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**RADIALLY EXCITED PSEUDOSCALAR  
STATES  
AND THE  $\tau(1440)$  PROBLEM**

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The recently discovered pseudoscalar isoscalar resonance  $\psi(1440)$ <sup>1,2/</sup> is either a gluonium state or a radially excited state  $\eta'_R$ , other candidates for the corresponding  $SU_3$  nonet  $P_R$  being  $\pi'(1205) = \pi_R^{3/}$ ,  $\zeta(1275) = \eta_R^{4/}$  and, possibly,  $K'(\sim 1400) = K_R^{5/}$ . Relying upon analogy with a pattern of  $\eta - \eta'$  mixing in the ground-state pseudoscalar nonet  $P = (\pi, \eta, \eta', K)$ <sup>6/</sup> we propose a natural Ansatz for  $\eta_R - \eta'_R$  mixing in the  $P_R$ , which seems to be consistent with the identification  $\psi(1440) = \eta'_R$ .

The most characteristic feature of the  $\eta - \eta'$  mixing which has been demonstrated in Ref.<sup>6/</sup> is the maximum  $\eta - \pi$  splitting. Due to a strong dependence of the mixing parameter  $\epsilon_P^2$  on particle masses (or, equivalently, on effective masses of constituent quarks), the maximum splitting condition (MSC) does not lead to any simple mass relation. In addition, the  $q\bar{q}$  components of the  $\eta$  and  $\eta'$  are slightly non-orthogonal. Neglecting this non-orthogonality the MSC predicts the singlet-octet mixing angle  $\theta_P = -\arctg(1/2\sqrt{2}) \approx 19.47^\circ$ , the corresponding prediction for the strong-non-strange mixing angle  $\theta_{\eta\eta'} = \theta_P - \theta_0 + 90^\circ$  being  $\theta_{\eta\eta'} = \theta_0 = \arctg(1/\sqrt{2}) \approx 35.26^\circ$ . By using the MSC the correct masses of the  $\eta'$  and  $K$  were obtained in Ref.<sup>6/</sup>, and the angles are in a very good agreement with the recent data on the  $\eta'/\eta$  production (here and in what follows we use the notation  $0.532(48) = 0.532 \pm 0.048$ , etc.):

$$K_{\eta\eta'} = \frac{\sigma(\pi^- p \rightarrow \eta' n)}{\sigma(\pi^- p \rightarrow \eta n)} = 0.532(48); \quad R_{\eta\eta'} = \frac{B(J/\psi \rightarrow \eta' \gamma)}{B(J/\psi \rightarrow \eta \gamma)} = 5.88(1.46). \quad /7/ \quad (1)$$

For  $K_{\eta\eta'}$  the weighted average of the three experiments<sup>8/</sup> with high statistics is given. The value of  $R_{\eta\eta'}$  obtained in<sup>7/</sup> does not significantly contradict several earlier measurements which give somewhat smaller values but have rather poor statistics for the  $\eta'$ . Relating  $K_{\eta\eta'}$  and  $R_{\eta\eta'}$  to the mixing angles,  $K_{\eta\eta'} = \tan \theta_{\eta\eta'}$ ,  $R_{\eta\eta'}^{-1} = (k_{\eta'}/k_{\eta})^3 \tan^2 \theta_P$ , we find from Eq. (1) that  $\theta_{\eta\eta'} = 36.0(13)^\circ$  and  $\theta_P = -20.8(24)^\circ$  (or  $\theta_{\eta\eta'} = 33.9(24)^\circ$ ). In Ref.<sup>9/</sup> we have shown that the predicted angle  $\theta_P$  is also consistent with all available data on radiative decays  $V \rightarrow P\gamma$ ,  $P \rightarrow V\gamma$ ,  $P \rightarrow \gamma\gamma$  of the vector,  $V = (\rho, \omega, \phi, K_V)$ , and the pseudoscalar ( $P$ ) light mesons except the width  $\Gamma(\eta \rightarrow \gamma\gamma)$ . Using new data on some of these decays ( $\eta \rightarrow \gamma\gamma$ ,  $\rho \rightarrow \pi\gamma$ ,  $\eta' \rightarrow \omega\gamma$ ) we now can obtain the best fit for  $\theta_{\eta\eta'}$  which is:  $\theta_{\eta\eta'} = 35.8(1.8)^\circ$ , i.e.,  $\theta_P = -18.9(1.8)^\circ$  ( $\Gamma(\eta \rightarrow \gamma\gamma)$  has not been

used in the fit). The weighted average of all three determinations is:

$$\bar{\theta}_{\eta\eta} = 35.6(1.0)^\circ, \quad \bar{\theta}_P = -(19.1 \pm 1.0)^\circ.$$

Disregarding an unlikely possibility that this impressive agreement of the essentially independent measurements of  $\theta_{\eta\eta}$  with each other and with the theoretical prediction is purely accidental, we conclude that the mixing model of Ref.<sup>/6/</sup> is experimentally confirmed.

For the  $\eta \rightarrow \gamma\gamma$  decay our prediction is<sup>/9/</sup>  $\Gamma(\eta \rightarrow \gamma\gamma) = 0.65 \pm 0.75$  keV, which is significantly larger than the value 0.323(46) keV quoted by PDG<sup>/5/</sup>. This value was obtained in one experiment, the result of another, earlier experiment (also using the Primakoff effect) has been: 1.00(22) keV<sup>/5/</sup>. Evidently,  $\Gamma(\eta \rightarrow \gamma\gamma)$  must be measured again, preferably by using a new method, e.g., two-photon production of the  $\eta$  in  $e^+e^-$  annihilation.

Let us pattern the  $P_R$  after the P nonet. To roughly estimate the  $\pi_R$  mass we use the relation  $a'_{ij}(M_{ij,R}^2 - M_{ij}^2) = \beta'^{6/}$ , where  $a'_{ij}$  is the Regge trajectory slope for the  $q_i\bar{q}_j$  states, and  $M_{ij}(M_{ij,R})$  is the mass of the ground (radially excited)  $q_i\bar{q}_j$  state. Using  $a'_{c\bar{c}} = 0.342'^{6/}$  we find  $\beta' = 1.36$ . Then, with  $\pi_R^2 = \pi^2 + \beta'/a'_{u\bar{u}}$  where  $a'_{u\bar{u}} = 0.91'^{6/}$ , we obtain  $\pi_R = 1.23$  GeV. Allowing for some dependence of  $\beta'$  on  $i$  and  $j$  and  $\sim 5\%$  uncertainty in  $a'_{u\bar{u}}$  one can more safely estimate  $\pi_R = 1.20 \pm 1.25$  GeV, which is in agreement with the value obtained in<sup>/3/</sup>. The mass formulae for the members of the  $P_R$ -nonet with mixing described by  $\epsilon_P^2$  are:

$$\pi_R^2 = m_R^2 - 2\Delta_R^2, \quad K_R^2 = m_R^2 - \frac{\Lambda_R^2}{K^2}, \quad \eta_R^2 = m_R^2 + 6\epsilon_R^2 - 2\delta_R^2, \quad \eta'_R{}^2 = \eta_R^2 + 4\delta_R^2, \quad (2)$$

where  $m_R^2$ ,  $\Delta_R^2$ ,  $\epsilon_R^2$ ,  $\delta_R^2$  are parameters and

$$\Delta_R^2 - \epsilon_R^2 = \delta_R^2 \cos(2\theta_{\eta\eta}^R), \quad 2\sqrt{2}\epsilon_R^2 = \delta_R^2 \sin(2\theta_{\eta\eta}^R); \quad (3)$$

$\theta_{\eta\eta}^R$  is the matrix mixing angle (in what follows  $\theta_{\eta\eta}^R = \theta$ ),  $\Delta_R^2 = s^2 - u^2 > fs^2 - u^2 = \Delta^2 \approx 0.11$  GeV<sup>2/6/</sup>. We also expect that

$$0 < \epsilon_R^2 \leq \epsilon_P^2 \quad \text{and correspondingly} \quad \theta \leq \theta_{\eta\eta}.$$

Consider the dependence of  $\pi_R$ ,  $\eta_R$  and  $\eta'_R$  on  $m_R$ ,  $\delta_R$ , and  $\theta$ , defined by Eqs. (2) and (3). The Jacobian  $\partial(\pi_R, \eta_R, \eta'_R) / \partial(m_R, \delta_R, \theta)$  vanishes if  $\tan(2\theta) = \sqrt{2}$ , i.e.,  $\theta = 1/2(90 - \theta_0)^\circ = \theta_1$

and  $\theta_P^R = \theta + \theta_0 - 90^\circ = -\theta_1 = -27.4^\circ$ . If the masses in the  $P_R$  nonet are such that  $\theta \sim \theta_1$ , the angle very rapidly varies with masses which, in fact, are close to their extreme values. The exact meaning of this statement is the following. Using Eqs.

(2), (3) one can easily obtain the mass formula

$$\eta'_R{}^2 + \eta_R^2 - 2\pi_R^2 = \sqrt{3}(\eta'_R{}^2 - \eta_R^2) \sin(2\theta + \theta_0). \quad (4)$$

As is now clear, the  $\eta_R$  mass has a maximum value for  $\theta = \theta_1$ , with  $\eta'_R$  and  $\pi_R$  fixed. Under similar conditions,  $\pi_R$  and  $\eta'_R$  have minimal values.

For  $\theta = \theta_1$  Eq. (4) reads

$$(\sqrt{3}-1)\eta'_R{}^2 + 2\pi_R^2 = (\sqrt{3}+1)\eta_R^2, \quad (5)$$

which is in striking agreement with masses of the  $\pi_R$ ,  $\zeta_R$ , and  $\iota$ :

$$\pi_R = 1.205(7)^{3/}, \quad \eta_R = \zeta = 1.275(15)^{4/}, \quad \eta'_R = \iota = 1.440(15)^{1/} \quad (6)$$

The equation (5) predicts one mass if the two others are known. With masses taken from Eq. (6) we obtain the predictions

$$\zeta = 1.272(8), \quad \pi_R = 1.209(33), \quad \iota = 1.449(55).$$

The prediction for the  $K_R$  mass is  $K_R \approx 1.285(5)$  and its identification with the  $K'(\sim 1400)$  candidate<sup>/5/</sup> being not impossible is as yet somewhat problematic.

Note that the mass formula (4) for  $|\theta - \theta_1| \leq 10^\circ$  essentially coincides with Eq. (5), and the masses are practically insensitive to variations of  $\theta$  in this interval. In contrast, the parameters  $K_{1\zeta}$  and  $R_{1\zeta}$ , defined by analogy with Eq. (1), exhibit very strong dependence on  $\theta$ . For  $|\theta - \theta_1| \leq 5^\circ$  we can estimate them by using the following linear approximations:

$$K_{1\zeta} \approx 0.27 + (\theta - \theta_1)^\circ / 40^\circ, \quad R_{1\zeta}^{-1} \approx 0.32 - (\theta - \theta_1)^\circ / 40^\circ.$$

From existing data on  $\iota$  and  $\zeta$  production one can only make an "educated guess" on their upper bounds  $R_{1\zeta}^{-1} \leq 0.25$ ,  $K_{1\zeta} \leq 0.4$  (see<sup>/1,2,4/</sup>), which gives  $30^\circ \leq \theta \leq 32^\circ$ . Any statistically significant violation of the inequality  $0.58 \leq R_{1\zeta}^{-1} + K_{1\zeta} \leq 0.65$  which must hold for  $25^\circ \leq \theta \leq 35^\circ$  will disprove our simple interpretation of the  $\iota(1440)$ . There remains a possibility of a strong mixing of the  $\eta'_R$  with gluonium, which is rather difficult to disprove. An applicable mixing of the  $\eta'_R$  with the  $\eta'$  is also allowed and can somewhat modify our quantitative estimates.

A possibility of placing the  $\iota$  and  $\zeta$  in the  $P_R$ -nonet was first discussed in Ref.<sup>/10/</sup> not considering mass formulae, quantitative mixing models, and the recent information on the  $\pi_R^{3/}$ . Relying upon qualitative estimates of  $K_{1\zeta}$  and  $R_{1\zeta}^{-1}$  the

author of ref./<sup>10</sup>/ concludes that the identification  $\iota(1400) = \eta'_R$  is unlikely.

In our opinion, the evidence given above demonstrates that this remarks to be proved "beyond any reasonable doubt". To uncover the real identity of the  $\iota(1440)$  significantly better upper bounds on  $R_{1\zeta}$  and  $K_{1\zeta}$  are needed. For quantitative understanding the structure of the  $P_R$ -nonet a confirmation of the  $\pi_R, \zeta, K$  and further information on their properties would be desirable.

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Псевдоскалярные радиально возбужденные состояния  
и проблема  $\iota(1440)$ -мезона

Предложена модель сильного смешивания в радиально возбужденном псевдоскалярном нонете, основанная на аналогии со смешиванием в основном нонете. Сравнение предсказываемых массовых формул и соотношений между вероятностями рождения изоскалярных частиц с экспериментальными данными о резонансах  $\pi'(1205), \zeta(1275), \iota(1440)$  указывает на возможность размещения этих состояний в одном радиальном нонете.

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Radially Excited Pseudoscalar States  
and the  $\iota(1440)$  Problem

A model of strong mixing in the radially excited pseudoscalar nonet, which is based on analogy with mixing in the ground-state nonet, is proposed. Comparing the predicted mass formulae and ratios of the isoscalar particle production with existing data on the  $\pi'(1205), \zeta(1275)$  and  $\iota(1440)$  resonances we conclude that all these states can be placed in the radial nonet.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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