2,14/. This Green function can be found from $\tilde{G}_0$ (4.8) as the coefficient of the first power in $\theta_1, \theta_2, \theta_3$ and θ_4. However, it is known [2,14] that the two-time fermionic Green function has no inverse in the whole 16-component spinor space. The resolvent operator can be found only in the 8-dimensional subspace only with equal sign of energy of both the particles*. This subspace can be separated using the projection operator $\Lambda_\pm$ onto subspaces with the positive and negative energy. Operators $\Lambda_\pm$ have a simple form in the Foldy-Wouthuysen representation. The transition to the F-W transformation is made by operators

$$T^{(1)}(\vec{q}) = \frac{m+w+\gamma^{(1)}\cdot\vec{q}}{\sqrt{2w(m+w)}} = \frac{1}{\sqrt{2w(m+w)}}\begin{pmatrix} m+w & \sigma^{(1)}\cdot\vec{q} \\ -\vec{\sigma}^{(1)}\cdot\vec{q} & m+w \end{pmatrix}$$
$$T^{(2)}(\vec{q}) = \frac{m+w-\gamma^{(2)}\cdot q}{\sqrt{2w(m+w)}} = \frac{1}{\sqrt{2w(m+w)}}\begin{pmatrix} m+w & -\vec{\sigma}^{(2)}\cdot\vec{q} \\ \vec{\sigma}^{(2)}\cdot\vec{q} & m+w \end{pmatrix}. \quad (4.10)$$

For the matrix γ_0 we use the representation $\gamma_0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$. The superscripts 1,2 in (4.10) indicate the particle on which $T(\vec{q})$ acts.

In the supersymmetric case the wave function (4.3) is decomposed in θ_1 and θ_2. The corresponding coefficients, the components of the superwave functions are denoted by $\Psi(a,b)$, where $(a,b=0,1,2)$.

Then, the Foldy-Wouthuysen transformation in the supersymmetric case is defined in the following way

$$\begin{aligned} &\tilde{\Psi}_F(\vec{q},1,1) = T^{(1)}(\vec{q})T^{(2)}(\vec{q})\tilde{\Psi}(q,1,1), \\ &\tilde{\Psi}_F(q,1,\alpha) = T^{(1)}(q)\tilde{\Psi}(q,1,\alpha) \\ &\tilde{\Psi}_F(q,\alpha,1) = T^{(2)}(q)\tilde{\Psi}(q,\alpha,1) \\ &\tilde{\Psi}_F(q,\alpha,\beta) = \tilde{\Psi}(\vec{q},\alpha,\beta) \qquad (\alpha,\beta = 0,2). \end{aligned} \quad (4.11)$$

The corresponding projection operators on the state with definite sign of energy in the F.W. representation are given by

*That is the case of Majorana spinors.

$$(\Lambda^F_\pm)^{1,2} = \frac{I^{(1,2)} \pm \gamma_0^{(1,2)}}{2} = \frac{1}{2}\begin{bmatrix} I^{1,2} & \pm\sigma_0^{(1,2)} \\ \pm\sigma_0^{(1,2)} & I^{(1,2)} \end{bmatrix} \quad (4.12)$$

which act on the fermionic components. For the Majorana spinor $\Psi(x)$ we have $\Lambda_-\Psi=0$ and consequently

$$\Lambda_-^{(1,2)}\Psi(1,1) = 0. \quad (4.13)$$

Then, without loss of invariance with respect to the spatial reflections, the following projection operators

$$\Pi_1 = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad \Pi_2 = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} \quad (4.14)$$

can be introduced, where I is the 2x2-identity matrix. Applying these operators to the components of the wave function (4.11) we have

$$\tilde{\Psi}_F^{(1,2)} = \Pi_{(1,2)}\tilde{\Psi}_F(q,a,b). \quad (4.15)$$

Then the following super-wave functions can be formed

$$\hat{\Psi}_F^{(1)} = \begin{bmatrix} \hat{\Psi}^+(x_1,\theta_1,x_2,\theta_2) \\ \hat{\Psi}^-(x_1\bar{\theta}_1,x_2,\theta_2) \end{bmatrix}, \quad (4.16)$$

and

$$\hat{\Psi}_F^{(2)} = \begin{bmatrix} \hat{\Psi}^-(x_1,\bar{\theta}_1,x_2,\theta_2) \\ \hat{\Psi}^+(x_1\bar{\theta}_1,x_2,\bar{\theta}_2) \end{bmatrix}, \quad (4.17)$$

where components of the super-wave functions $\hat{\Psi}^\pm$ are given by (4.15). Corresponding two-time Green functions are given by

$$\hat{G}^1_{0F} = \begin{bmatrix} {}^1\hat{G}^{++}_{0F} & {}^1\hat{G}^{+-}_{0F} \\ {}^1\hat{G}^{-+}_{0F} & {}^1\hat{G}^{--}_{0F} \end{bmatrix} = \begin{bmatrix} \tilde{G}^{++,++}_{0F} & \tilde{G}^{++-+}_{0F} \\ \tilde{G}^{-+,++}_{0F} & \tilde{G}^{-+,-+}_{0F} \end{bmatrix}, \quad (4.18)$$

and

$$\hat{G}^2_{0F} = \begin{bmatrix} {}^2\hat{G}^{++}_{0F} & {}^2\hat{G}^{+-}_{0F} \\ {}^2\hat{G}^{-+}_{0F} & {}^2\hat{G}^{--}_{0F} \end{bmatrix} = \begin{bmatrix} \tilde{G}^{+-,+-}_{0F} & \tilde{G}^{+-,--}_{0F} \\ \tilde{G}^{--,+-}_{0F} & \tilde{G}^{--,--}_{0F} \end{bmatrix} \quad (4.19)$$

Here $\tilde{G}_{0F}^{\alpha\beta\gamma\delta}$ can be obtained from (4.8) by the substitution

$$\exp 2\theta_j \vec{q}\bar{\theta}_k \to 1 + 4\vec{q}^2 \delta^{\Gamma}(\theta_j)\delta^{\Gamma}(\bar{\theta}_k)$$
$$m\delta^{\Gamma}(\theta_j - \theta_k) \to m\delta^{\Gamma}(\theta_j) + m\delta^{\Gamma}(\theta_k) + w\theta_j \epsilon \theta_k , \tag{4.20}$$

where $w = \sqrt{q^2+m^2}$, i.e., the F.W. transformation is performed.

From the condition (3.10) we can determine the corresponding to $\hat{G}_{0F}^{(1,2)}$ resolvents. The explicit form of $\hat{G}_{0F}^{1}$ is given by

$$({}^1\hat{G}_{0F}^{-1})^{++} = \frac{i}{\pi}\{w[\delta^{\Gamma}(\theta_1)\delta^{\Gamma}(\theta_2) + \delta^{\Gamma}(\theta_1)\delta^{\Gamma}(\theta_4) + \delta^{\Gamma}(\theta_2)\delta^{\Gamma}(\theta_3)$$
$$+ \delta^{\Gamma}(\theta_3)\delta^{\Gamma}(\theta_4)] + \frac{2w^2}{m}\theta_1\epsilon\theta_3[\delta^{\Gamma}(\theta_2) + \delta^{\Gamma}(\theta_4)] + m\theta_2\epsilon\theta_4$$
$$\times[\delta^{\Gamma}(\theta_1) + \delta^{\Gamma}(\theta_3)] - 2w\theta_1\epsilon\theta_3\theta_2\epsilon\theta_4\},$$
$$({}^1G_{0,F}^{-1})^{-+} = \frac{i}{\pi}\{\frac{w}{2m}[\delta^{\Gamma}(\theta_2) + \delta^{\Gamma}(\theta_4)] - \frac{w}{2m}[m^2 + 2(\frac{E^2}{4} - w^2)]$$
$$\times\delta^{\Gamma}(\bar{\theta}_1)\delta^{\Gamma}(\theta_3)[\delta^{\Gamma}(\theta_2) + \delta^{\Gamma}(\theta_4)] + \frac{1}{2}\theta_2\epsilon\theta_4 - \frac{Ew}{m}(\bar{\theta}_1\theta_3)$$
$$\times[\delta^{\Gamma}(\theta_2) + \delta^{\Gamma}(\theta_4)] + E(\bar{\theta}_1\theta_3)\theta_2\epsilon\theta_4 +$$
$$+ 2[m^2 + 2(\frac{E^2}{4} - w^2)]\theta_2\epsilon\theta_4\delta^{\Gamma}(\bar{\theta}_1)\delta^{\Gamma}(\theta_3)\}. \tag{4.21}$$

Remaining elements of ${}^1\hat{G}_{0F}^{-1}$ can be found from (4.21) by complex conjugation of variables θ_1 and θ_3. The Green function ${}^2\tilde{G}_{0F}^{-1}$ can be obtained from ${}^1\hat{G}_{0F}^{-1}$ by complex conjugation of all variables θ.

Now we can write a supersymmetric quasipotential equation of the Logunov-Tavkhelidze type for the two-particle super-wave-functions. In the super-center-of-mass system it is given by

$$\underbrace{{}^{(1,2)}\hat{G}_{0F}^{-1}\hat{\Psi}_F^{(1,2)}} = \underbrace{\hat{V}_{(1,2)}\hat{\Psi}_F^{(1,2)}}, \tag{4.22}$$

where integration over the intermediate momentum and spinor variables θ should be taken into account. Here the quasipotential $\hat{V}$ can be determined in a perturbative way from quantum field theory, as in the ordinary case. These potentials

have the matrix structure as the Green functions (4.18) and (4.19), respectively. The explicit form of the potential depends on the interaction Lagrangian. Because of a cumbersome structure of the equations corresponding to (4.22) for the components of the super-wave function they are not written here. Note only that after eliminating the nondynamical components $\Psi(2,a)$ and $\Psi(a,2)$ $(a = 0,1,2)$, the equations for scalar and spin 1/2 particles coincide in form with the corresponding quasipotential equations in the ordinary theories [1,2]. We also pointed out that when the supersymmetry breaking terms are present in the Lagrangian, these terms are added to the quasipotential $\hat{V}(x,\theta)$ only. In the last case after eliminating the nondynamical components of the wave-function we have a suitable mass splitting between bosonic and fermionic masses. As in the ordinary theories, the supersymmetric three-dimensional eqs. (4.22) can be used for investigation of the bound states, as well as the scattering in various supersymmetric theories.

APPENDIX A

The simplest scalar chiral superfields are determined by the equations

$$\bar{D}_{\dot{a}}\Phi^{+}(x,\theta) = 0,$$
$$D_a\Phi^{-}(x,\bar{\theta}) = 0, \quad (a,\dot{a} = 1,2). \tag{A.1}$$

Here D_a, $\bar{D}_{\dot{a}}$ are supercovariant derivatives (see ref. [13]). For our purposes it is convenient to use the two-component spinor formalism. In the nonsymmetric representations the fields Φ^+ and Φ^- are given by

$$\Phi^{+}(x,\theta) = \frac{1}{2}(A(x) - iB(x)) + \theta^a\phi_a(x) + \frac{1}{2}\theta\epsilon\theta\,(E(x) + iG(x)),$$
$$\Phi^{-}(x,\bar{\theta}) = (\Phi^{+}(x,\theta))^{*}, \tag{A.2}$$

where A and F are real scalar fields, B and G are real pseudoscalar fields and ϕ two-component spinor fields. The corresponding supersymmetric propagators are given by:

$$D^{++}(x_1-x_2,\theta_1,\theta_2)=\langle 0|T(\Phi^+(x_1,\theta_1)\Phi^+(x_2,\theta_2))|0\rangle=$$

$$=m\,\delta^{\Gamma}(\theta_1-\theta_2)\,\Delta_c(x_1-x_2,m) \qquad (A.3)$$

$$D^{+-}(x_1-x_2,\theta_1,\bar{\theta}_2)=\frac{1}{2}\exp(-2i\theta\,\underset{\sim}{\partial}\vec{\theta}_2)\,\Delta_c(x_1-x_2,m),$$

where $\Delta_c(x,m)$ is the Feynman propagator and $\underset{\sim}{\partial}=\sigma_\mu\partial^\mu$, σ_0 is the identity 2x2 matrix and $\sigma_j\,(j=1,2,3)$ are the Pauli matrices.

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Зайков Р.П. E2-82-442
Суперсимметричные квазипотенциальные уравнения

В работе предлагается суперсимметричное обобщение квазипотенциального подхода Логунова-Тавхелидзе. Исходным является, как и в обычном случае, суперсимметричное уравнение Бете-Солпитера. Переход от четырехвременной к двухвременной функции Грина делается в фиксированной системе в суперпространстве. Резольвентный оператор найден с использованием майорановского характера двухфермионной волновой функции. Записано суперсимметричное квазипотенциальное уравнение и обсуждаются возможные нарушающие суперсимметрию члены в квазипотенциале.

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Zaikov R.P. E2-82-442
Supersymmetric Quasipotential Equations

In the paper supersymmetric generalization of the Logunov-Tavkhelidze quasipotential approach is supposed. As in the ordinary case, the starting point is the supersymmetric Bethe-Salpeter equation. Corresponding transition from the four-time Green function to the two-time one is done in the fixed reference frame in the superspace. The resolvent operator is find using the Majorana character of the two-fermionic wave-function. The supersymmetric quasipotential equation is written down and the symmetry breaking terms are discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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