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V.N.Pervushin, J.Hořejší

SPINOR ANALYSIS OF YANG-MILLS THEORY IN THE MINKOWSKI SPACE

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1. INTRODUCTION

In recent papers [1,2] a new approach to non-abelian gauge theories has been developed, which incorporates the description of the vacuum in terms of a complex self-dual (SD) Yang-Mills field in the Minkowski space. Within this approach the problem of solution of the Dirac equation for fermions interacting with an external SD gauge field arises naturally. The main aim of the present paper is to extend and improve the discussion in refs. [1,2] concerning this point. To this end, we have employed the spinorial (Newman-Penrose) method [3,4] in analogy with the analysis of Jackiw and Rebbi [5] performed in the four-dimensional Euclidean space.

The paper is organized as follows: In Sec.1 the self-duality equations (SDE) for SU(2) gauge field are investigated. It is shown that a Minkowski-space version of the Yang construction of Euclidean SD fields [6] emerges naturally in the spinorial formalism. The main results of the paper are presented in Sec.3 where we deal with the solutions of the Dirac equation for fermions with an arbitrary isospin, interacting with an external SD field. We restrict ourselves to the (complex) SD fields which are the Minkowskian analog of the Corrigan-Fairlie - 't Hooft-Wilczek (CFTW) solution (see, e.g., refs. [7,8]) in the Euclidean space (cf. also refs. [1,2,8]). For the fermion isospin T=1/2 and 1 the solutions of the Dirac equation are expressed in terms of the solutions of the d'Alembert equation.

2. SELF-DUALITY EQUATIONS IN THE MINKOWSKI SPACE

In this section we shall deal with the equations

$$F^{(\pm)}_{\mu\nu} = \pm i \, {}^*F^{(\pm)}_{\mu\nu} \qquad (1)$$

(the superscript (+) and (-) refers to the self-dual and anti-self-dual (ADS) solution, resp.), where

$${}^*F_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta} ,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu , A_\nu] ,$$

$$A_\mu = A^a_\mu \frac{\sigma_a}{2i} .$$

with σ_a, $a = 1,2,3$ being the Pauli matrices. From (1) it is obvious that A^a_μ cannot be all real. The solutions of eqs. (1) have been studied in many papers by means of various Ansätze (see, e.g., the review [8]). In the present paper we employ for the solution of eqs. (1) the Newman-Penrose spinorial method [3,4]. The basic idea of this method consists in passing to the spinor basis according to

$$A^\mu \to A^{B\dot{X}} = \sigma_\mu^{B\dot{X}} A^\mu ,$$
$$F^{\mu\nu} \to F^{B\dot{X}C\dot{Y}} = \sigma_\mu^{B\dot{X}} \sigma_\nu^{C\dot{Y}} F^{\mu\nu} \qquad (2)$$

(and analogously for any higher-rank Lorentz tensor), where the transition coefficients $\sigma_\mu^{B\dot{X}}$ ($\mu = 0,1,2,3$ and $B, \dot{X} = 1,2, \dot{1},\dot{2}$) are given by

$$\sigma_\mu^{B\dot{X}} = \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] . \qquad (3)$$

Note that (2) includes the definition of the "spinor gradient"

$$\partial^{B\dot{X}} = \sigma_\mu^{B\dot{X}} \partial^\mu ;$$

explicitly, according to (3)

$$\partial^{1\dot{1}} = \frac{1}{\sqrt{2}}(\partial^0 + \partial^3),\ \partial^{1\dot{2}} = \frac{1}{\sqrt{2}}(\partial^1 - i\partial^2),\ \partial^{2\dot{1}} = \frac{1}{\sqrt{2}}(\partial^1 + i\partial^2),\ \partial^{2\dot{2}} = \frac{1}{\sqrt{2}}(\partial^0 - \partial^3). \qquad (4)$$

The transition coefficients $\sigma_\mu^{B\dot{X}}$ are treated as SL(2, C) spinors and Lorentz four-vectors w.r.t. indices $B, \dot{X}$ and μ, respectively. This means that spinor indices are raised and lowered by means of the two-dimensional Levi-Civita symbol

$$\epsilon_{AB} \equiv \epsilon_{\dot{X}\dot{Y}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \epsilon^{AB} = \epsilon^{\dot{X}\dot{Y}}$$

and the Lorentz index μ by means of the metric tensor $g_{\mu\nu} \equiv \mathrm{diag}\ (+1,-1,-1,-1)$. Using the rules of the spinor algebra following from the above definitions (see ref. [4]), eqs. (1) may be reduced to

$$F^{(+)}_{B\dot{W}}{}^{B}{}_{\dot{X}} = 0 , \qquad (5a)$$

$$F^{(-)}_{A\dot{X}}{}^{\dot{X}}{}_{B} = 0 , \qquad (5b)$$

(summation over repeating indices is tacitly assumed). Let us consider the individual components of eq. (5a):

$$F^{(+)}_{1\dot{1}2\dot{1}} = 0 , \qquad (6)$$

$$F^{(+)}_{1\dot{2}2\dot{2}} = 0 , \qquad (7)$$

$$F^{(+)}_{A\dot{2}B\dot{1}}\ \epsilon^{AB} = 0 . \qquad (8)$$

Eqs. (6) and (7) mean that the potential $A^{(+)}_{B\dot{X}}$ in the directions $(B\dot{1})$ and $(B\dot{2})$ is pure gauge and may be therefore written as

$$A^{(+)}_{B\dot{1}} = \mathfrak{D}^{-1} \partial_{B\dot{1}} \mathfrak{D} , \qquad (6a)$$

$$A^{(+)}_{B\dot{2}} = \mathfrak{R}\, \partial_{B\dot{2}} \mathfrak{R}^{-1} , \qquad (7a)$$

where the "generating matrices" $\mathfrak{D}$ and $\mathfrak{R}$ are 2x2 matrices with determinant unity. Eq. (8) then takes the form

$$\epsilon^{AB} \partial_{A\dot{2}} [(\mathfrak{D}\mathfrak{R})^{-1} \partial_{B\dot{1}} (\mathfrak{D}\mathfrak{R})] = 0 . \qquad (8a)$$

For eq. (5b) (i.e., for the ASD solutions) one may proceed in the same way. We have thus obtained a Minkowski-space version of the Yang construction [6] formulated first in the Euclidean space. Explicit solutions of SDE may be now constructed in full analogy with the Euclidean case. In particular, an analog of the "Ansatz $\mathcal{A}_1$" (which is equivalent to the well-known CFTW Euclidean solution - see, e.g., ref. [7]) yields the gauge potentials (cf. also [1,2,8])

$$A^{(\pm)}_\mu = \Sigma^{(\pm)}_{\mu\nu} \partial^\nu \ln\rho \ ; \quad \Box\rho = 0 , \qquad (9)$$

where

$$\Sigma^{(\pm)}_{\mu\nu} = \frac{1}{4} [a^{(\pm)}_\mu a^{(\mp)}_\nu - a^{(\pm)}_\nu a^{(\mp)}_\mu] \qquad (10)$$

with (cf. (3))

$$a^{(\pm)}_\mu = (I, \pm\sigma_k) .$$

Note that $\Sigma^{(\pm)}_{\mu\nu}$ are the two-dimensional Lorentz generators in the representations (0, 1/2) and (1/2, 0), resp. It is straightforward to show that, (cf. [8])

$$\Sigma^{(\pm)}_{\mu\nu} = \Sigma^{(\pm)a}_{\mu\nu} \frac{\sigma_a}{2i} , \qquad (11)$$

where

$$\Sigma^{(\pm)a}_{\mu\nu} = \epsilon_{0a\mu\nu} \mp i g_{a\mu} g_{\nu 0} \pm i g_{a\nu} g_{\mu 0} \ . \qquad (12)$$

For the purpose of the next section it is convenient to rewrite the solutions (9) in the following form:

$$A^{(+)}_{B\dot X;UV} = \frac{1}{2}\epsilon_{BU}\partial_{V\dot X}\ln\rho + \frac{1}{2}\epsilon_{BV}\partial_{U\dot X}\ln\rho ; \qquad \Box\rho = 0 \ , \qquad (13a)$$

where (cf.[5])

$$A^{(+)}_{B\dot X;UV} = A^{(+)}_{B\dot X;}{}^{W}{}_{V}\,\epsilon_{WU} \ ; \qquad (14a)$$

$$A^{(+)}_{B\dot X;}{}^{U}{}_{V} = A^{(+)a}_{B\dot X}\left(\frac{\sigma_a}{2i}\right)_{UV}$$

and

$$A^{(-)}_{B\dot X;\dot U\dot V} = -\frac{1}{2}\epsilon_{\dot X\dot U}\partial_{B\dot V}\ln\rho + \frac{1}{2}\epsilon_{\dot X\dot V}\partial_{B\dot U}\ln\rho ; \qquad \Box\rho = 0 \ , \qquad (13b)$$

where

$$A^{(-)}_{B\dot X;\dot U\dot V} = A^{(-)}_{B\dot X;}{}^{\dot W}{}_{\dot V}\,\epsilon_{\dot W\dot U} \ , \qquad (14b)$$

$$A^{(-)}_{B\dot X;}{}^{\dot U}{}_{V} = A^{(-)a}_{B\dot X}\left(\frac{\sigma_a}{2i}\right)_{\dot V\dot U}$$

The introduction of dotted and undotted isospinor indices in (13a) and (13b) is necessary in order to maintain proper labelling of the Levi-Civita symbols and spinor gradients and can be traced back to the mixing of Lorentz and isospin indices in (12). Notice that (14a) and (14b) are related via complex conjugation. Indeed, for Pauli matrices transposition is equivalent to complex conjugation and according to (12) $\Sigma^{(-)a}_{\mu\nu} = (\Sigma^{(+)a}_{\mu\nu})^*$.

3. DIRAC EQUATIONS

In this section we shall deal with Dirac equations for massless fermions with an arbitrary isospin, interacting with external (A)SD fields (13a), (13b). We proceed in close analogy with the spinorial methods worked out in ref.[5] for the solution of Euclidean equations, although in Minkowski space a qualitatively different picture emerges[2].

In the spinorial formalism the wave function of a fermion with isospin T and definite chirality is represented by multi-indexed objects (spinors w.r.t. SL(2,C) × SU(2))

$\psi_{(+)B;\,U_1\dots U_{2T}}$ (positive chirality)

$\psi_{(-)\dot X;\,U_1\dots U_{2T}}$ (negative chirality)

entirely symmetric in the isospin indices $U_1\dots U_{2T}$ (which according to (13a) through (14b) may be dotted); B, X are SL(2, C) (Lorentz) indices. The Dirac equation with external field (13a) or (13b) may be then written in analogy with[5] as (the symbol {P} in the following means the permutations of $U_1 \dots U_{2T}$)

$$\frac{1}{2T}\partial_{A\dot X}\psi^{\dot X}_{(-);U_1\dots U_{2T}} + A^{(+)}_{A\dot X;U_1W}\psi^{\dot X\,W}_{(-);\ \ U_2\dots U_{2T}} + \{P\} = 0 \ , \qquad (15a)$$

$$\frac{1}{2T}\partial_{A\dot X}\psi^{\dot X}_{(-);U_1\dots U_{2T}} + A^{(-)}_{A\dot X;\dot U_1\dot W}\psi^{\dot X\,\dot W}_{(-);\ \ \dot U_2\dots\dot U_{2T}} + \{P\} = 0 \ , \qquad (15b)$$

$$\frac{1}{2T}\partial_{A\dot X}\psi^{A}_{(+);U_1\dots U_{2T}} + A^{(+)}_{A\dot X;U_1W}\psi^{A\,W}_{(+);\ \ U_2\dots U_{2T}} + \{P\} = 0 \ , \qquad (15c)$$

$$\frac{1}{2T}\partial_{A\dot X}\psi^{A}_{(+);\dot U_1\dots\dot U_{2T}} + A^{(-)}_{A\dot X;\dot U_1\dot W}\psi^{A\,\dot W}_{(+);\ \ \dot U_2\dots\dot U_{2T}} + \{P\} = 0. \qquad (15d)$$

It is sufficient to deal, e.g., only with eqs. (15a), (15b) since the only point that matters in the solution of eqs. (15a) through (15d) is whether ψ carries both types of indices (i.e., both dotted and undotted) or not.

We shall start with eq. (15a). Let us remark first that it may be written in the equivalent form (for brevity we omit everywhere the subscript (−))

$$\partial_{A\dot X}\psi^{\dot X}{}_{;U_1\dots U_{2T}} + A^{(+)}_{A\dot X;\underline{U}_1W}\psi^{\dot X\,W}{}_{;\underline{U}_2\dots \underline{U}_{2T}} = 0 \ ; \qquad (16)$$

in eq. (16) we have employed the notation analogous to ref.[5]: Let the multi-index object $\xi_{A_1\dots A_n}$ be already symmetric in $A_2\dots A_n$, we define $\xi_{\underline{A}_1\dots\underline{A}_n} = \xi_{A_1A_2\dots A_n} + \xi_{A_2A_1\dots A_n} + \dots + \xi_{A_nA_2\dots A_1}$ (n terms). The equivalence of the forms (15a) and (16) will be used frequently in the subsequent discussion.

Substituting (13a) into (15a) yields

$$\frac{1}{2T}\partial_{A\dot X}\psi^{\dot X}{}_{;U_1\dots U_{2T}} + \frac{1}{2}\epsilon_{AU_1}(\partial_{B\dot X}\ln\rho)\psi^{\dot X\,B}{}_{;\ U_2\dots U_{2T}} - \qquad (17)$$

$$-\frac{1}{2}(\partial_{U_1\dot X}\ln\rho)\psi^{\dot X}{}_{;AU_2\dots U_{2T}} + \{P\} = 0 .$$

For the solution of the last equation all substitutions of ref. [5] may be adopted. We shall describe them only briefly displaying the changes associated with Minkowski space.

The redefinition

$$\psi = \rho^T f \tag{18}$$

simplifies eq. (17):

$$\partial_{A\dot X} f^{\dot X}{}_{;U_1\dots U_{2T}} + \epsilon_{A\underline{U}_1}(\partial_{B\dot X}\ln\rho) f^{\dot X B}{}_{;\underline{U}_2\dots\underline{U}_{2T}} = 0. \tag{19}$$

Multiplying eq. (19) by ϵ^{AU_1} and performing the summation over indices one gets

$$\partial_{B\dot X} f^{\dot X B}{}_{;U_2\dots U_{2T}} + (2T+1)(\partial_{B\dot X}\ln\rho) f^{\dot X B}{}_{;U_2\dots U_{2T}} = 0. \tag{20}$$

Inserting (20) back into eq. (19) gives the following kinematical condition

$$\partial_{A\dot X} f^{\dot X}{}_{;U_1\dots U_{2T}} - \frac{1}{2T+1}\epsilon_{A\underline{U}_1}\partial_{B\dot X} f^{\dot X B}{}_{;\underline{U}_2\dots\underline{U}_{2T}} = 0 .$$

This condition can be automatically satisfied if we set

$$f_{\dot X;U_1\dots U_{2T}} = \partial_{\underline{U}_1\dot X}\eta_{\underline{U}_2\dots\underline{U}_{2T}} + V_{\dot X;U_1\dots U_{2T}} \tag{21}$$

with

$$\partial^{A\dot Y} V_{\dot Y;U_1U_2\dots U_{2T}} = 0. \tag{22}$$

Substitution of (21) into (20) yields the equation for η:

$$\frac{1}{2}\Box\eta_{U_2\dots U_{2T}} + (\partial^{B\dot X}\ln\rho)(\partial_{B\dot X}\eta_{U_2\dots U_{2T}} + \partial_{\underline{U}_2\dot X}\eta_{B\underline{U}_3\dots\underline{U}_{2T}} + V_{\dot X;BU_2\dots U_{2T}}) = 0 . \tag{23}$$

Eq. (23) can be simplified by the substitution

$$\eta = \rho^{-2T}\chi. \tag{24}$$

Using the manipulations analogous to ref. [5] we obtain the following equation for χ:

$$\frac{1}{2}\Box\chi_{U_2\dots U_{2T}} - (\partial_{\underline{U}_2\dot Y}\ln\rho)\partial^{C\dot Y}\chi_{C\underline{U}_3\dots\underline{U}_{2T}} = \rho^{2T}[-(\partial^{B\dot X}\ln\rho)V_{\dot Y;B\underline{U}_2\dots U_2}$$

which can be rewritten in the form (25)

$$\partial_{U_2\dot Y}\left(\frac{1}{\rho^{2T-1}}\partial^{C\dot Y}\chi_{CU_3\dots U_{2T}}\right) + \{P\} = -\rho[(\partial^{B\dot Y}\ln\rho)V_{\dot Y;BU_2\dots U_{2T}}]. \tag{26}$$

The solution of eq. (17) may be therefore written as (see (18), (21), (24))

$$\psi_{\dot X;U_1\dots U_{2T}} = \rho^T[\partial_{U_1\dot X}(\rho^{-2T}\chi_{U_2\dots U_{2T}}) + V_{\dot X;U_1U_2\dots U_{2T}}], \tag{27}$$

where χ and V are solutions of eqs. (22) and (26), resp. In particular, for $T=1/2$ eq. (25) reads

$$\frac{1}{2}\Box\chi = -(\partial^{B\dot X}\rho)V_{\dot Y B} \tag{28}$$

and the solution (27) with χ given by (28) coincides with the one found in ref. [2] by direct methods. As is shown in ref. [2], the boundary conditions imposed on ψ at the singularity points of $A^{(+)}_\mu$ may be satisfied only for the solution with $V\equiv 0$. In what follows we shall set for simplicity $V\equiv 0$. For $T=1$ there are two types of solutions of eq. (26). They may be written in terms of solutions of the d'Alembert equations as follows

$${}^{(1)}\chi_{U_2} = \partial_{U_2\dot X}{}^{(1)}\phi^{\dot X};\quad \Box^{(1)}\phi^{\dot X} = 0 , \tag{29a}$$

$${}^{(2)}\chi_{U_2} = \partial_{U_2\dot X}[\Box^{-1}(\rho\,\partial^{A\dot X}{}^{(2)}\phi_A)];\quad \Box^{(2)}\phi_A = 0. \tag{29b}$$

Here and in what follows the symbol $\Box^{-1}$ denotes the integral operator defined by means of a definite Green function of $\Box$.

We thus see that the problem of solution of eq. (17) is converted to the solution of the d'Alembert equation in analogy with Euclidean solutions presented in ref. [5]. The difference between Euclidean and Minkowski space enters just in the last stage, in the solution of the d'Alembert equation.

Let us now turn to the solution of eq. (15b). Note that an analogous problem in the Euclidean space has not been discussed in ref. [5] since it has no relevance for the problem of zero modes of the Euclidean Dirac equation - as is shown in ref. [5] there are no normalizable zero modes with positive (negative) chirality for an SD (ASD) external field A_μ.

Substituting (13b) into (15b) gives

$$\frac{1}{2T}\partial_{A\dot Y}\psi^{\dot Y}{}_{;\dot U_1\dots\dot U_{2T}} + \frac{1}{2}(\partial_{A\dot W}\ln\rho)\psi_{\dot U_1;}{}^{\dot W}{}_{\dot U_2\dots\dot U_{2T}} + \frac{1}{2}(\partial_{A\dot U_1}\ln\rho)\psi_{\dot W;}{}^{\dot W}{}_{\dot U_2\dots\dot U_{2T}} + \{P\} = 0 . \tag{30}$$

Using the well-known rule of spinor algebra

$$\xi_{AB} - \xi_{BA} = \xi_{AB}\,\xi_C{}^C \tag{31}$$

and making the substitution

$$\psi = \rho^{-T} f \qquad (32)$$

one may rewrite eq. (30) in the following form

$$\partial_{A\dot Y} f^{\dot Y}{}_{;\dot U_1\ldots\dot U_{2T}} + (\partial_{A\dot{\underline U}_1}\ln\rho) f_{\dot W;}{}^{\dot W}{}_{\dot{\underline U}_2\ldots\dot{\underline U}_{2T}} = 0. \qquad (33)$$

We shall now decompose f in the following way (symmetrization)

$$f_{\dot Y;\dot U_1\ldots\dot U_{2T}} = \eta_{\dot Y;\dot U_1\ldots\dot U_{2T}} + \epsilon_{\dot Y\dot{\underline U}_1}\rho\chi_{\dot{\underline U}_2\ldots\dot{\underline U}_{2T}}, \qquad (34)$$

where

$$\eta_{\dot Y;\dot U_1\ldots\dot U_{2T}} = \frac{1}{2T+1} f_{\dot{\underline Y};\dot{\underline U}_1\ldots\dot{\underline U}_{2T}} \qquad (35)$$

and

$$\rho\chi_{\dot U_2\ldots\dot U_{2T}} = \frac{1}{2T+1} f_{\dot W;}{}^{\dot W}{}_{\dot U_2\ldots\dot U_{2T}} . \qquad (36)$$

The identity (34) together with (35), (36) may be easily verified with the help of (31). Substituting now (34) and (36), into eq. (33) we obtain, after some manipulations

$$\partial_A{}^{\dot Y}\eta_{\dot Y;\dot U_1\ldots\dot U_{2T}} = -\rho\partial_{A\dot{\underline U}_1}\chi_{\dot{\underline U}_2\ldots\dot{\underline U}_{2T}} + 2T(\partial_{A\dot{\underline U}_1}\rho)\chi_{\dot{\underline U}_2\ldots\dot{\underline U}_{2T}}. \qquad (37)$$

Acting on both sides of eq. (37) by $\partial^A{}_{\dot X}$ we get (using $\partial^A{}_{\dot X}\partial_A{}^{\dot Y} = -\frac{1}{2}\delta_{\dot X}{}^{\dot Y}\Box$)

$$\frac{1}{2}\Box\eta_{\dot X;\dot U_1\ldots\dot U_{2T}} = \partial^A{}_{\dot X}[\rho\partial_{A\dot{\underline U}_1}\chi_{\dot{\underline U}_2\ldots\dot{\underline U}_{2T}} - 2T(\partial_{A\dot{\underline U}_1}\rho)\chi_{\dot U_2\ldots\dot U_{2T}}]. \qquad (38)$$

Let us denote the r.h.s. of eq. (35) by $R_{\dot X;\dot U_1\ldots\dot U_{2T}}$. The symmetrization of the R yields

$$R_{\dot X;\dot U_1\ldots\dot U_{2T}} = S_{\dot X;\dot U_1\ldots\dot U_{2T}} + \epsilon_{\dot{\underline U}_1\dot X} D_{\dot{\underline U}_2\ldots\dot{\underline U}_{2T}}, \qquad (39)$$

where

$$S_{\dot X;\dot U_1\ldots\dot U_{2T}} = Q_{\dot{\underline X};\dot{\underline U}_1\ldots\dot{\underline U}_{2T}} \qquad (40)$$

with

$$Q_{\dot X;\dot U_1\ldots\dot U_{2T}} = (\partial^A{}_{\dot X}\rho)(\partial_{A\dot{\underline U}_1}\chi_{\dot{\underline U}_2\ldots\dot{\underline U}_{2T}}) \qquad (41)$$

and

$$D_{\dot U_2\ldots\dot U_{2T}} = \frac{1}{2}\rho\Box\chi_{\dot U_2\ldots\dot U_{2T}} - (\partial^A{}_{\dot{\underline U}_2}\rho)(\partial_{A\dot Y}\chi^{\dot Y}{}_{\dot{\underline U}_3\ldots\dot{\underline U}_{2T}}). \qquad (42)$$

Since both the l.h.s. of eq. (38) and the S in (39) are entirely symmetric, the expressions (42) must vanish

$$\frac{1}{2}\rho\Box\chi_{\dot U_2\ldots\dot U_{2T}} - (\partial^A{}_{\dot{\underline U}_2}\rho)(\partial_{A\dot Y}\chi^{\dot Y}{}_{\dot{\underline U}_3\ldots\dot{\underline U}_{2T}}) = 0. \qquad (43)$$

Eq: (43) coincides with eq. (25) for V≡0, which we have already solved for T=1/2, 1. We have thus obtained a compatibility condition for eq. (37); if (43) were not satisfied, eq. (37) would not have solution. Let $\chi_{\dot U_2\ldots\dot U_{2T}}$ be a solution of eq. (43). Denote the r.h.s. of eq. (37) by $\mathcal{R}_{A;\dot U_1\ldots\dot U_{2T}}$. Obviously, a solution of eq. (37) may be written as

$$\eta_{\dot X;\dot U_1\ldots\dot U_{2T}} = -2\partial^A{}_{\dot X}(\Box^{-1}\mathcal{R}_{A;\dot U_1\ldots\dot U_{2T}}); \qquad (44)$$

to verify this, one has only to employ the relation $\partial_A{}^{\dot X}\partial^B{}_{\dot X} = -\frac{1}{2}\delta_A{}^B\Box$. However, (cf. (38), (39))

$$\partial^A{}_{\dot X}\mathcal{R}_{A;\dot U_1\ldots\dot U_{2T}} = -R_{\dot X;\dot U_1\ldots\dot U_{2T}}$$

and hence, in view of (43)

$$\eta_{\dot X;\dot U_1\ldots\dot U_{2T}} = 2\Box^{-1}S_{\dot X;\dot U_1\ldots\dot U_{2T}}. \qquad (45)$$

The general solution of eq. (37) is the sum of (45) and an arbitrary solution $\eta^{(0)}$ of the homogeneous equation corresponding to (37), i.e.,

$$\partial_A{}^{\dot X}\eta^{(0)}_{\dot X;\dot U_1\ldots\dot U_{2T}} = 0. \qquad (46)$$

Concluding this section we may write the explicit solutions of eq. (30) (up to an arbitrary solution of (46)) for T=1/2 and 1. For T=1/2 we have, according to (32), (34), (40), (41), (43) and (45)

$$\psi_{\dot X;\dot U_1} = \rho^{-1/2}\{2\Box^{-1}[(\partial^A{}_{\dot{\underline X}}\rho)(\partial_{A\dot{\underline U}_1}\chi)] + \epsilon_{\dot X\dot U_1}\rho\chi\} \qquad (47)$$

with $\Box\chi = 0$.

The expression (47) is equivalent to the result obtained in ref.[2] by direct methods. For T=1 we may write two types of solutions

$${}^{(i)}\psi_{\dot X;\dot U_1\dot U_2} = \rho^{-1}[2\Box^{-1}S_{\dot X;\dot U_1\dot U_2}(\rho, {}^{(i)}\chi) + \epsilon_{\dot X\dot{\underline U}_1}\rho\,{}^{(i)}\chi_{\dot{\underline U}_2}],$$

where i=1,2 the $S(\rho, \chi)$ is given by (40), (41) and

$${}^{(1)}\chi_{\dot U_2} = \partial_{A\dot U_2}{}^{(1)}\phi^A; \qquad \Box\,{}^{(1)}\phi^A = 0$$

$${}^{(2)}\chi_{\dot U_2} = \partial_{A\dot U_2}\Box^{-1}(\rho\partial^{A\dot X}\,{}^{(2)}\phi_{\dot X}); \qquad \Box\,{}^{(2)}\phi_{\dot X} = 0.$$

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Первушин В.Н., Горжейши И. Спинорный анализ теории Янга-Миллса в пространстве Минковского E2-82-316

Спинорные методы применяются к решению уравнений самодуальности для полей Янга-Миллса и уравнения Дирака с внешним самодуальным полем в пространстве Минковского. Рассматривается случай калибровочной группы SU(2). Показано, что в рамках спинорного формализма естественно получается аналог конструкции Янга для самодуальных полей. Получены решения уравнения Дирака для безмассового фермиона с произвольным изоспином, взаимодействующего с внешним самодуальным или антисамодуальным полем. В качестве внешнего поля берется аналог евклидовой подстановки Хоофта. Показано, что для изоспина 1/2 и 1 решения уравнения Дирака выражаются через решения уравнения Даламбера. Полученные решения можно использовать в рамках недавно предложенного подхода к калибровочным теориям, основанного на аналогии с теорией сверхтекучести; в таком подходе самодуальные решения уравнений Янга-Миллса играют роль вакуумных полей.

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Pervushin V.N., Hořejší J. Spinor Analysis of Yang-Mills Theory in the Minkowski Space E2-82-316

Spinorial methods are applied to the solution of self-duality equations for Yang-Mills field and the Dirac equation with an external self-dual field in the Minkowski space. Gauge group SU(2) is considered. It is shown that in the spinorial formalism an analog of the Yang construction of self-dual fields emerges naturally. Solutions of the Dirac equation for massless fermion with an arbitrary isospin, interacting with an external self-dual or anti-self-dual field are obtained. The external field is chosen to be the Minkowskian analog of the Euclidean 't Hooft Ansatz. It is shown that for the isospin 1/2 and 1 the solutions of the Dirac equation may be expressed in terms of the solutions of d'Alembert equation. The solutions obtained in this paper may be employed in the approach to gauge theories proposed recently, which is based on an analogy with the superfluidity theory; in such an approach the self-dual solutions of the Yang-Mills equations represent the vacuum.

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