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PION FORM FACTOR IN THE VIRTON-QUARK MODEL

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1. Introduction

Pion electromagnetic form factor $F_\pi(t)$, where t is a squared transfer momentum, is measured now - directly or indirectly - in the region $-10\ \mathrm{GeV}^2 < t < 10\ \mathrm{GeV}^2$ [1]. More than 150 experimental points are obtained in different experiments at different energies [2]. Analysing the form factor behaviour, the following conclusions have been drawn:

1) The resonance is determined by the ρ-meson;
2) effects of the $\rho-\omega$ interference are perceptible in the resonance region;
3) the form factor asymptotically approaches to zero with $t \to \pm\infty$.

Until now no theoretical model exists, that would be able to describe the pion form factor in the whole region of experimentally measured values of t.

The Vector Dominance Model (VDM) [3] describes $F_\pi(t)$ not well enough. There exist various modifications of VDM, in particular, in ref. [4], where the behaviour of the form factor near the resonance is defined.

Except for VDM, all approaches to the explanation of the behaviour of $F_\pi(t)$ can be divided into two parts:

1) the dispersion-relation method;
2) field-theoretical models (chiral models, σ-models with quarks, quantum chromodynamics, etc.).

In the framework of the dispersion approach the form factor $F_\pi(t)$ was studied in detail [5-10]. The best description of

$F_\pi(t)$ in the region $-4\ \mathrm{GeV}^2 < t < 9\ \mathrm{GeV}^2$ is given by the model [8] which pretends also [9] to predict the asymptotical behaviour in the limit $t \to \pm\infty$. Although models, using methods of dispersion relations, describe $F_\pi(t)$ well both in time-like $(t>0)$ and space-like $(t<0)$ regions, they essentially depend on ad hoc hypotheses, like the choice of suitable functions with a definite number of parameters.

In the field-theoretical models there is a somewhat different situation. In general, these models can be distinguished by the regions of t, where the form factor behaviour can be described by the given model.

Let us list the models, which are able to describe $F_\pi(t)$ near the point $t=0$ and can predict the value of the rms pion radius with a good fit to the experimental data. These are: the chiral model with hadrons[11], σ-model with quarks [12], chiral model with quark loops[13], model[14] based on the idea of global duality between quark loops and hadron resonances, model[15] that uses four-quark Lagrangian. These models provide a good agreement of the rms mean square pion radius $\langle r_\pi^2 \rangle$ with that of VDM and with experimental data[16-18].

The region of the asymptotic behaviour of $F_\pi(t)$ as $t \to \pm\infty$ has been intensively studied in the framework of the quantum-field models in the last ten years[19]. It was found that in the limit $t \to \infty$ the power of the form factor decrease depends on the number of constituents of a hadron n_H, i.e.,on the number of quarks, and is equal to $F_\pi(t) \sim t^{1-n_H}$ [20]. Such a simple formula, obtained by the dimensional analysis methods, provides a good fit to the experimental data.

The results, obtained in the framework of the so-called nongauge theories,like that in[21] confirm that the form factor of pion as $t \to \pm\infty$ behaves itself like $F_\pi(t) \sim 1/t$.

Quantum chromodynamics (QCD),that pretends to be a theory of strong interactions, also gives the asymptotical behaviour of F_π like $F_\pi(t) \sim 1/t$. Besides, in the framework of QCD the methods were elaborated, that allow one to study the behaviour of the form factor in the intermediate regions at $t<0$. So, in ref.[23] the behaviour of $F_\pi(t)$ in the region $-5\ \mathrm{GeV}^2 < t < -1\ \mathrm{GeV}^2$ and in ref.[24] – in the region $-3\ \mathrm{GeV}^2 < t < -0{,}5\ \mathrm{GeV}^2$ is obtained.

The nonlocal quark model[25] or the virton-quark model of hadronic interactions belongs to the theoretical-field models. This model[26] pretends to supply a unified description of the hadronic interactions in the confinement region, i.e., in the low energy region (till 2 GeV). In this paper we compute the pion form factor in the framework of that model. It turned out that the virton-quark model describes the form factor in the region $-5\ \mathrm{GeV}^2 < t < 1\ \mathrm{GeV}^2$. The rms pion mean square radius obtaned here is proved to be $\langle r_\pi^2 \rangle = 0{,}37\ \mathrm{fm}^2$.

2. Pion Electromagnetic Form Factor

The Lagrangian, describing electromagnetic and strong interactions of a system, consisting of chraged pions, ρ-meson, charged quarks, and photons, can be written as follows[26]:

$$\mathcal{L}_I(x) = \mathcal{L}_{qq\pi}(x) + \mathcal{L}_{qq\rho}(x) + \mathcal{L}_{q\,em}(x) + \mathcal{L}_{\pi\,em}(x) . \tag{1}$$

Here

$$\mathcal{L}_{qq\pi}(x) = i g_\pi \left[\pi^+(\bar q \gamma_5 \tau^+ q) + \pi^-(\bar q \gamma_5 \tau^- q)\right] , \tag{2}$$

$$\mathcal{L}_{qq\rho}(x) = \frac{g_\rho}{\sqrt{2}} \rho^0_\mu (\bar q \gamma_\mu \tau_3 q) ,$$

$$\mathcal{L}_{\pi\,em}(x) = -ie(\pi^+ \partial_\mu \pi^- - \partial_\mu \pi^+ \pi^-) A_\mu ,$$

$$\mathcal{L}_{q\,em}(x) = e A_\mu J_\mu^{em} ,$$

where the quark electromagnetic current $\mathcal{J}_\mu^{em}$ in the regularized form (see[26]) is:

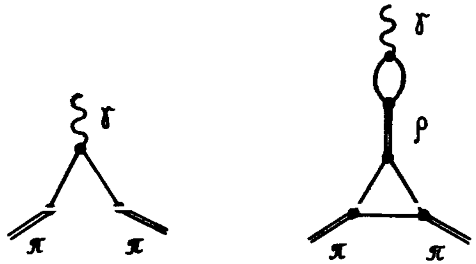
$$\left(\mathcal{J}_\mu^{em}\right)^\delta = \sum_{j=1}^{\infty} (-1)^j \left(\bar{q}_j^\delta \gamma_\mu Q \, q_j^\delta\right) .$$

The following notes were introduced:

$$q = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}; \quad \tau^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}; \quad \tau^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}; \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$Q = \frac{1}{6} I + \frac{1}{2} \tau_3 .$$

Taking into consideration the bound-state condition[26], the electromagnetic form factor is determined in our approach by the diagrams, shown in Fig. 1.



(a) (b)

Fig. 1. Diagrams, determing the pion form factor.

$$F_\pi(t) = F_a(t) + F_b(t) . \tag{3}$$

Here the contributions of diagrams (a) and (b) in Fig. 1 are denoted by $F_a(t)$ and $F_b(t)$.

Calculations give for $F_a(t)$:

$$F_a(t) = \frac{R(t)}{R(0)} , \tag{4}$$

where

$$R(t) = R_1(t) + R_2(t) + R_3(t)$$

$$R_1(t) = \frac{1}{\pi^4} \int d^4K \left\{1 + \frac{2}{T}\left[(KP_+) + (KP_-)\right]\right\} \tilde{A}(K^2) \frac{\tilde{A}((K+P_+)^2) - \tilde{A}((K+P_-)^2)}{(K+P_+)^2 - (K+P_-)^2} \tag{5}$$

$$R_2(t) = \frac{1}{\pi^4} \int d^4K \, \frac{(KP_+) + (KP_-)}{T} \tilde{B}(K^2) \frac{(K+P_+)^2 \tilde{B}((K+P_+)^2) - (K+P_-)^2 \tilde{B}((K+P_-)^2)}{(K+P_+)^2 - (K+P_-)^2}$$

$$R_3(t) = \frac{1}{\pi^4} \int d^4K \cdot K^2 \left\{1 + \frac{1}{T}\left[(KP_+) + (KP_-)\right]\right\} \tilde{B}(K^2) \frac{\tilde{B}((K+P_+)^2) - \tilde{B}((K+P_-)^2)}{(K+P_+)^2 - (K+P_-)^2} .$$

Integration is carried out over the Euclidean space. It is convenient to take the momenta P_+ and P_- in the form:

$$P_\pm = \left(\frac{L\sqrt{t}}{4}, \ \pm i\frac{L\sqrt{t}}{4}, \ 0, \ 0\right) . \tag{6}$$

$$T = t \cdot \frac{L^2}{4} ;$$

$$\tilde{A}(K^2) = \cos(\xi\sqrt{K^2}) \, e^{-K^2} ;$$

$$\tilde{B}(K^2) = \frac{\sin(\xi\sqrt{K^2})}{\sqrt{K^2}} e^{-K^2} . \tag{7}$$

Parameters of the model[26] L and ξ equal

$$L^{-1} = 320 \text{ MeV} ; \quad \xi = 1{,}4 . \tag{8}$$

The contribution of diagram (b) in Fig. 1 after the standard calculations[26] is written as

$$F_b(t) = \frac{16 \lambda_\rho}{R(0)} V(t) \frac{t}{m_\rho^2 - t - i m_\rho \Gamma_\rho(t)} W(t) . \tag{9}$$

Here the function $V(t)$ describes the block $\rho \to 2\pi$ in the diagram (b) and equals:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{10}$$

$$V_1(t) = \frac{1}{\pi^4} \int d^4K \left\{1 + \frac{2}{T}\left[(KP_+) + (KP_-)\right]\right\} \tilde{A}(K^2) \tilde{A}((K+P_+)^2) \tilde{B}((K+P_-)^2)$$

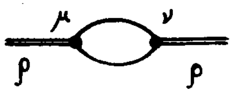


Fig. 2. The ρ-meson own mass diagram.

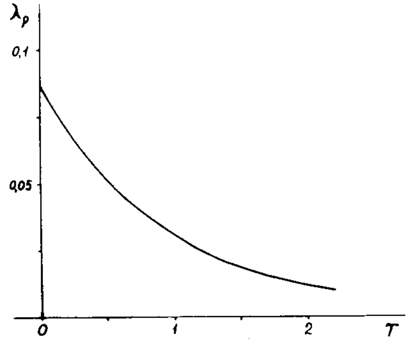


Fig. 3. Effective constant $\lambda_\rho(t)$ as a function of T.

$$V_2(t) = \frac{(-1)}{\pi^4}\int d^4k \left[(kP_+)+(kP_-)\right] \tilde{B}(k^2)\,\tilde{A}\left((k+P_+)^2\right)\tilde{A}\left((k+P_-)^2\right)$$

$$V_3(t) = \frac{1}{\pi^4}\int d^4k \cdot k^2 \left\{-1+\frac{1}{T}\left[(kP_+)+(kP_-)\right]\right\} \tilde{B}(k^2)\,\tilde{B}\left((k+P_+)^2\right)\tilde{B}\left((k+P_-)^2\right)$$

Function $W(t)$ describes the block $\rho \to \gamma$ and looks as

$$W(t) = 12\int_0^1 \alpha(1-\alpha)\,d\alpha \int_0^\infty \tilde{B}(u-\alpha(1-\alpha)T)\,du. \qquad (11)$$

Function $\Gamma_\rho(t)$ describes the decay $\rho^0 \to \pi^+\pi^-$ with the ρ-meson "mass" $\sqrt{t}$ and is equal to

$$\Gamma_\rho(t) = \frac{3\cdot 2^{10}}{\pi^3}\lambda_\pi^2\,\lambda_\rho(t)\cdot\sqrt{t}\left(1-\frac{4m_\pi^2}{t}\right)^{3/2}\cdot V^2(t)\cdot\theta\left(1-\frac{4m_\pi^2}{t}\right). \qquad (12)$$

Here $\lambda_\pi = g_\pi^2/(4\pi)^2 \approx 0{,}13$ is an effective pion-quark coupling constant. The effective coupling constant $\lambda_\rho(t) = g_\rho^2(t)/(4\pi)^2$ is calculated from the bound-state condition /26/ with the ρ-meson "mass" $\sqrt{t}$:

$$1 - \lambda_\rho \cdot \Sigma'(t) = 0. \qquad (13)$$

Here $\Sigma(t)$ is a coefficient of $g_{\mu\nu}$ in the representation of the ρ-meson mass operator, defined by the diagram in Fig. 2

$$\Sigma_{\mu\nu}(t) = g_{\mu\nu}\,\Sigma(t) + q_\mu q_\nu\,\Sigma_1(t)\,,$$

where q_μ is a transfer momentum $(t = q^2)$. It is convenient to carry out the calculation in the x-representation; we have:

$$\Sigma'(t) = 12\int_0^\infty du\, u^3 \left\{ \frac{I_2(u\sqrt{T})}{2T}\left[A^2(u^2)+\frac{1}{3}u^2B^2(u^2)\right] + \right.$$
$$\left. + \frac{1}{3}\frac{1}{T^2}\left[u^2T\cdot I_0(u\sqrt{T}) - 5u\sqrt{T}\cdot I_1(u\sqrt{T}) + 12\,I_2(u\sqrt{T})\right]B^2(u^2)\right\}, \qquad (14)$$

where

$$A(u^2) = 4\int_0^\infty s^2\cos(\xi s)\,e^{-s^2}\,\frac{J_1(s\sqrt{u^2})}{\sqrt{u^2}}\,ds\ ;$$

$$B(u^2) = 4\int_0^\infty s^2\sin(\xi s)\,e^{-s^2}\,\frac{J_2(s\sqrt{u^2})}{u^2}\,ds\ ;$$

J_1, J_2 are Bessel functions of the first kind; I_0, I_1, I_2 - modified Bessel functions of the first kind.

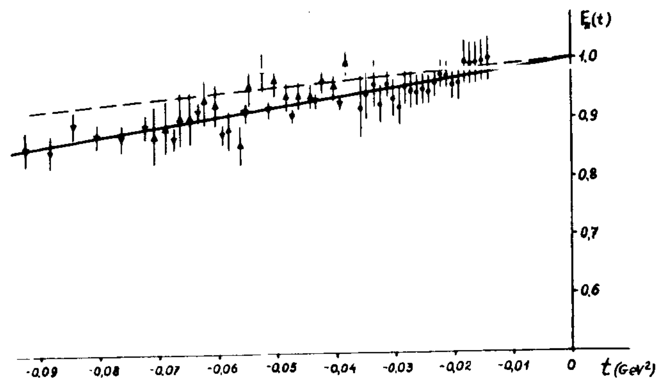


Fig. 4. Behaviour of the form factor at the small negative t. The $F_\pi(t)$-dependence, computed in the present paper is given by the full line. The broken line denotes the contribution of $F_a(t)$. The measured values of $F_\pi(t)$ are indicated by the following symbols: ● - Serpukhov experiment (1977) /16/; ▲ - Fermilab experiment (1977) /17/; ▼ - Fermilab experiment (1982) /18/.

The effective coupling constant $\lambda_\rho(t)$ as a function of T is shown in Fig. 3. At the point $t = m_\rho^2$ the decay width Γ_ρ is equal to

$$\Gamma_{\rho\to 2\pi} = \Gamma_\rho(m_\rho^2) = 145 \text{ MeV},$$

that is in a good agreement with the experimental value

$$(\Gamma_{\rho\to 2\pi})_{exp} = (152 \pm 4) \text{ MeV}.$$

Formulae (4) and (9) obtained for $F_a(t)$ and $F_b(t)$ give us the representation of the form factor $F_\pi(t)$ both in space-like $(t<0)$ and in time-like $(t>0)$ regions. The computed results are shown in Figs.4-6.

The behaviour of the form factor $F_\pi(t)$ at a small negative t, i.e.,in the region, which is determined by the pion square radius is shown in Fig. 4. We have

$$\langle r_\pi^2 \rangle = \langle r_\pi^2 \rangle_a + \langle r_\pi^2 \rangle_b = 0{,}37 \text{ fm}^2, \tag{15}$$

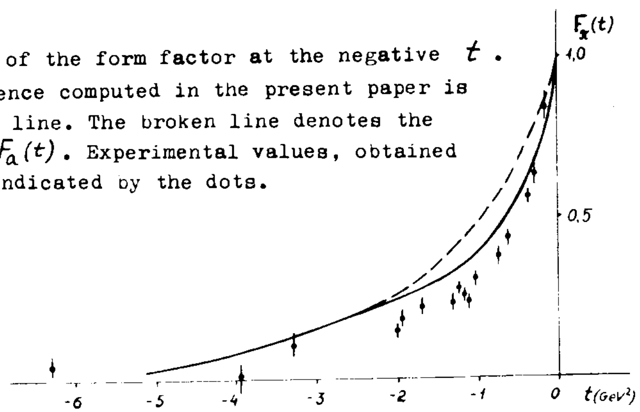
where

$$\langle r_\pi^2 \rangle_a = 0{,}21 \text{ fm}^2;$$

$$\langle r_\pi^2 \rangle_b = 0{,}16 \text{ fm}^2.$$

It is seen from Fig. 4, that our curve gives a good fit to experimental data[16-18]. The value of the rms radius,

Fig. 5. Behaviour of the form factor at the negative t. The $F_\pi(t)$-dependence computed in the present paper is given by the full line. The broken line denotes the contribution of $F_a(t)$. Experimental values, obtained in ref.[27] are indicated by the dots.



obtained from the experimental data, depends on the processing. It is shown in ref.[18] that this value fluctuates in the region

$$0{,}31 \text{ fm}^2 \le \langle r_\pi^2 \rangle \le 0{,}61 \text{ fm}^2.$$

The behaviour of the form factor $F_\pi(t)$ in the space-like region $t<0$ is represented in Fig. 5. In this region the form factor decreases in a good agreement with experimental data till $(-5 \div -6)$ GeV^2. In the region $t < -6$ GeV^2 our form factor becomes twice equal to zero, and then goes to the asymptotical behaviour

$$F_\pi(t) = \frac{0{,}1}{(-T)}\left(1 + O\left(\frac{1}{T^2}\right)\right) \sim \frac{0{,}041 \text{ GeV}^2}{(-t)}. \tag{16}$$

Indeed, $F_b(t)$ decreases as $t \to -\infty$ like $\exp\{-\frac{1}{3}|t|\}$ owing to the decrease of the strong block $\rho \to 2\pi$ in the diagram (b) Fig. 1. Therefore the asymptotical behaviour is determined only by $F_a(t)$. One can prove, that when $t \to -\infty$, integrals (5) behave like

$$S(t) = \int d^4k \left\{ \begin{matrix} 1 \\ \frac{1}{(-T)}\left[(kP_+)+(kP_-)\right] \end{matrix} \right\} X(k^2) \frac{Y((k+P_+)^2) - Y((k+P_-)^2)}{(k+P_+)^2 - (k+P_-)^2} \to$$

$$\xrightarrow[t\to-\infty]{} \frac{\pi^2}{2(-T)} \int_0^\infty ds\, s^2 \int_0^1 d\alpha\, \alpha^3 \int_{-1}^{1} d\nu \sqrt{1-\nu^2} \left\{ \begin{matrix} 1 \\ i\nu \frac{\alpha}{2\sqrt{1-\alpha^2}} \end{matrix} \right\} X(s\alpha^2)\, Y'(s\alpha^2 + 2is\alpha\nu\sqrt{1-\alpha^2}).$$

Using this formula for the functions $R_j(t)$ in (5) and carrying out some numerical calculations, we obtain (16). The behaviour of the form factor $F_\pi(t)$ in the time-like region $t>0$ is shown in Fig. 6. In this region we have a good agreement with experimental data[28-32] till $t = (0{,}8 \div 0{,}9)$ GeV^2. In the region $t > 1$ GeV^2 the increase of our form factor begins, that is, we come into the strong coupling region in our model

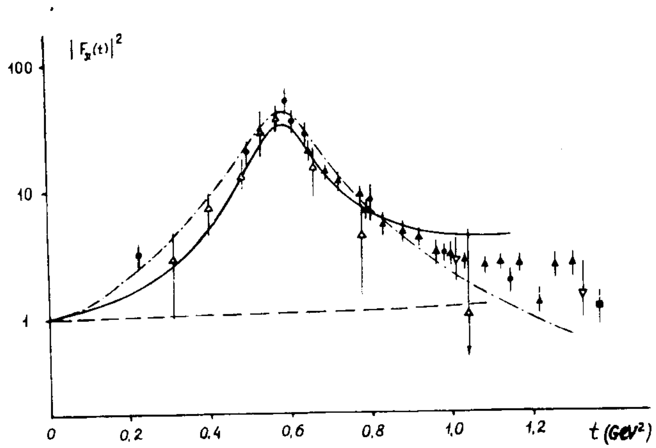


Fig. 6. Behaviour of the form factor at the positive t . The $|F_\pi(t)|^2$-dependence computed in the present paper is given by the full line. The broken line denotes the contribution of $F_a(t)$. The touch-broken line represents the $|F_\pi(t)|^2$ -dependence, obtained in the framework of VDM in ref./4/. Experimental results are indicated by: ● - Orsay (1968-1976)/28/; ■ - Frascati (1973) /29/; △ - Novosibirsk (1967)/30/; ▽ - Novosibirsk (1970) /31/; ▲ - Novosibirsk (1978) /32/.

and have to take into consideration the whole perturbation expansion.

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Дубничкова А.З., Ефимов Г.В., Лобанов Ю.Ю. E2-82-373
Формфактор пиона в виртон-кварковой модели

Вычислен формфактор пиона $F_\pi(t)$ в виртон-кварковой модели адронных взаимодействий. Получено согласие с экспериментальными данными в области -5 ГэВ$^2 < t <$ 1 ГэВ^2. Вычислен среднеквадратичный радиус пиона $\langle r_\pi^2 \rangle$ =0,37fm^2.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Pion Form Factor in the Virton-Quark Model

In the framework of the virton-quark model of the hadronic interactions the pion form factor $F_\pi(t)$ is studied. A good fit to experimental data in the region -5 GeV$^2 < t <$ 1 GeV^2 is obtained. The rms radius of pion is calculated: $\langle r_\pi^2 \rangle$ =0.37 fm^2.

The investigation has been performed at the Laboratory of the Theoretical Physics, JINR.

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