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ON TOPOLOGICAL VACUUM DEGENERACY IN GAUGE THEORIES

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1. Introduction

In the present paper we shall discuss several methods for the description of the topological vacuum degeneracy in gauge theories[1]. The main conclusion of the paper consists in that the nontrivial topology of the nonAbelian gauge symmetry group leads to the infrared renormalization of S-matrix in QCD.

The topological degeneracy of the classical vacuum in Yang--Mills theory has been first discovered in ref.[1]. It stimulated a great amount of works of papers investigating the vacuum degeneracy in QCD by semiclassical methods (see review[2]).

The essence of the vacuum degeneracy consists in the discrete ambiguity of the phase of nonAbelian gluon fields. The manifold of the classical gluon vacua has the same properties as that of one-dimensional paths on a two-dimensional plane with a hole: each path belongs to one of the equivalence classes characterized by an integer giving the number of windings around the hole.

There is the same ambiguity of the phase of the wave-function in the experimentally observable Aharonov-Bohm effect, i.e., electron scattering on an infinitly long and thin solenoid. All electron trajectories lie in the space, where the magnetic field is equal to zero and the scattering occurs owing to the point of the discrete variation of the phase.

The phase jumps lead to the undamped current in the ground state of a superconductive ring around the solenoid, that is the essence of the Josephson effect (if we abstract ourselves from the way of preparation of the phase jump). Our task is to show that the nontrivial topology of the gauge fields leads to non-

vanishing vacuum fields in the ground state of the quantum gauge theory (i.e., the field Josephson effect). The cause of this effect is the nonlocalizability of the "wave function" which is given in the whole field space and "feels" its topological structure[5].

In section 2 we shall consider in detail the topological vacuum degeneracy in the two-dimensional QED and we shall show that the same local dynamics (Hamiltonian and the local gauge condition) with different topological properties of gauge transformations leads to different physical pictures in quantum theory.

In section 3 we try to answer the question of why the semiclassical description of topological vacuum degeneracy does not work.

In section 4 we discuss the vacuum degeneracy in QCD starting with the identical definition of the physical vacuum both for Abelian and nonAbelian theories.

2. Quantum Electrodynamics in Two-Dimensional Space-Time

Let us consider first the simplest gauge field theory, the free "electromagnetic field", in the one-dimensional space

$$S = \int dt\,dx\,\frac{1}{2}F_{01}^2 \quad ; \qquad F_{01} = \partial_0 A_1 - \partial_1 A_0 \tag{1}$$

From the point of view of the usual four-dimensional QCD_{3+1} the theory (1) has no physical degree of freedom as the free field is transverse and the transverse field is absent in the one-dimensional space.

We shall show here that this result is not always correct and depends on the gauge symmetry group.

Theory (1) is invariant under gauge transformations

$$A_\mu \longrightarrow A_\mu(x,t) + \partial_\mu \Lambda(x,t). \tag{2}$$

We use this ambiguity for removing the temporal field component

$$A_0' = A_0 + \partial_0 \Lambda = 0 \tag{3}$$

up to the integration constant $\lambda(x)$

$$\Lambda(x,t) = -\int^t dt'\, A_0(x',t) + \lambda(x).$$

As a result, we obtain the Lagrangian

$$L = \frac{1}{2} \int dx\, \dot{A}_1^2 \quad ; \qquad \dot{A}_1 \equiv \frac{\partial}{\partial t} A_1 = \partial_0 A_1 \tag{4}$$

which is invariant under the stationary gauge transformations

$$A_1(x,t) \longrightarrow A_1'(x,t) = A_1(x,t) + \partial_1 \lambda(x). \tag{5}$$

These transformations by definition of gauge fields belong to the gauge group $U(1)$

$$A_1'(x,t) = e^{i\lambda(x)} \left(A_1(x,t) + i\partial_1 \right) e^{-i\lambda(x)} \tag{6}$$

and have to be chosen in the class of nonsingular functions vanishing at infinity

$$\lim_{|x|\to\infty} e^{i\lambda(x)} = 1 \quad ; \qquad \lambda(+\infty) - \lambda(-\infty) = 2\pi n, \quad n = \pm(0, 1, 2, \ldots). \tag{7}$$

The total gauge group of nonsingular transformations differs from the one of QED_{3+1} by a discrete ambiguity. The function $exp\{i\lambda(x)\}$ is a map of line $R(1)$ into circle $U(1)$. The map is characterized by the index (n) called the degree of mapping, which indicates how many times the line $R(1)$ turns around $U(1)$. Therefore the total gauge group is a product of the gauge group $G_{n=0} = G_0$ and the group of integers Z :

$$G = G \times Z.$$

The classical vacuum $A_1 = \partial_1 \lambda$ is degenerated in the number ($n = \frac{1}{2\pi}\int dx \partial_1 \lambda(x)$). As is shown in ref. /1/, the Yang-Mills theory gauge symmetry group has the same topological properties.

For both theories there is a relativistic invariant quantity, called the Pontryagin index, which for gauge fields $\partial_1 \lambda = A_1$ as $t = \pm\infty$ has a form of the difference of the degrees of mapping. In our case the Pontryagin index is defined by the expression

$$\nu[A] = \frac{1}{2\pi} \int dx\, dt\, F_{01} = \tag{8}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} dt \frac{\partial}{\partial t} \left[\int dx A_1 \right] = \int dx A_1 \Big|_{t=-\infty}^{t=+\infty} = n_{(+)} - n_{(-)} \neq 0 \tag{9}$$

The quantization of the theory (4) consists in the change of the electric field by its operator

$$\dot{A} \longrightarrow \frac{1}{i} \frac{\delta}{\delta A(x)} = \hat{E}(x) \quad ; \quad [E(x), A(y)] = -i\delta(x-y) \tag{10}$$

and in the solution of the Schrödinger equation

$$H\Psi(A) = \mathcal{E}\Psi(A) \quad ; \qquad H = \frac{1}{2}\int dx E^2 \tag{11}$$

and the gauge invariance condition

$$\Psi(A'^{(n)}) = \Psi(A) \quad ; \qquad n = 0, 1, 2, \ldots \tag{12}$$

The last equation means also the invariance under the infinitesimal transformations from subgroup G_0 .

$$\delta_\lambda \Psi = \int dx \frac{\delta \Psi(A)}{\delta A} \partial_1 \lambda^{(0)}(x) = 0 \quad \mapsto \quad \partial_1 \hat{E} \Psi(A) = 0 \tag{13}$$

Eq. (13) is an analog of the transversality condition in QED_{3+1}. It is easy to show that the common solution of eqs. (11),(12) is a continual product of plane waves with the same momentum

$$\Psi(A) = C\, exp\left\{ ik \int dx A_1(x) \right\} = C\, exp\left\{ ikN \right\}, \tag{14}$$

$$N = \int dx A_1 , \tag{15}$$

where C is a normalization constant, k is an eigenvalue of the electric field operator, which does not depend on the coordinate x_1 due to condition (13), $N[A]$ is the collective variable, which is transformed covariantly under the nontrivial topological transformations (6,7):

$$N[A'^{(n)}] = N[A] + 2\pi n = T^n N$$
$$T = exp\,(2\pi\, d/dN). \tag{16}$$

Thus, the gauge invariance condition may be written in the operator form

$$T\Psi(N) \equiv \Psi(N+2\pi) = \Psi(N). \qquad (17)$$

From the point of view of the usual QED_{3+1} the solution (14) is physically unacceptable as the constant electric field appears without <u>external sources</u> of the field (for example, - plates of a condensator). If we limited ourselves to the transversality condition (13) and the topologically trivial gauge transformations from G_0, the contradiction would consist in that we should obtain the representation of the group of translations (plane wave) without the corresponding group. Therefore system of equations (11), (13) has a trivial solution

$$\left.\begin{array}{l} H\Psi = \mathcal{E}\Psi \\ \partial_1 E\Psi = 0 \end{array}\right\} \Rightarrow \Psi = C \;; \quad \mathcal{E} = 0 \qquad (A)$$

in the correspondence with results of the usual quantization of QED_{3+1} [8].

However, in the theory with the topological vacuum degeneracy (8), (17) both contradictions disappear as the group of translational invariance exists and the condition (17) means physical identity of points $N, N+2\pi, \ldots$, i.e., the system is closed w.r.t. N. In the closed system the collective excitations exist and correspond to the "circular" motion of the field without the external sources. This situation is realized in quantum liquids [9,10,11]. Thus, the discrete gauge ambiguity leads to a physical picture different from (A)

$$\left.\begin{array}{l} H\Psi = \mathcal{E}\Psi \\ \partial_1 E\Psi = 0 \\ T\Psi = e^{i\theta}\Psi \end{array}\right\} \Rightarrow \Psi = C e^{ikN} \;; \quad \begin{array}{l} k = \ell + \frac{\theta}{2\pi} \\ \ell = \pm(0,1,2,\ldots). \end{array} \qquad (B)$$

The condition (17) is here written in a covariant form where θ is called the quasimomentum, and ℓ is the number of the Brillouin zone. Vacuum of such a theory (which is usually called the θ-vacuum) is the stationary electric field with the finite energy density

$$\mathcal{E}/\int dx = k^2/2 \qquad (18)$$

There is a charge confinement in the theory as stationary states of the charged particles in the electric field are absent.

It is not difficalt to be convinced of that the solution (14) describes the infrared dynamics and does not depend on the choice of gauge.

It is known that the clasical and quantum dynamics of gauge fields are completely defined by the action functional

$$S = \frac{1}{2}\int dt\, dx\, F_{01}^2 \qquad (19)$$

given on classical solutions of equation for the component (see refs. [12,13])

$$\delta S/\delta A_0 = 0 \Rightarrow \partial_1^2 A_0 = \partial_1 \partial_0 A_1. \qquad (20)$$

The temporal component, which has no canonical momentum, in principle, cannot be considered as a quantum operator in contrast to the component A_1. Therefore eq. (20) is treated as an auxiliary condition or a constraint equation.

The general solution of eq. (20) is

$$A_0(x,t) = C_0(t) + C_1(t)x + \int^x dx'\, \partial_0 A_1(x',t), \qquad (21)$$

where $C_0(t)$ and $C_1(t)$ are the integration constants. From eq. (21) we obtain for F_{01} the expression

$$F_{01}(x,t) = -C_1(t). \qquad (22)$$

If we neglect the infrared singular solutions of the homogeneous eq. (20)

$$\partial_1^2 A_{0(inf)} = 0 \Rightarrow A_{0(inf)} = C_0(t) + C_1(t)x, \qquad (23)$$

we obtain for the action (19) zero: $S \equiv 0$ that corresponds to the usual electrodynamics.

However, the condition of the topological vacuum degeneration (9)

$$\nu = \frac{1}{2\pi}\int dt\, dx\, F_{01} \equiv \frac{1}{2\pi}\int dt\, \dot{N} = -\frac{1}{2\pi}\int dt\, C_1(t)\left(\int dx\right) \neq 0$$

leads to the Lagrangian

$$L = \frac{1}{2}\dot{N}^2\left[\int dx\right]^{-1} \qquad (24)$$

expressed in terms of a "covariant variable"

$$\dot{N} = -C_1(t)\left(\int dx\right) \qquad (25)$$

It is easy to show that the results of quantization of the theory (24) are completely equivalent to the ones obtained in the gauge $A_0 = 0$.

Thus the topological vacuum degeneration in the considered theory is described by the introduction of singular infrared fields into the explicit solutions of the constraint equation [20]. Therefore the construction of the classical Lagrangian in terms of the physical variables requires an infrared regularization $\left(\int dx\right) < \infty$. The removal of the regularization in eq. (24) leads, generally speaking, to the known trivial result $S \equiv 0$. However, the size of the region of validity of the quantum theory L_a , which is defined by the effective mass in Lagrangian (24) $m \sim \left(\int dx\right)^{-1}$, coincides with the macroscopic size of the space, where the gauge field is given

$$L_a \sim \left(\int dx\right). \qquad (26)$$

Therefore, no classical analogy of infrared topological dynamics does exist. It is physically clear, as we describe the "zero" frequence field excitation.

3. Topological Vacuum Degeneration in QCD (Instantons)

The topological vacuum degeneration of gauge fields was first considered in the nonAbelian theory

$$S = -\frac{1}{4}\int d^4x \left(F^a_{\mu\nu}\right)^2; \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^{abc} A^b_\mu A^c_\nu \qquad (1)$$

In ref. [1] classical solutions with a finite Euclidean action and zero energy (instantons) were found with the global characteristics, the Pontryagin index,

$$\nu[A] = \frac{g^2}{32\pi^2}\int d^4x\, F^a_{\mu\nu}\, {}^*F^{a\mu\nu} \equiv \frac{g^2}{8\pi^2}\int d^4x\, E^a_i B^a_i \neq 0\,, \qquad (2)$$

where

$${}^*F_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta},$$

E^a_i, B^a_i – are electric and magnetic fields

$$E^a_i = \partial_0 A^a_i - \nabla^{ab}_i(A)A^b_0\,; \quad \nabla^{ab}_i(A) = \delta^{ab}\partial_i - g\varepsilon^{abc}A^c_i\,,$$
$$B^a_i = \frac{1}{2}\varepsilon_{ijk}F^a_{jk}\,. \qquad (3)$$

The instantons in gauge $A_0 = 0$ are transitions between classical vacua

$$\lim_{t\to\pm\infty} A_i(\vec{x},t) = v_{(\pm n)}(\vec{x})\,\partial_i v^{-1}_{(\pm n)}(\vec{x})\,; \quad A_i = g\frac{\tau^a A^a}{2i}\,, \qquad (4)$$

where the matrix $v(\vec{x})$ satisfies the condition

$$\lim_{|x|\to\infty} v_{(n)}(\vec{x}) = I\,. \qquad (5)$$

It is known [1,2] that $SU(2)$ -valued matrices $v(\vec{x})$ and the condition (5) define the regular (smooth) mapping of space $R(3)$ into $SU(2)$ and are characterized by the integer index (degree of mapping)

$$n = -\frac{1}{12\pi^2}\int d^3x\, \varepsilon_{ijk}\, \mathrm{tr}\left(v^{-1}\partial_i v\right)\left(v^{-1}\partial_j v\right)\left(v^{-1}\partial_k v\right) \qquad (6)$$

indicating how many times the space $R(3)$ turns around $SU(2)$. The total group of the gauge transformations

$$A'_i(\vec{x},t) = v(\vec{x})\left(A_i + \partial_i\right) v(\vec{x})^{-1} \qquad (7)$$

is topologically disconnected and it is a product of a "small" gauge group and the infinite cyclic group of all integers

$$G = G_{(n=0)} \times Z\,.$$

The classical vacuum is degenerated in number (6). For fields with asymptotics (4) it is easy to show that the expression (2) has the form of the difference of the degrees of mapping $(n_{(+)}, n_{(-)})$

$$\nu = \int dt\, d^3x\, \partial_0 A^a_i \left[\frac{g^2}{8\pi^2}B^a_i\right] = \int dt\, d^3x\, \partial_0 A^a_i \cdot$$
$$\cdot \left(\delta N[A]/\delta A^a_i\right) = \int dt\, \partial_0 N = n_{(+)} - n_{(-)}\,, \qquad (8)$$

where $N[A]$ is a functional

$$N[A]=\frac{g^2}{8\pi^2}\int d^3x\,\varepsilon_{ijk}\left(\frac{1}{2}\partial_i A_j^a A_k^a+\frac{g}{6}\varepsilon^{abc}A_i^a A_j^b A_k^c\right) \qquad (9)$$

which is equal to the integer (6) for the gauge fields (4). According to (8) a variational derivative of the functional N equals magnetic field (3)

$$\frac{\delta N}{\delta A_i^a(x)}=\frac{g^2}{8\pi^2}B_i^a(x). \qquad (10)$$

Like the "collective variable" N in section 2, the functional (10) is transformed covariantly under the topologically nontrivial gauge transformation[14]

$$N[A^{((n)}]=N[A]+n=(T)^n N[A] \qquad (11)$$

$$T=\exp\left(d/dN\right)\;;\quad dN=\int d^3x\,\frac{\delta N}{\delta A}\,\delta A. \qquad (12)$$

From the point of view of invariance principles this theory does not differ from QED [illegible]. The problem is the construction of the quantum nonAbelian theory with transformation group $G=G_0\times Z$.

The investigation of the topological quantum-vacuum degeneration is usually based on the analogy with quantum mechanics and on the interpretation of instantons as tunnellings between different classical vacua. In quantum mechanics the semiclassical expansion of the Green function around these solutions allows one to calculate the ground state energy. The small parameter of the expansion (α) is proportional to the difference of the ground state energy $(\mathcal{E}_0)$ and the tunnelling energy

$$\alpha\sim\Delta\mathcal{E}=\mathcal{E}_0-\mathcal{E}_T. \qquad (13)$$

To calculate $\Delta\mathcal{E}$, one takes the limit of the Green function spectral representation as $t\to i\infty$:

$$\lim_{T\to\infty}\langle A_i^{(1)}(x)|e^{iTH}|A_j^{(2)}(x)\rangle=\lim_{T\to\infty}\sum_{\mathcal{E}}e^{iT\mathcal{E}}\Psi_{\mathcal{E}}^*[A_i^{(1)}]\Psi_{\mathcal{E}}[A_j^{(2)}]\Big|_{T=i\tau}\sim e^{-\tau\mathcal{E}_0}; \qquad (14)$$

where $\Psi_{\mathcal{E}}$, $\Psi_{\mathcal{E}}^*$ are physical solutions of the system of equations

$$\left.\begin{aligned} H\Psi_{\mathcal{E}}&=\mathcal{E}\Psi_{\mathcal{E}}\\ \nabla_i\hat{E}_i\Psi_{\mathcal{E}}&=0\\ T\Psi_{\mathcal{E}}&=e^{i\theta}\Psi_{\mathcal{E}}\end{aligned}\right\}\qquad H=\frac{1}{2}\int d^3x\,(E^2+B^2)\qquad E_i^a=\frac{1}{i}\,\delta/\delta A_i^a \qquad (B^1),\ (B^2),\ (B^3)$$

This system differs from the analogous one (B) in section 2 by the transverse degrees of freedom with the potential of oscillator type $V(A)=B^2/2$ and by the dependence of the covariant "collective variable" $N[A]$ on the oscillator transverse variables locally, at each point of space (10). According to equation (10) the operators of energy and topological translation do not commute $[T,H]\neq 0$. These operators may have common eigenfunctions, if only an eigenvalue of one of these operators is equal to zero. Such an operator may be only H (as $|e^{i\theta}|=1$). Therefore we have

$$\mathcal{E}=0. \qquad (15)$$

However, this eigenvalue is unphysical. It is easy to construct the corresponding wave function, using eq. (10)

$$\Psi_0=\exp\{\pm i(2\pi\ell+\theta)N[A]\}\;;\quad(2\pi\ell+\theta)=i\,\frac{8\pi^2}{g^2}. \qquad (16)$$

The plane wave (16) is not normalizable in the field space. Thus the system of equations (B^1)-(B^3) has no physical solution and the Green function (14) is identically equal to zero. One can say that the quantum tunnelling energy $\mathcal{E}_T=0$, which is an eigenvalue for the dual wave function (16) ($\hat{E}\Psi_0\sim B\Psi$) is situated in the deeply unphysical region of the possible spectrum for infinite number of oscillators ($\mathcal{E}_0-\mathcal{E}_T=\infty$). The small parameter of the instanton approximation in gauge theory does not exist. This fact is beginning to be more recognized by the followers of the semiclassical approach[2].

The absence of exact physical solutions of system (B) does not still mean the same for the system

$$H\Psi=\mathcal{E}\Psi \qquad (A^1)$$

The properties of the infrared gluons resemble more those of nonideal Bose-gas in the microscopic Bogolubov theory of superfluidity [10].

The difficulty of the physical interpretation and understanding of the obtained results consists in the extension of the quantum representations to the whole macroscopic observable region, where the nonAbelian fields are given. Following the quantum liquid ideology we should propose the unique macroscopic wave function for the observable world, which is defined not only by the local equations, but also by the symmetry group topology

$$\Psi_{vac} \sim \exp\{ip\nu\} \;; \quad p = (2\pi \ell + \theta)$$

$$\ell = \pm(0,1,2,\dots)\;; \quad 0 \leqslant \theta \leqslant \pi \,.$$

We have shown in section 2 that in the "quantum world" the classical fields with internal singularities without external sources may exist, that is forbidden in the classical Yang-Mills theory. These singularities are conditioned by "rotation" of the system of the nonAbelian fields as a whole over the circular "zero" variable ν. The equations of quantum hydrodynamics for such QCD suprefluid vacuum may be written in the form

$$E_i \sim \nabla_i \Phi \tag{3}$$

$$B_i \sim \nabla_i \Phi \;; \quad \nabla_i E_i = \nabla_i B_i = \nabla^2 \Phi = 0\,, \tag{4}$$

where E and B are electric and magnetic vacuum fields and Φ is the singular classical solution of the constraint equation (1). Equation (3) is a consequence of the stationarity of the quantum liquid and equation (4) follows from the relativistic covariance (if one neglects the interaction with matter fields, then the electric and magnetic fields should enter into the theory on equal footing).

The unique solution of eqs. (3), (4), satisfying the conditions of isotropy and stability (i.e., the quasiparticly being energetically disfavoured) is the singular field

$$(A^a_\mu)_{vac} = \frac{1}{g}\sum^a_{\mu\nu} \partial^\nu \ln\rho \;; \quad \Box\rho = 0 \tag{5}$$

$$\rho = e^{ik_0 t}\,\frac{\sin k_0 \tau}{\tau} \;; \quad \tau = |\vec{x}| \tag{6}$$

$$\sum^a_{\mu\nu} = \varepsilon_{0a\mu\nu} + i g_{a\mu} g_{\nu 0} - i g_{a\nu} g_{\mu 0}\,.$$

This vacuum field minimizes the effeotive action, expressed in terms of the conserved topological momentum.

As is shown in ref. [21] the physical picture of the asymptotic states of coloured fields in such a vacuum is a relativistic version of the hadron bag model [21,22]. The nonAbelian vacuum represents the manifold of "empty" hadron bag obtained from (5), (6) by various Poincare group transformations. All physical observables are equal to zero [21] and in this sense the physical vacuum is relativistically invariant.

The analogy of the "size" of a physical device in QED for QCD is the bag size (K_0^{-1}), which should be considered as the phenomenological parameter, because of the necessity to consider the system "as a whole" (for example, for the calculation of singularity parameters in the superfluid rotating helium. it is necessary to take into account the macroscopic-rotation energy, vessel size, and the microscopic parameter of an average distance between the helium atoms [11], i.e., to give a physical mechanism of the infrared regularization of an expression of the type of $[\int d^3x (\nabla\Phi)^2]$).

In QCD for results to be finite such details may not be needed, as there are eigenvalues of the topological momentum

$$\left(\ell + \frac{\theta}{2\pi}\right)^{-1} = g^2/4\pi$$

for which the action and the Pontryagin index are finite and physical results do not depend on parameters and the mechanism of infrared regularization.

The topological description of zero mode ν allows one to make the infrared renormalization in QCD in the spirit of ref. [19], in which the QED S-matrix is formulated without infrared divergences.

The infrared factorization in QCD, like in QED, is justified by the general solution of the Dirac equation in arbitrary vacuum fields (5); the solution is obtained in ref. [23] by the Newman-Penrose method.

Conclusion

In the paper we have shown that the topological vacuum degeneration in QCD may lead to the infrared renormalization of QCD expressed in the redefinition of the classical action by surface terms. In such a theory the quark confinement emerges as a macroscopic quantum phenomenon which cannot be obtained by the renormgroup methods, by the analytic calculation at a computer, or by the exact calculation of the usual Faddeev-Popov functional integral.

The confinement might be understood if we proceeded from the definition of the physical vacuum of any gauge theory as the manifold of all infrared fields which have no physical observables, but the interactions with them must be necessarily taken into account for removing the infrared catastrophe. (For example, in QED the infrared divergences are removed in the long run by taking into account the interaction with all long-wave photons). Whereas in QED the vacuum is the gas of free photons, in QCD, owing to the strong interaction of infrared gluons, the vacuum is the quantum liquid, singularities of which form the hadron bags.

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Appendix

We present the results of the formal calculation of the nonAbelian action functional on extremals

$$\delta S/\delta A_0^a = 0 \implies (\nabla^2 A_0)^a = (\nabla_i \partial_0 A_i)^a . \tag{A.1}$$

Into the solution of equation (A.1)

$$A_0^a = C(t)\phi^a + \left(\frac{1}{\nabla^2}\right)^{ab} (\nabla_i \partial_0 A_i)^b \tag{A.2}$$

we include the solution of the homogeneous equation (A.1): $\nabla^2\phi = 0$ with coefficient $C(t)$, defined by the Pontryagin index

$$\nu \equiv \int dt\, \dot{\nu} = \frac{g^2}{16\pi^2} \int d^4x\, F_{\mu\nu}{}^{*}F^{\mu\nu} \Big|_{A_0 = A_0[c(t), \partial_0 A_i]} . \tag{A.3}$$

Then the action expressed in terms of the topological conserving variable

$$P = \delta S/\delta \dot{\nu} = (2\pi \ell + \theta)$$

is

$$S(P, \delta S/\delta A_0 = 0) = \frac{1}{2}\int dt \left\{ \int d^3x \left(E_{\mathcal{D}}^2 - B_{\mathcal{D}}^2\right) + \left[\left(P\frac{g^2}{8\pi^2}\right)^2 - 1\right] \chi_B^2 \, \chi_\phi^{-1} \right. \tag{A.4}$$

$$E_{\mathcal{D}} = E_T - \nabla\phi(\chi_E \chi_\phi^{-1}) \; ; \qquad B_{\mathcal{D}} = B - \nabla\phi\,\chi_B(\chi_\phi^{-1}) \tag{A.5}$$

$$E_{Ti} = \left(\delta_{ij} - \nabla_i \frac{1}{\nabla^2}\nabla_j\right)\partial_0 A_j \; ; \quad B_i = \frac{1}{2}\varepsilon_{ijk} F_{jk} .$$

The functionals

$$\chi_E = \int d^3x (E_T \nabla\phi); \quad \chi_B = \int d^3x (B\nabla\phi); \quad \chi_\phi = \int d^3x (\nabla\phi)^2 . \tag{A.6}$$

Owing to the transversality conditions

$$\nabla E = \nabla B = \nabla(\nabla\phi) = 0$$

are surface terms.

References

1. Belavin A.A. et al. Phys.Lett. 1975, 59B, p. 85.
2. Вайнштейн А. и др. Препринт ИТЭФ-2, 1981."Инстантонная азбука".
3. Aharonov Y., Bohm D.Phys.Rev. 1959, 115, p. 485.
   Файнберг Е.Л. УФН 1962, 78, стр. 53.
4. Любошиц В.Л., Смородинский Я.А. ЖЭТФ 1978, 75, стр. 40.
5. Блохинцев Д.И. УФН 1950, 42, вып. 1.
6. Polyakov A.M. Nucl.Phys., 1976, B120, p.429.
   Phys.Lett. 1975, 59B, p. 83.
7. Callan C.G. Jr. et al. Phys.Rev. 1978, D17, p. 2717.
8. Славнов А.А., Фаддеев Л.Д. Введение в теорию квантованных полей, М., Наука, 1978.
9. Ландау Л.Д. ЖЭТФ, 1941, 11, 592; ДАН СССР, 1948, 61, с. 253.
10. Bogolubov N.N. Journ. of Phys., 1947, 2, p. 23.

11. Халатников И.М.Введение в теорию сверхтекучести. М., Наука, 1963.
12. Полубаринов И.В. ОИЯИ 2-Р-2421, Дубна, 1965.
13. Бьеркен Дж.Д., Дрелл С.Д. "Релятивистские квантовые поля". т. 2, М., Наука, 1978.
14. Фаддеев Л.Д. Материалы IУ Межд. Сов. по нелокальным теориям поля. 1976, ОИЯИ Д1-9788, Дубна 1977, с.207.
15. Polyakov A.M. Nucl.Phys., 1976, B120, p.429.
16. t' Hooft G. Phys.Rev. 1976, D14, 3432.
17. Первушин В.Н. ТМФ, 1980, 45, 394. ОИЯИ Р2-12225, Дубна, 1979.
18. Efremov A.V., Radyushkin A.V. Rivista del. Nuovo Cim. 1980, 3, N 2.
19. Кулиш П.П., Фаддеев Л.Д. ТМФ 1970, 4, с. 153.
20. Первушин В.Н. ЯФ 1982, 36, с.263. Pervushin V.N. JINR E2-81-525, Dubna, 1981.
21. Bogolubov P.N. Ann.Inst. H.Poincare, 1967, 8, p. 163.
22. Chodos. A. et al. Phys.Rev., 1974, D19, p. 2391.
23. Pervushin V.N., Horjeisi I. JINR E2-82-316, Dubna, 1982.

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Первушин В.Н. E2-82-365

О топологическом вырождении вакуума в калибровочных теориях

Топологическое вырождение вакуума в калибровочных теориях рассматривается как квантовое явление, не имеющее аналога в классической физике. Показано, что нетривиальная топология калибровочной группы дает возможность проводить инфракрасную перенормировку неабелевой теории, такую же как в КЭД. Предлагается единое определение физического вакуума для абелевых и неабелевых теорий.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Pervushin V.N. E2-82-365

On Topological Vacuum Degeneracy in Gauge Theories

It is shown that the nontrivial topology of gauge fields leads to the Josephson effect in the field space, i.e., to nonvanishing vacuum fields. The same definition is proposed for the physical (infrared) vacuum for Abelian (QED) and nonAbelian (QCD) theories. The equations and the topological Josephson effect for the gluon vacuum are discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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