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FOUR-LOOP BETA-FUNCTION IN THE WESS-ZUMINO MODEL

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Recently an effective method [1] of computing momentum integrals in the dimensional regularization has been proposed. This method allows one to calculate the renormalization group functions up to the four-loop level in any renormalizable theory. It seems interesting to apply the method to the simplest supersymmetric theory, the Wess-Zumino (WZ) model.

In this model the β-function has been calculated in the three- [2] and four-loop [3] approximation. But there are some mistakes in these calculations: for instance, the presence of $\zeta(2)$ in the β-function coefficients [3] is impossible [4,1].

The supersymmetric dimensional regularization (regularization by dimensional reduction) [5] is known to be inconsistent [6]. However, in ref. [7] its unambiguous modification has been proposed. Extending the arguments of ref. [7] to the WZ model leads to the conclusion that the regularization by dimensional reduction is quite correct in four-loop calculations. It justifies the use of this regularization in our calculations.

The action of the massless WZ model written in terms of the chiral superfield of type one $\Phi_1(x,\theta)$ is:

$$S = \int dx\, d^2\theta\, d^2\bar{\theta}\, \bar{\Phi}_1(x,\bar{\theta})\, e^{-2i\theta\hat{\partial}\bar{\theta}}\, \Phi_1(x,\theta)$$

$$+ \frac{\lambda}{3!}\left(\int dx\, d^2\theta\, \Phi_1^3(x,\theta) + \int dx\, d^2\bar{\theta}\, \bar{\Phi}_1^3(x,\bar{\theta}) \right).$$

We use the following notation:

$$\theta\psi = \theta_A \epsilon_{AB} \psi_B, \qquad \bar{\theta}\bar{\psi} = \bar{\theta}_{\dot{A}}\, \bar{\epsilon}_{\dot{A}\dot{B}}\, \bar{\psi}_{\dot{B}},$$

$$\theta^2 = \theta_A \epsilon_{AB} \theta_B, \qquad \theta\sigma_\mu \bar{\psi} = \theta_A \epsilon_{AB} (\sigma_\mu)_{B\dot{B}}\, \bar{\epsilon}_{\dot{B}\dot{A}}\, \bar{\psi}_{\dot{A}},$$

$$(\tilde{\sigma}_\mu)_{\dot{A}A} = \epsilon_{AB} (\sigma_\mu)_{B\dot{B}}\, \bar{\epsilon}_{\dot{B}\dot{A}}, \qquad \epsilon_{AB}\epsilon_{BC} = -\delta_{AC},$$

$$\hat{p} = \sigma_\mu p_\mu, \qquad \tilde{p} = \tilde{\sigma}_\mu p_\mu, \text{ etc.}$$

The σ-matrices obey the commutation relations:

$$\sigma_\mu \tilde{\sigma}_\nu + \sigma_\nu \tilde{\sigma}_\mu = 2g_{\mu\nu} \mathbf{1},$$

$$\tilde{\sigma}_\mu \sigma_\nu + \tilde{\sigma}_\nu \sigma_\mu = 2g_{\mu\nu} \mathbf{1}.$$

The trace conditions are:

$$\mathrm{tr}\,\mathbf{1} = \delta_{AA} = 2, \quad g_{\mu\mu} = 4-2\epsilon .$$

The θ-integration formulae are:

$$\int d^2\theta = 0, \quad \int d^2\theta\, \theta_A = 0, \quad \int d^2\theta\, \theta^2 = 1,$$

and the same are formulae for the $\bar{\theta}$-variables.

In this model only propagator-type diagrams can diverge [8]. Hence, to calculate the β-function we only need the field renormalization constant Z_Φ which in the minimal subtraction scheme has the form:

$$Z_\Phi(h,\epsilon) = 1 + \sum_{n=1}^{\infty} \frac{Z_\Phi^{(n)}(h)}{\epsilon^n}, \tag{1}$$

where

$$h = \left(\frac{\lambda}{4\pi}\right)^2$$

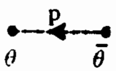
Then

$$\beta(h) = -3h^2 \frac{\partial}{\partial h} Z_\Phi^{(1)}(h). \tag{2}$$

The diagrams contributing to Z_Φ and the results of their calculations are presented in the Table.

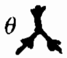
We use the technique of ref. [8] to reduce the supergraphs to momentum integrals. For propagator diagrams this technique turns out to be more convenient than that of ref. [9], used in refs. [2,3]. The reason consists in the possibility of making and exploiting the most convenient choice of independent momenta of loop integrations.

In the massless case the only propagator in the WZ model is $<\Phi_1(x,\theta)\bar{\Phi}_1(y,\bar{\theta})>$, which in momentum space has the form:

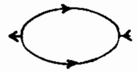
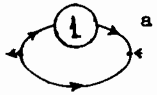
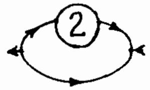
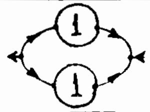
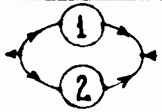
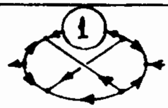
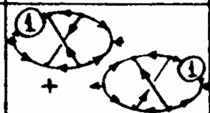


$$\Rightarrow i\,\frac{\exp(2\theta \hat{p} \bar{\theta})}{p^2 + i0}.$$

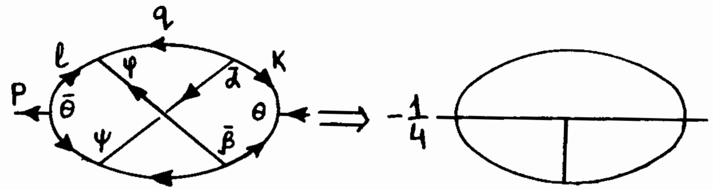
The θ and $\bar{\theta}$ vertices are represented by the factors:

θ  $\Rightarrow i\lambda$, $\quad \bar{\theta}$  $\Rightarrow i\lambda$.

Results of computations

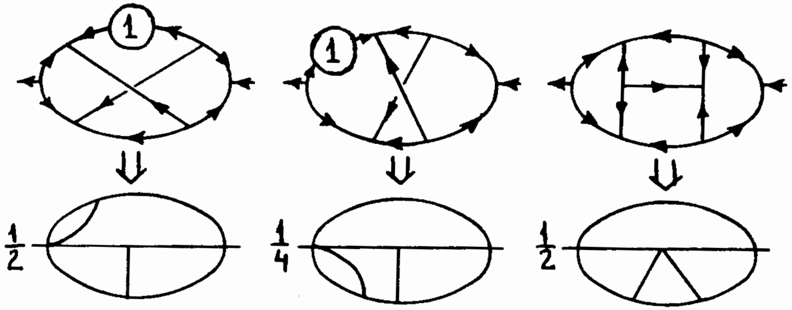
N	Supergraphs	Contributions to Z_Φ
1.1		$h\cdot\left[-\frac{1}{2\epsilon}\right]$
2.1	 a)	$h^2\left[-\frac{1}{4\epsilon^2}+\frac{1}{4\epsilon}\right]$
3.1		$h^3\left[-\frac{1}{6\epsilon^3}+\frac{1}{3\epsilon^2}-\frac{1}{4\epsilon}\right]$
3.2		$h^3\left[-\frac{1}{24\epsilon^3}+\frac{1}{24\epsilon^2}+\frac{1}{24\epsilon}\right]$
3.3		$h^3\left[-\frac{\zeta(3)}{2\epsilon}\right]$
4.1		$h^4\left[\frac{7}{48\epsilon^4}+\frac{5}{12\epsilon^3}-\frac{1}{2\epsilon^2}\;\frac{3\zeta(3)}{8\epsilon^2}+\frac{1}{3\epsilon};\frac{7\zeta(3)}{16\epsilon}+\frac{3\zeta(4)}{16\epsilon}\right]$
4.2		$h^4\left[-\frac{1}{16\epsilon^4}+\frac{11}{96\epsilon^3}-\frac{1}{48\epsilon^2}-\frac{5}{96\epsilon}-\frac{\zeta(3)}{16\epsilon}\right]$
4.3		$h^4\left[-\frac{\zeta(3)}{4\epsilon^2}+\frac{3\zeta(3)}{4\epsilon}-\frac{3\zeta(4)}{8\epsilon}\right]$
4.4		$h^4\left[-\frac{\zeta(3)}{4\epsilon^2}+\frac{3\zeta(3)}{4\epsilon}-\frac{3\zeta(4)}{8\epsilon}\right]$
4.5		$h^4\left[+\frac{5\zeta(5)}{2\epsilon}\right]$

a) —L— is the L-loop contribution to the connected propagator.



a) b)

Fig.1



a) b) c)

Fig.2

As an example of calculations we consider the diagram 3.3 of the Table. The choice of the loop momenta is pictured in Fig.1a.

The numerator of this diagram can be written in the following form:

$$\exp\{2(\psi-\theta)\hat{k}(\bar{\beta}-\bar{\alpha})+2(\psi-\phi)\hat{q}(\bar{\beta}-\bar{\alpha})$$

$$+2(\psi-\phi)\hat{\ell}(\bar{\beta}-\bar{\theta})+2(\psi-\theta)\hat{p}(\bar{\beta}-\bar{\theta})-2\theta\hat{p}\bar{\theta}\}.$$

Performing the change of the θ-variables

$$\psi-\theta=\psi', \quad \bar{\beta}-\bar{\theta}=\bar{\beta}', \quad \psi-\phi=\phi', \quad \bar{\beta}-\bar{\alpha}=\bar{\alpha}'$$

we get:

$$\exp(-2\theta\hat{p}\bar{\theta})\exp(2\psi'\hat{k}\bar{\alpha}')\exp(2\phi'\hat{q}\bar{\alpha}')\exp(2\phi'\hat{\ell}\bar{\beta}')\exp(2\psi'\hat{p}\bar{\beta}')$$

and after the θ-integration we have:

$$\exp(-2\theta\hat{p}\bar{\theta})\cdot[k^2\cdot\ell^2+p^2\cdot q^2-\mathrm{tr}(\hat{p}\hat{k}\hat{q}\hat{\ell})].$$

All expressions containing the external momentum in numerators are finite due to the logarithmic [8] divergency of the propagator diagrams in the WZ model. Hence, we can set to zero the external momentum in numerators from the very beginning, if we are interested in divergencies only. In this way the divergent part of the diagram 3.3 is reduced to the scalar diagram pictured in Fig.1b (the factor 1/4 takes into account the symmetry of the diagram).

The divergent parts of the diagrams 4.3, 4.4, 4.5 of the Table are analogously reduced to the scalar diagrams of Figs.2a-c, respectively.

The method of ref. [1] for calculating momentum integrals is based on a set of algebraic recurrent relations. This set allows one to represent any three-loop propagator integral as an expansion over a fixed basis. The elements of this basis are primitive integrals (which can be reduced to a number of one-loop integrations) and two nontrivial ones which can be calculated once and for all by the Gegenbauer polynomial x-space technique [10]. As a result we get a tool for the analytical evaluation of any three-loop integral of the propagator type up to and including $O(\epsilon^0)$ order, which is sufficient [4,1] for the calculation of four-loop renormalization constants.

The main advantage of the used method is the existence of a relatively simple calculational algorithm that allows one to write an effective computer program on the system of analytical evaluations SCHOONSCHIP [11]. Any three-loop integral with one external momentum can be computed by this program to finite order in ϵ. The four-loop calculation in the WZ model is one of the first and simplest applications of the program. With the help of it the computation of the diagrams 4.1-4.5 of the Table has been carried out. The use of the program was the most essential point to facilitate the calculation of the diagram 3.3, the finite part of which contributes to the divergent part of 4.1.

The final expression for Z_Φ is:

$$\begin{aligned} Z_\Phi = 1 &+ h\left(-\frac{1}{2\epsilon}\right)+h^2\left(-\frac{1}{4\epsilon^2}+\frac{1}{4\epsilon}\right) \\ &+h^3\left(-\frac{5}{24\epsilon^3}+\frac{3}{8\epsilon^2}-\frac{5}{24\epsilon}-\frac{\zeta(3)}{2\epsilon}\right) \\ &+h^4\left(-\frac{5}{24\epsilon^4}+\frac{17}{32\epsilon^3}-\frac{25}{48\epsilon^2}-\frac{7}{8\epsilon^2}\zeta(3)\right. \\ &\left.+\frac{9}{32\epsilon}+\frac{15}{8\epsilon}\zeta(3)-\frac{9}{16\epsilon}\zeta(4)+\frac{5}{2\epsilon}\zeta(5)\right)+O(h^5). \end{aligned} \quad (3)$$

Then formulae (1), (2) give:

$$\beta(h) = \frac{3}{2}h^2 - \frac{3}{2}h^3 + \left(\frac{15}{8} + \frac{9}{2}\zeta(3)\right)h^4$$
$$-\left(\frac{27}{8} + \frac{45}{2}\zeta(3) - \frac{27}{4}\zeta(4) + 30\zeta(5)\right)h^5 + O(h^6). \qquad (4)$$

Note that the coefficients of h^3 and h^4 in (3), and consequently, the coefficients of h^4 and h^5 in (4) differ from those obtained in [2] and [3].

The validity [3] of the consistency condition

$$\beta(h)\left(\frac{1}{3} - h\frac{\partial}{\partial h}\right) Z_{\Phi}^{(m)}(h) = -h^2 \frac{\partial}{\partial h} Z_{\Phi}^{(m+1)}(h) \qquad (5)$$

for the coefficient of ϵ^{-1} in the $O(h^3)$ term of (3) simply follows from the fact that the results of three-loop calculations has been used to calculate the four-loop contribution. Of course, the other coefficients of (3) also obey the condition (5).

Notice the increase and the sign-alternation of the coefficients in (4), that seems to indicate that the perturbative expansion for the β-function is an asymptotic alternating series and that the Borel-like summation methods [12] can in principle be applied to it.

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Авдеев Л.В. и др. E2-82-342

β-функция модели Весса-Зумино в четырехпетлевом приближении

Метод расчета импульсных интегралов, предложенный Четыркиным и Ткачевым, применен к вычислению β-функции в модели Весса-Зумино в четырехпетлевом приближении.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Four-Loop Beta-Function in the Wess-Zumino Model

A method for calculating momentum integrals, proposed by Chetyrkin and Tkachov, is applied to the four-loop calculations of the β-function in the Wess-Zumino model.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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