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ON THE DESCRIPTION OF MULTIPLICITY DISTRIBUTIONS IN MULTIPLE PRODUCTION PROCESSES IN TERMS OF COMBINANTS

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I. INTRODUCTION

It has been shown since long that multiplicity distributions in multiple production processes contain valuable information on the production mechanism itself. This, together with a relative ease in obtaining this kind of data in track detectors such as bubble chambers, streamer chambers or nuclear emulsions, leads to the accumulation of a large number of data on the multiplicity of secondary particles in various high energy reactions. By "multiplicity distribution" is usually meant a set of probabilities $P(n)=\sigma_n/(\Sigma\sigma_n)$, where σ_n is the cross section for the production of n particles of a given kind.

Multiplicity distributions are usually described in terms of their statistical moments or other parameters closely related to the moments. These are: average multiplicity $\langle n\rangle = \sum_n n P(n)$, dispersion $D=\sqrt{\mu_2}$, skewness $\gamma=\mu_3/D^3$, kurtosis $\delta=\mu_4/D^4$, etc., where μ_k denotes the k-th central moment, $\mu_k=\langle(n-\langle n\rangle)^k\rangle$.[1] Other choices are: normalized moments $c_k=\langle n^k\rangle/\langle n\rangle^k$ [2], binomial moments F_k, and integrated correlation functions f_k [3].

Experimental investigation of the multiplicity distributions of secondary particles in various reactions shows that parameters related to higher-order moments ($k>3$) are almost energy-independent, and the first three parameters: the average multiplicity, the dispersion and the skewness may be sufficient to parametrize the multiplicity distributions[1]. In practice, especially for not very big samples, only the first two, average multiplicity and dispersion, are usually quoted. The dispersion, D, is often replaced by the integrated correlation function $f_2=\langle n(n-1)\rangle-\langle n\rangle^2=D^2-\langle n\rangle$.

Experimental data on multiplicities, obtained by counting tracks in a track detector, could pertain to all charged secondaries, $P(n_{ch})$, or to a given type of secondaries. Most often the multiplicity distributions of negatively charged secondaries, $P(n_-)$, are quoted. There are several reasons for this choice:

a) selecting particles of definite sign prevents us from constraints resulting from charge conservation (the distribution $P(n_{ch})$ must vanish for certain n_{ch} and thus can never be a Poisson distribution; the f_k^{ch} are non-zero even in the absence of any dynamical correlations)[4,5];

b) experimentally, to a first approximation, all the negative particles produced in nucleon-nucleon collisions can be regarded as negative pions (the same is also for nucleus-nucleus collisions);
c) predictions of many theoretical models take a simple form in terms of n_- .

The aim of the paper is a description of experimental multiplicity distributions in multiple production processes in terms of a new set of parameters introduced in ref. [6].

Taking the above arguments into account, we shall restrict our analysis to multiplicity distributions of negative secondaries, $P(n_-)$.

II. COMBINANTS, THEIR DEFINITION AND PROPERTIES

Standard parameters used to describe the multiplicity distributions of secondary particles in high-energy reactions (see INTRODUCTION) are related to the cumulant expansion in probability theory *. The cumulants, the first two of which are the mean and the variance, are expressible in terms of the moments, each of which involves an infinite "cumulative" sum over all the $P(n)$. Thus, the cumulants and other related parameters are calculated from the <u>entire</u> multiplicity distributions.

In ref. [6] it is proposed to analyze multiplicity distributions in terms of another set of probability coefficients, $C(k)$, related to the probability ratios $P(m)/P(n)$; $C(k)$ are the coefficients in the particular expansion of the generating function of the multiplicity distribution.

Consider the distribution $P(n)$ with properties: $P(0) > 0$, $P(n) \geq 0$ for $n = 1, 2, \ldots$, $\sum_{n=0}^{\infty} P(n) = 1$, and its generating function **

$$F(\lambda) = \sum_{n=0}^{\infty} \lambda^n P(n).$$

It can be shown that for the Poisson distribution

$$P(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

* The cumulant expansion is the power series expansion of the logarithm of the generating function.

** Note that the Mellin transform is taken instead of the Fourier or Laplace transform which is used to obtain the cumulant expansion.

the generating function is

$$F(\lambda) = \exp[(\lambda - 1)\langle n \rangle].$$

As multiplicity distributions observed in multiple production processes are usually not very different from the Poisson distribution, the authors of ref. [6] propose to take the generating function in the form

$$F(\lambda) = \exp\left[\sum_{k=1}^{\infty} C(k)(\lambda^k - 1)\right],$$

where the expansion coefficients, $C(k)$, for $k \geq 2$, characterize the deviation of the distribution from the Poisson form. For the Poisson distribution $C(1) = \langle n \rangle$, $C(2) = C(3) = \ldots = 0$.

Taking the Poisson distribution as a reference is certainly justified, as for a wide class of theoretical models assuming weak correlation between secondary particles (thermodynamical models, Fermi-gas model, Regge pole model with exchange degeneracy) a Poisson multiplicity distribution is expected both for hadron-hadron collisions [5,6] and for nucleus-nucleus collisions with fixed impact parameter [7].

The expressions for $C(k)$ are the following:

$$C(1) = P(1)/P(0),$$

$$C(2) = P(2)/P(0) - \frac{1}{2}[P(1)/P(0)]^2,$$

$$C(3) = P(3)/P(0) - [P(1)/P(0)][P(2)/P(0)] + \frac{1}{3}[P(1)/P(0)]^3,$$

etc. We note that each $C(k)$ is expressible in terms of just the first k probability ratios: $P(1)/P(0)$, $P(2)/P(0), \ldots, P(k)/P(0)$. This stands in contrast to "cumulants", each of which involves the infinite number of $P(n)$ in its definition. It is proposed to call $C(k)$ "combinants".

Combinants possess an "additivity property": any random variable, which is composed of a sum of independent random variables, has for its combinants just the sum of the respective combinants of its components, $C(k) = \sum_i C_i(k)$. Cumulants also have this property.

III. PROTON-PROTON DATA

In <u>Fig. 1a</u> we present the values of the first three combinants $C(1)$, $C(2)$ and $C(3)$ for the multiplicity distributions of negative pions produced in proton-proton collisions for energies between 6 and 400 GeV [8-19]. Dashed lines show the behaviour expected for the Poisson distribution: $C(1) = \langle n \rangle$, $C(2) = C(3) = 0$.

In the investigation of pp collisions it has been established that at low energies (below $\langle n \rangle \simeq 1.7$ or $E_p \simeq 50$ GeV) the

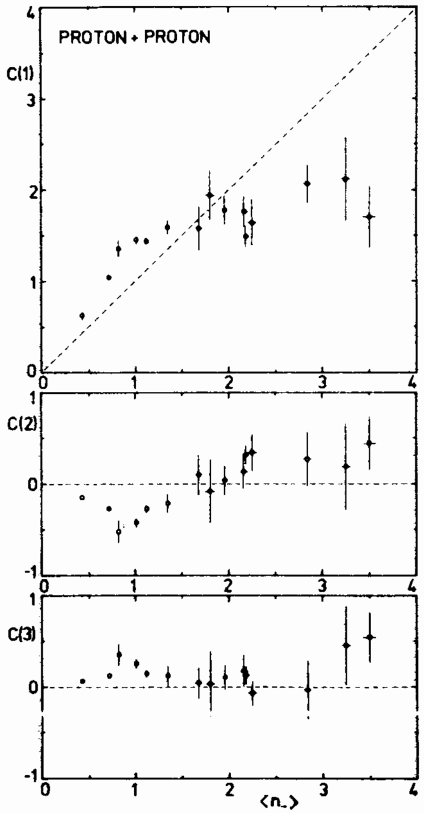

Fig.1a

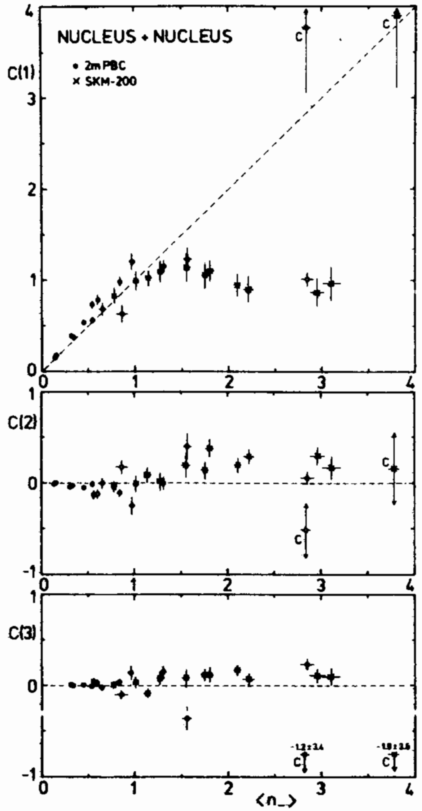

Fig.1b

pion multiplicity distribution is narrower than the Poisson distribution($D^2 < \langle n \rangle$, or $f_2 < 0$), while at higher energies the distribution becomes wider ($D^2 > \langle n \rangle$, or $f_2 > 0$). These features are clearly recognizable in the behaviour of the combinants, at least in C(1) and C(2). Reminding the very simple formula for C(1), it is amusing to find that the qualitative information about the shape of the entire multiplicity distribution is contained already in the first two probabilities! One cannot, however, exclude an accidental character of this observation, as for pp collisions a relative contribution of the diffractive component to the low-multiplicity part of the P(n) distribution changes with energy, and for Fermilab energies it becomes dominant.

IV. NUCLEUS-NUCLEUS DATA

In Fig.1b we present the values of the first three combinants for the multiplicity distributions of negative pions produced in collisions of relativistic light nuclei with nuclear targets. The data come from the Dubna 2 m propane bubble chamber (PBC) with thin tantalum target plates installed inside the chamber volume, exposed to beams of p, d, He and C nuclei at p/A = 2.2 to 5.4 GeV/c /20-22/ and from the Dubna 2 m streamer chamber (SKM-200) exposed to He and C beams at p/A = = 4.5 GeV/c /23,24/. The numbers of events recorded in various exposures of these detectors are given in Tables 1 and 2.

Table 1

Numbers of events recorded in the propane bubble chamber with tantalum target plates exposed to beams of relativistic nuclei /20-22/

p_0/A, GeV/c	2.2		4.2		5.4	
Beam \ Target	C	Ta	C	Ta	C	Ta
P	1212	842	1620	1475	1220	390
d	–	1235	699	2308	–	–
He	–	1250	995	1676	–	–
C	–	221	686	1587	–	–

Table 2

Numbers of events recorded in the streamer chamber exposed to beams of relativistic nuclei /23,24/

p_0/A, GeV/c	4.5							
Beam \ Target	Li	C	Ne	Al	Si	Cu	Zr	Pb
He	4026	1099	988	1239	–	804	–	1048
C	–	2033	2224	–	1553	1170	883	–

The non-Poisson behaviour of the combinants for $\langle n \rangle \geq 1$ is clearly seen. This dependence is very different from that for pp collisions and reflects the fact that for nucleus-nucleus collisions the multiplicity distributions are very wide due to fluctuations in the number of interacting nucleons [21]. In the case of P(n) showing KNO scaling to a smooth curve, $\Psi(z)$, with $\Psi(0) \neq 0$, one would asymptotically expect C(1)=1, C(2)=1/2, C(3)=1/3, etc. The data seem not to contradict this conjecture.

The points marked with letter "C" in Fig.1b are for "central"CC collisions (for selection criteria see refs. [25,26]). The first combinant, C(1), for these events does show a very different behaviour from that for inclusive samples, while the C(2) and C(3) have errors too big to make any significant statements.

V. CONCLUSIONS

1. It has been shown that for pp collisions the combinants reveal the known behaviour of the shape of multiplicity distribution with changing energy.

2. Description of multiplicity distributions in terms of the combinants may be useful, especially for a detailed study of the low-multiplicity part of the distribution. Also, combinants appear more appropriate than the standard parameters for the description of the multiplicity distributions of rare particles (antiprotons, charmed particles, etc.).

3. Such a description needs high statistics and, especially, a high accuracy in determining P(0) what is experimentally difficult as P(0) is subject to systematic errors connected with the separation of elastic and quasielastic scattering, diffraction dissociation, etc.

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Об описании распределений по множественности в процессах множественного рождения в терминах комбинантов

Обсуждается новый метод описания распределений по множественности вторичных частиц в процессах множественного рождения. Метод применен к процессам соударений протон-протон и ядро-ядро при высоких энергиях.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

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On the Description of Multiplicity Distributions in Multiple Production Processes in Terms of Combinants

A new method of analyzing multiplicity distributions of secondary particles in multiple production processes is discussed and applied to high-energy proton-proton and nucleus-nucleus collisions.

The investigation has been performed at the Laboratory of High Energies, JINR.

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