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S.S.Sautbekov, A.B.Shvachka

STABILITY AND DYNAMICAL PROPERTIES OF SOLITONS IN THE FRAMEWORK OF TWO-FIELD MODEL OF THE CLASSICAL FIELD THEORY

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Below we investigate the structural stability and dynamical properties of soliton-type solution of the two-field model of the classical field theory proposed by Makhankov [1]. It is shown that the types of soliton interaction are the same as for solitons in the range of the Klein-Gordon equation with cubic non-linearity [2].

The model under investigation consists of a complex ϕ-field (with the zero boundary conditions at both infinities) interacting with a scalar η-field with a degenerate vacuum:

$$\begin{aligned} &\phi_{tt} - \phi_{xx} - \phi(1+\eta) = 0, \\ &\eta_{tt} - \eta_{xx} + \nu\eta - g\eta^2 - |\phi|^2 = 0. \end{aligned} \tag{1}$$

This system of equations may be obtained from the variation principle $\delta\int \mathcal{L}\, dx\, dt = 0$ with the Lagrangian density

$$\mathcal{L} = |\phi_t|^2 - |\phi_x|^2 + |\phi|^2 + \frac{1}{2}(\eta_t^2 - \eta_x^2 - \nu\eta^2 + \frac{2}{3}g\eta^3 + 2|\phi|^2\eta) \tag{2}$$

The energy density corresponding to (2) is

$$\mathcal{H} = |\phi_t|^2 + |\phi_x|^2 + (1-\eta)|\phi|^2 + \frac{1}{2}(\eta_t^2 + \eta_x^2 + \nu\eta^2 - \frac{2}{3}g\eta^3). \tag{3}$$

Suppose that a particular solution of (1) in the soliton rest frame is as follows:

$$\begin{aligned} &\phi(x, \tau) = \frac{1}{\sqrt{2}}\, \varphi(x) e^{-i\omega_0 \tau}, \\ &\eta(x, \tau) = \eta(x). \end{aligned} \tag{4}$$

Because of the U(1) symmetry of the model the "charge" of the soliton

$$Q = \omega_0 \int_{-\infty}^{\infty} \varphi^2\, dx \tag{5}$$

is a constant of motion.

Particular solutions of (1) are as follows:

a) at $\omega_0 = \sqrt{1-\nu}$

$$\phi_1 = \frac{3\nu}{2}\sqrt{1-g}\,\mathrm{ch}^{-2}\frac{\sqrt{\nu}}{2}x\,e^{-i\omega_0\tau},$$

$$\eta_1 = \frac{3\nu}{2}\mathrm{ch}^{-2}\frac{\sqrt{\nu}}{2}x; \qquad (6)$$

b) at $\omega_0 = \sqrt{1-\nu_0/6}$, $g=3$

$$\phi_2 = 2(1-\omega_0^2)\,\mathrm{ch}^{-1}(\sqrt{\omega_0^2-1}\,x)\,e^{-i\omega_0\tau},$$

$$\eta_2 = 2(1-\omega_0^2)\,\mathrm{th}^2(\sqrt{\omega_0^2-1}\,x). \qquad (7)$$

These solitons may be considered as a "bag" of the η-field in which charged "mesons" of the ϕ-field are locked [1].

To determine the stability region of the charged soliton we use the so-called theorem of the Q-stability, proposed by Makhankov [2]. It reads as follows:

$$\frac{\omega_0}{Q}\frac{dQ}{d\omega_0} < 0. \qquad (8)$$

Putting (5) in the last expression we get the following condition for the soliton-like solution (6) to be stable:

$$0.5 < |\omega_0| < 1 \quad \text{or} \quad 0 < \nu < \sqrt{3}/2 \simeq 0.866.$$

The stability region of (6) can be found from the minimum energy density condition $\delta^2 \mathcal{H}|_{Q=\mathrm{const}} > 0$, where

$$\mathcal{H} = |\phi_t|^2 + |\phi_x|^2 + (1-\eta+\eta_{vac})|\phi|^2 + \frac{1}{2}[\eta_t^2 + \eta_x^2 + \nu(\eta^2-\eta_{vac}^2)] - \frac{2}{3}g(\eta^3-\eta_{vac}^3)] \qquad (9)$$

and

$$\eta_{vac} = \lim_{x\to\infty}\eta_2 = \frac{\nu}{g}.$$

If $(\delta\eta, \delta\varphi) \approx \mathrm{ch}^{-2}\frac{\sqrt{\nu}}{2}x$, then the stability regions are $0<\nu<\frac{7}{18}(2-\frac{1}{1-g})$ for φ and $0<\nu<0.75$ for η. The stability region of the solution (6) is defined by the following inequalities:

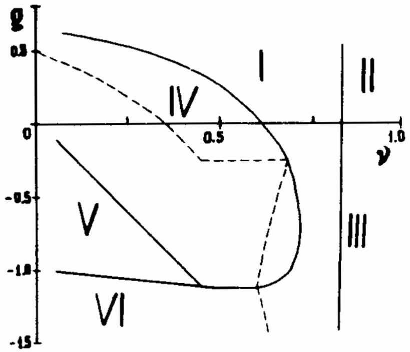

Fig. The types of quasi-soliton (6) interactions in the (ν, g) plane. I-III, VI-the regions where the singularity of the field arises before the quasi-solitons interaction; IV - the region where the singularity of the field arises as a result of the interaction of the quasi-solitons; V - the region of the quasi-elastic interaction.

$$0<\nu<10/(10+3\sqrt{2(1-g)}), \qquad g<(1-\sqrt{5})/4. \qquad (10)$$

As is shown in the Figure the stability region of the solution (6) found from the minimum energy density condition is smaller than calculated from the Q-stabily theorem [2].

3. Dynamical properties of the quasi-solitons (6) were studied by means of the computer simulation. Using Lorentz transformation we can easily obtain the quasi-soliton moving with velocity v (in the light velocity units) along the x-axis:

$$\phi_1 = \frac{3\nu}{2}\sqrt{1-g}\,\mathrm{ch}^{-2}\left(\frac{\sqrt{\nu}}{g}\gamma(x-vt)\right)e^{i(\omega\gamma(vx+t)+\theta_1)},$$

$$\eta_1 = \frac{3\nu}{2}\mathrm{ch}^{-2}\left(\frac{\sqrt{\nu}}{g}\gamma(x-vt)\right), \qquad (11)$$

where γ being the relativistic factor and $\omega=\sqrt{1-\nu}$.

Depending on the frequency ν and the coupling constant g the following three types of the quasi-soliton interaction take place in the framework of the model (1) (see the Figure):

I) quasi-elastic and weak inelastic interaction;

II) arasing of the field singularity at the moment of the quasi-solitons overlapping;

III) arasing of the field singularity before the quasi-solitons interaction.

We want to stress that the quasi-elastic interaction takes place in the stability region of the single quasi-soliton and the collapse occurs on the boundary of the stability region before the interaction of quasi-solitons. As a result of the collisions of quasi-solitons in the region IV (see the Figure) the collapse of the quasi-solitons arises at the moment of the quasi-solitons overlapping, when there is maximal disturbance of each of them.

In the case of the phase shift $\Delta\theta = \pi$ the elastic interaction of quasi-solitons becomes of the repulsing character. Otherwise at $\Delta\theta \neq 0$ the amplitudes of the quasi-solitons pulsate and the field configuration becomes asymmetric. When $\omega \to 1 (\nu \to 0)$ the region of the elastic interaction of the quasi-solitons expands (see region V in the Fig.). So as a result of interaction of "heavy" solitons (6) (i.e., with large amplitude) field singularity arises in contrast with "light" quasi-solitons elastic interaction.

Computer simulation indicates that the interaction process slightly depends on the quasi-solitons velocity ($0.09 \leq v \leq 0.9$).

The types of quasi-solitons interaction in the framework of the model (1) are nearly the same as found earlier in the range of the Klein-Gordon equation with cubic non-linearity/8/. According to above discussion we come to the conclusion that the quasi-solitons are of the pulson type and are stable for the limited range of ν, g.

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Саутбеков С.С., Швачка А.Б. E2-82-413
Устойчивость и динамические свойства солитонов в рамках двухполевой модели классической теории поля

Исследованы динамические свойства и найдена область устойчивости солитоноподобного решения двухполевой скалярной модели классической теории поля. Динамические свойства квазисолитонов и типы их взаимодействий изучены методом численного эксперимента. Показано, что типы взаимодействий квазисолитонов практически совпадают с обнаруженными ранее при исследовании свойств пульсонов в рамках уравнения Клейна-Гордона с кубической нелинейностью.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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Sautbekov S.S., Shvachka A.B. E2-82-413
Stability and Dynamical Properties of Solitons in the Framework of Two-Field Model of the Classical Field Theory

The stability region of the soliton solution in the range of two-field scalar model of the classical field theory was investigated. Dynamical properties of solitons and the types of soliton interaction have been studied by means of the computer simulations. It is shown that the types of solution interaction are the same as for solitons in the range of the Klein-Gordon equation with cubic nonlinearity.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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