

ОБЪЕДИНЕННЫЙ Институт ядерных исследований

дубна

19/2-82

3300/82

E2-82-316

V.N.Pervushin, J.Hořejší

SPINOR ANALYSIS OF YANG-MILLS THEORY IN THE MINKOWSKI SPACE

Submitted to "XXI International Conference on High Energy Physics (Paris, 1982)

1982

1. INTRODUCTION

In recent papers $^{/1,2'}$ a new approach to non-abelian gauge theories has been developed, which incorporates the description of the vacuum in terms of a complex self-dual (SD) Yang-Mills field in the Minkowski space. Within this approach the problem of solution of the Dirac equation for fermions interacting with an external SD gauge field arises naturally. The main aim of the present paper is to extend and improve the discussion in refs. $^{/1,2'}$ concerning this point. To this end, we have employed the spinorial (Newman-Penrose) method $^{/3,4'}$ in analogy with the analysis of Jackiw and Rebbi $^{/5'}$ performed in the four-dimensional Euclidean space.

The paper is organized as follows: In Sec.1 the self-duality equations (SDE) for SU(2) gauge field are investigated. It is shown that a Minkowski-space version of the Yang construction of Euclidean SD fields $^{/6/}$ emerges naturally in the spinorial formalism. The main results of the paper are presented in Sec.3 where we deal with the solutions of the Dirac equation for fermions with an arbitrary isospin, interacting with an external SD field. We restrict ourselves to the (complex) SD fields which are the Minkowskian analog of the Corrigan-Fairlie - 't Hooft-Wilczek (CFTW) solution (see, e.g.,) in the Euclidean space (cf. also refs. /1.2.8 refs.). For the fermion isospin T=1/2 and 1 the solutions of the Dirac equation are expressed in terms of the solutions of the d'Alembert equation.

2. SELF-DUALITY EQUATIONS IN THE MINKOWSKI SPACE

In this section we shall deal with the equations -(t) (t)

$$\mathbf{F}_{\mu\nu}^{(-)} = \pm \mathbf{i} * \mathbf{F}_{\mu\nu}^{(-)}$$

(the superscript (+) and (-) refers to the self-dual and anti-self-dual (ADS) solution, resp.), where

 $*F_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}$,

 $\mathbf{F}_{\mu\nu} = \partial_{\mu} \mathbf{A}_{\nu} - \partial_{\nu} \mathbf{A}_{\mu} + [\mathbf{A}_{\mu}, \mathbf{A}_{\nu}],$

Иемание во составляет Раз Росса БИБЛИОТЕКА (1)

L

$$A_{\mu} = A_{\mu}^{a} \frac{\sigma_{a}}{2i} .$$

with σ_a , a =1,2,3 being the Pauli matrices. From (1) it is obvious that $A^a_{\ \mu}$ cannot be all real. The solutions of eqs. (1) have been studied in many papers by means of various Ansätze (see, e.g., the review ⁽⁸⁾). In the present paper we employ for the solution of eqs. (1) the Newman-Penrose spinorial method ^(3,4). The basic idea of this method consists in passing to the spinor basis according to

$$A^{\mu} \rightarrow A^{BX} \equiv \sigma_{\mu}^{BX} A^{\mu} ,$$

$$F^{\mu\nu} \rightarrow F^{BXCY} \equiv \sigma_{\mu}^{BX} \sigma_{\nu}^{CY} F^{\mu\nu}$$
(2)

(and analogously for any higher-rank Lorentz tensor), where the transition coefficients σ_{μ}^{BX} (μ =0,1,2,3 and **B**, X =1,2, 1,2) are given by

$$\sigma_{\mu}^{\mathbf{BX}} = \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] .$$
(3)

Note that (2) includes the definition of the "spinor gradient" $\partial^{\mathbf{B}\dot{\mathbf{X}}} = \sigma_{\mu}^{\mathbf{B}\dot{\mathbf{X}}} \partial^{\mu}$;

explicitly. according to (3)

$$\partial^{11} = \frac{1}{\sqrt{2}} (\partial^{0} + \partial^{3}), \ \partial^{12} = \frac{1}{\sqrt{2}} (\partial^{1} - i\partial^{2}), \ \partial^{21} = \frac{1}{\sqrt{2}} (\partial^{1} + i\partial^{2}), \ \partial^{22} = \frac{1}{\sqrt{2}} (\partial^{0} - \partial^{3}).$$
(4)

The transition coefficients σ_{μ}^{BX} are treated as SL(2, C) spinors and Lorentz four-vectors w.r.t. indices B,X and μ . respectively. This means that spinor indices are raised and lowered by means of the two-dimensional Levi-Civita symbol

$$\epsilon_{AB} \equiv \epsilon_{XY} \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \equiv \epsilon^{AB} \equiv \epsilon_{XY}$$

and the Lorentz index μ by means of the metric tensor $g_{\mu\nu} \equiv diag (+1,,-1,-1,-1)$. Using the rules of the spinor algebra following from the above definitions (see ref. '4'), eqs. (1) may be reduced to

$$F_{BW}^{(+)} B_{X}^{B} = 0,$$
(5a)

$$F_{C}^{(-)} \dot{X} = 0,$$
(5b)

$$A\dot{X} B$$
(5b)

(summation over repeating indices is tacitly assumed). Let us consider the individual components of eq. (5a):

$$F_{1igi}^{(+)} = 0$$
, (6)

$$F_{1222}^{(+)} = 0, \qquad (7)$$

$$\mathbf{F}_{\mathbf{A}\mathbf{\hat{2}}\mathbf{B}\mathbf{\hat{1}}}^{(+)} \quad \boldsymbol{\epsilon}^{\mathbf{A}\mathbf{B}} = \mathbf{0} \,. \tag{8}$$

Eqs. (6) and (7) mean that the potential $A_{BX}^{(+)}$ in the directions (Bi) and (B2) is pure gauge and may be therefore written as

$$A_{Bi}^{(+)} = \mathfrak{D}^{-1} \partial_{Bi} \mathfrak{D}, \qquad (6a)$$
$$A_{Bi}^{(+)} = \mathfrak{R} \partial_{Bi} \mathfrak{R}^{-1},$$

where the "generating matrices" \mathfrak{D} and \mathfrak{R} are 2x2 matrices with determinant unity. Eq. (8) then takes the form

$$\epsilon^{AB} \partial_{A\dot{z}} \left[(\mathfrak{D} \mathfrak{R})^{-1} \partial_{B\dot{z}} (\mathfrak{D} \mathfrak{R}) \right] = 0.$$
(8a)

For eq. (5b) (i.e., for the ASD solutions) one may proceed in the same way. We have thus obtained a Minkowski-space version of the Yang construction 6 formulated first in the Euclidean space. Explicit solutions of SDE may be now constructed in full analogy with the Euclidean case. In particular, an analog of the "Ansatz \mathfrak{A}_1 " (which is equivalent to the well-known CFTW Euclidean solution - see, e.g., ref. 77) yields the gauge potentials (cf. also 1,2,8)

$$A^{(\pm)}_{\mu} = \sum_{\mu\nu} {}^{(\pm)}_{\rho\nu} \partial^{\nu} \ln\rho ; \quad \Box\rho = 0 , \qquad (9)$$

where

٠

$$\Sigma_{\mu\nu}^{(\pm)} = \frac{1}{4} \left[a_{\mu}^{(\pm)} a_{\nu}^{(\mp)} - a_{\nu}^{(\pm)} a_{\mu}^{(\mp)} \right]$$
(10)

with (cf. (3))

$$a_{\mu}^{(\pm)} = (\mathbf{I}, \pm \sigma_{\mathbf{k}}).$$

Note that $\Sigma_{\mu\nu}^{(-)}$ are the two-dimensional Lorentz generators in the representations (0, 1/2) and (1/2, 0), resp. It is straightforward to show that, (cf. $^{/8/}$)

$$\Sigma_{\mu\nu}^{(\pm)} = \Sigma_{\mu\nu}^{(\pm)a} \frac{\sigma_{a}}{2i} , \qquad (11)$$

(7a)

$$\Sigma_{\mu\nu}^{(\pm)a} = \epsilon_{0a\mu\nu} + ig_{a\mu}g_{\nu0} \pm ig_{a\nu}g_{\mu0}$$
 (12)

For the purpose of the next section it is convenient to rewrite the solutions (9) in the following form:

$$A_{BX;UV}^{(+)} = \frac{1}{2} \epsilon_{BU} \frac{\partial}{\partial x} \ln \rho + \frac{1}{2} \epsilon_{BV} \frac{\partial}{\partial uX} \ln \rho; \quad \Box \rho = 0, \quad (13a)$$

where $(cf.^{/5/})$

$$A_{BX;UV}^{(+)} = A_{BX;V}^{(+)} \stackrel{W}{=} K_{WU}^{(+)} \stackrel{W}{=} K_{WU}^{$$

and

$$A_{BX;UV}^{(-)} = \frac{1}{2} \epsilon_{XU} \frac{\partial}{\partial BV} \ln \rho + \frac{1}{2} \epsilon_{XV} \frac{\partial}{\partial BU} \ln \rho; \quad \Box \rho = 0, \quad (13b)$$

where

$$A_{\mathbf{B}\dot{\mathbf{X}};\dot{\mathbf{U}}\dot{\mathbf{V}}}^{(-)} = A_{\mathbf{B}\dot{\mathbf{X}};\dot{\mathbf{V}}}^{(-)} \dot{\mathbf{W}} + A_{\mathbf{B}\dot{\mathbf{X}};\dot{\mathbf{V}}}^{(-)} \dot{\mathbf{W}} \dot{\mathbf{V}} , \qquad (14b)$$

$$A_{\mathbf{B}\mathbf{X};\dot{\mathbf{V}}}^{(-)} \dot{\mathbf{U}} = A_{\mathbf{B}\dot{\mathbf{X}}}^{(-)a} \cdot \frac{(\sigma_{a})}{2i} \dot{\mathbf{V}} \dot{\mathbf{U}}$$

The introduction of dotted and undotted isospinor indices in (13a) and (13b) is necessary in order to maintain proper labelling of the Levi-Civita symbols and spinor gradients and can be traced back to the mixing of Lorentz and isospin indices in (12). Notice that (14a) and (14b) are related via complex conjugation. Indeed, for Pauli matrices transposition is equivalent to complex conjugation and according to (12) $\Sigma_{\mu\nu}^{(-)a} = (\Sigma_{\mu\nu}^{(+)a})^*$.

3. DIRAC EQUATIONS

In this section we shall deal with Dirac equations for massless fermions with an arbitrary isospin, interacting with external (A)SD fields (13a), (13b). We proceed in close analogy with the spinorial methods worked out in ref. $^{/5/}$ for the solution of Euclidean equations, although in Minkowski space a qualitatively different picture emerges $^{/2/}$.

٦,

In the spinorial formalism the wave function of a fermion with isospin T and definite chirality is represented by multiindexed objects (spinors w.r.t. $SL(2, C) \times SU(2)$)

$$\psi_{(+)B; U_1 \dots U_{2T}}$$
 (positive chirality)
 $\psi_{(-)x; U_1 \dots U_{2T}}$ (negative chirality)

entirely symmetric in the isospin indices $U_1 \dots U_{2T}$ (which according to (13a) through (14b) may be dotted); B,X are SL(2, C) (Lorentz) indices. The Dirac equation with external field (13a) or (13b) may be then written in analogy with $^{/5/}$ as (the symbol {P} in the following means the permutations of $U_1 \dots U_{2T}$)

$$\frac{1}{2T} \partial_{A\dot{X}} \psi (-); U_{1} \dots U_{2T} + A_{A\dot{X}}^{(+)}; U_{1} \psi (-); U_{2} \dots U_{2T}^{(+)} + \{P\} = 0, \quad (15a)$$

$$\frac{1}{2T} \partial_{A\dot{X}} \psi_{(-); U_{1}} \cdots U_{2T} + A_{A\dot{X}; \dot{U}_{1}}^{(-)} \psi_{(-); \dot{U}_{2}} \cdots \dot{U}_{2T}^{+} \{P\} = 0, \quad (15b)$$

$$\frac{1}{2T}\partial_{A}\dot{x}\,\psi^{A}_{(+)};\,U_{1}\,...\,U_{gT}^{}+A^{(+)}_{A}\dot{x};\,U_{1}W\,\psi^{A}_{(+)};\,U_{2}\,...\,U_{gT}^{}+\{P\}=0,\qquad(15c)$$

$$\frac{1}{2\mathbf{T}} \partial_{\mathbf{A}} \dot{\mathbf{X}} \psi_{(+)}^{\mathbf{A}}; \dot{\mathbf{U}}_{1} \dots \dot{\mathbf{U}}_{2\mathbf{T}} + \mathbf{A}_{\mathbf{A}} \dot{\mathbf{X}}; \dot{\mathbf{U}}_{1} \dot{\mathbf{W}} \psi_{(+)}^{\mathbf{A}}; \overset{W}{\mathbf{U}}_{2} \dots \dot{\mathbf{U}}_{2\mathbf{T}}^{\mathbf{A}} + \{\mathbf{P}\} = \mathbf{0}.$$
(15d)

It is sufficient to deal, e.g., only with eqs. (15a), (15b) since the only point that matters in the solution of eqs. (15a) through (15d) is whether ψ carries both types of indices (i.e., both dotted and undotted) or not.

We shall start with eq. (15a). Let us remark first that it may be written in the equivalent form (for brevity we omit everywhere the subscript (-))

 $\begin{array}{l} \partial_{\mathbf{A}} \dot{\psi} \stackrel{\mathbf{X}}{} + \stackrel{\mathbf{A}}{} \stackrel{(+)}{} \psi \stackrel{\mathbf{X}}{} = 0; \quad (16) \\ \text{Ax} : U_1 \dots U_{2T} \quad \mathbf{Ax}; U_1 \psi \quad : U_2 \dots U_{2T} \quad (16) \\ \text{in eq. (16) we have employed the notation analogous to ref.} \\ \text{Let the multi-index object } \xi_{\mathbf{A}_1 \dots \mathbf{A}_n} \quad \text{be already symmetric} \\ \text{in } \mathbf{A}_2 \dots \mathbf{A}_n , \text{ we define } \xi_{\mathbf{A}_1 \dots \mathbf{A}_n} = \xi_{\mathbf{A}_1 \mathbf{A}_2 \dots \mathbf{A}_n} \quad \text{be already symmetric} \\ + \xi_{\mathbf{A}_n \mathbf{A}_2} \dots \mathbf{A}_1 \quad (n \text{ terms}). \text{ The equivalence of the forms (15a)} \\ \text{and (16) will be used frequently in the subsequent discussion.} \\ \text{Substituting (13a) into (15a) yields} \end{array}$

$$\frac{1}{2T} \partial_{\mathbf{A}\dot{\mathbf{X}}} \psi^{\dot{\mathbf{X}}} : \mathbf{U}_{1} \cdots \mathbf{U}_{2T} + \frac{1}{2} \epsilon_{\mathbf{A}\mathbf{U}_{1}} (\partial_{\mathbf{B}\dot{\mathbf{X}}} \ln \rho) \psi^{\dot{\mathbf{X}}} \overset{\mathbf{B}}{;} \mathbf{U}_{2} \cdots \mathbf{U}_{2T} -$$
(17)

4

5

$$-\frac{1}{2}(\partial_{U_1} x \ln \rho) \psi^{\dot{X}} = A U_2 \cdots U_{2T} + \{P\} - 0.$$

For the solution of the last equation all substitutions of ref. '5' may be adopted. We shall describe them only briefly displaying the changes associated with Minkowski space.

The redefinition

$$\psi = \rho^{\mathrm{T}} \mathrm{f} \tag{18}$$

11

N.

X

simplifies eq. (17):

$$\partial_{\mathbf{A}\dot{\mathbf{X}}} \mathbf{f}^{\mathbf{X}} : \mathbf{U}_{1} \cdots \mathbf{U}_{2\mathbf{T}}^{+} \mathbf{f}_{\mathbf{A}} \underline{\mathbf{U}}_{1} (\partial_{\mathbf{B}\dot{\mathbf{X}}} \ln \rho) \mathbf{f}^{\mathbf{X}} : \overset{\mathbf{B}}{\underbrace{\mathbf{U}}_{2}} \cdots \underline{\mathbf{U}}_{2\mathbf{T}}^{\pm} \mathbf{0}.$$
(19)

Multiplying eq. (19) by e^{AU1} and performing the summation over indices one gets

$$\partial_{\mathbf{B}\dot{\mathbf{X}}} \mathbf{f} \stackrel{\mathbf{X}}{:} \frac{\mathbf{B}}{\mathbf{U}_{2}...\mathbf{U}_{2\mathbf{T}}} + \frac{(2\mathbf{T}+1)(\partial_{\mathbf{B}\dot{\mathbf{X}}}\ln\rho)\mathbf{f}}{\mathbf{B}\dot{\mathbf{X}}} \frac{\mathbf{B}}{\mathbf{U}_{2}...\mathbf{U}_{2\mathbf{T}}} = \mathbf{0}.$$
 (20)

Inserting (20) back into eq. (19) gives the following kinematical condition

$$\partial_{\mathbf{A}\dot{\mathbf{X}}} \mathbf{f} \overset{\mathbf{X}}{:} \mathbf{U}_{1} \cdots \mathbf{U}_{\mathbf{2T}} - \frac{1}{\mathbf{2T}+1} \mathcal{A}_{\mathbf{U}_{1}} \partial_{\mathbf{B}\dot{\mathbf{X}}} \mathbf{f} \overset{\mathbf{X}}{:} \overset{\mathbf{B}}{\underbrace{\mathbf{U}}_{\mathbf{2}}} \cdots \overset{\mathbf{U}_{\mathbf{2T}}}{:} = \mathbf{0}$$

This condition can be automatically satisfied if we set

$${}^{f}\dot{\mathbf{x}}: \mathbf{U}_{1} \dots \mathbf{U}_{2\mathbf{T}} \stackrel{=}{\longrightarrow} {}^{d}\underline{\mathbf{U}}_{1}\dot{\mathbf{x}}^{\eta} \underline{\mathbf{U}}_{2} \dots \underline{\mathbf{U}}_{2\mathbf{T}} \stackrel{+}{\longrightarrow} {}^{v}\dot{\mathbf{x}}: \mathbf{U}_{1} \dots \mathbf{U}_{2\mathbf{T}}$$
(21)

with .

$$\partial^{\mathbf{AY}} \mathbf{V} : \mathbf{U}_1 \mathbf{U}_2 \dots \mathbf{U}_{2\mathbf{T}} = \mathbf{0}.$$
 (22)

Substitution of (21) into (20) yields the equation for η :

$$\frac{1}{2} \Box \eta_{U_2} \dots U_{2T} + (\partial^{\mathbf{B}X} \ln \rho) (\partial_{\mathbf{B}X} \eta_{U_2} \dots U_{2T} + \partial_{\underline{U}_2} \dot{x}^{\eta} \underline{B} \underline{U}_{3} \dots \underline{U}_{2T} + V_{\dot{X}; \underline{B}U_2} \dots U_{2T}) = 0.$$
(23)

Eq. (23) can be simplified by the substitution

$$\eta = \rho^{-2\mathbf{T}} \chi. \tag{24}$$

Using the manipulations analogous to ref. $^{'b'}$ we obtain the following equation for χ :

 $\frac{1}{2} \Box \chi_{U_2 \dots U_{2\mathbf{T}}} - (\partial_{U_2} \dot{\mathbf{y}} \ln \rho) \partial^{C \dot{\mathbf{Y}}} \chi_{C \underline{U}_3 \dots \underline{U}_{2\mathbf{T}}} = \rho^{2\mathbf{T}} [-(\partial^{B \dot{\mathbf{X}}} \ln \rho) V_{\mathbf{Y}; B U_2 \dots U_2}]$ which can be rewritten in the form (25)

$$\partial_{\mathbf{U}_{2}\dot{\mathbf{Y}}} \left(\frac{1}{\rho^{2\mathbf{T}-1}} \partial^{\mathbf{C}\dot{\mathbf{Y}}} \times_{\mathbf{C}\mathbf{U}_{3}} \dots \mathbf{U}_{2\mathbf{T}}\right) + \{\mathbf{P}\} = -\rho \left[\left(\partial^{\mathbf{B}\dot{\mathbf{Y}}} \ln \rho \right) \mathbf{V}_{\dot{\mathbf{Y}}; \mathbf{B}\mathbf{U}_{2}} \dots \mathbf{U}_{2\mathbf{T}} \right]. (26)$$

The solution of eq. (17) may be therefore written as (see (18), (21), (24))

 $\psi_{\dot{\mathbf{x}};\,\mathbf{U}_{1}\,...\,\mathbf{U}_{2\,\mathrm{T}}} = \rho^{\mathrm{T}} [\partial_{\mathbf{U}_{1}} \dot{\mathbf{x}}^{(\rho^{-2\mathrm{T}}} \chi_{\mathbf{U}_{2}\,...\,\mathbf{U}_{2\,\mathrm{T}}}) + \mathbf{V}_{\dot{\mathbf{x}};\,\mathbf{U}_{1}\mathbf{U}_{2}\,...\,\mathbf{U}_{2\,\mathrm{T}}}], \quad (27)$ where χ and \bar{V} are solutions of eqs. (22) and (26), resp. In particular, for T=1/2 eq. (25) reads

$$\frac{1}{2} \Box \chi = -(\partial \mathbf{B} \mathbf{X} \rho) \mathbf{V}_{\mathbf{Y}\mathbf{B}}$$
(28)

and the solution (27) with χ given by (28) coincides with the one found in ref. 2^{\prime} by direct methods. As is shown in ref. 2^{\prime} , the boundary conditions imposed on ψ at the singularity points of $A_{u}^{(+)}$ may be satisfied only for the solution with $V \equiv 0$. In what follows we shall set for simplicity $V \equiv 0$. For T=1 there are two types of solutions of eq. (26). They may be written in terms of solutions of the d'Alembert equations as follows

$${}^{(1)} \chi_{U_{2}} = \partial_{U_{2} \dot{X}} {}^{(1)} \phi \dot{X} ; \Box^{(1)} \phi \dot{X} = 0 , \qquad (29a)$$

$${}^{(2)}\chi_{U_2} = \partial_{U_2}\dot{x} \left[\Box^{-1} \left(\rho \partial^A \dot{x} \left(2 \right) \phi_A \right) \right]; \ \Box^{(2)}\phi_A = 0.$$
 (29b)

Here and in what follows the symbol \Box^{-1} denotes the integral operator defined by means of a definite Green function of D.

We thus see that the problem of solution of eq. (17) is converted to the solution of the d'Alembert equation in analogy with Euclidean solutions presented in ref. 757. The differen-Ce between Euclidean and Minkowski space enters just in the last stage, in the solution of the d'Alembert equation.

Let us now turn to the solution of eq. (15b). Note that an analogous problem in the Euclidean space has not been discussed in ref. '5' since it has no relevance for the problem of zero modes of the Euclidean Dirac equation - as is shown in ref. there are no normalizable zero modes with positive (negative) chirality for an SD (ASD) external field A_{μ} . Substituting (13b) into (15b) gives

$$\frac{1}{2\mathrm{T}}\partial_{\mathbf{A}}\dot{\mathbf{y}}\psi^{\mathbf{Y}};\dot{\mathbf{U}}_{1}\ldots\dot{\mathbf{U}}_{2\mathrm{T}}+\frac{1}{2}(\partial_{\mathbf{A}}\dot{\mathbf{w}}^{\mathrm{ln}\rho})\psi\dot{\mathbf{U}}_{1};\overset{\mathbf{W}}{\mathbf{U}}_{2}\ldots\dot{\mathbf{U}}_{2\mathrm{T}}^{+}$$
(30)

$$+\frac{1}{2}(\partial_{A}\dot{U}_{1}\ln\rho)\psi_{W}; \overset{W}{\psi}_{2}...\dot{U}_{2T} + \{P\} = 0.$$

Using the well-known rule of spinor algebra

$$\xi_{AB} - \xi_{BA} = \xi_{AB} \xi_{C}^{C}$$
(31)

6

7

and making the substitution

$$\psi = \rho^{-T} f \tag{32}$$

one may rewrite eq. (30) in the following form

$$\partial_{\mathbf{A}\dot{\mathbf{Y}}} \mathbf{f}^{\mathbf{Y}}; \dot{\mathbf{U}}_{1}...\dot{\mathbf{U}}_{2\mathbf{T}} + (\partial_{\mathbf{A}\dot{\underline{U}}_{1}} \ln \rho) \mathbf{f}_{\mathbf{W}}; \quad \mathbf{\underline{U}}_{2}...\underline{\mathbf{U}}_{2\mathbf{T}} = \mathbf{0}.$$
(33)

We shall now decompose f in the following way (symmetrization)

$${}^{\mathbf{r}}\dot{\mathbf{y}}; \dot{\mathbf{u}}_{1} \dots \dot{\mathbf{u}}_{2} \overline{\mathbf{T}} {}^{\eta} \dot{\mathbf{y}}; \dot{\mathbf{u}}_{1} \dots \dot{\mathbf{u}}_{2T} {}^{+} {}^{\boldsymbol{\ell}} \dot{\mathbf{y}} \underline{\dot{\mathbf{u}}}_{1} {}^{\rho} \chi \ \underline{\dot{\mathbf{u}}}_{2} \dots \underline{\dot{\mathbf{u}}}_{2T} , \qquad (34)$$
where

 $\eta \dot{\mathbf{y}} \cdot \mathbf{i}_{f} = \dot{\mathbf{i}}_{f} = \frac{1}{2m+1} \mathbf{f} \dot{\mathbf{y}} \cdot \dot{\mathbf{y}} = \dot{\mathbf{y}}$

and
$$\mathbf{1}, \mathbf{0}_1 \dots \mathbf{0}_{2\mathbf{T}} \quad \mathbf{2}^{\mathbf{T}+1} \quad \mathbf{1}, \mathbf{0}_1 \dots \mathbf{0}_{2\mathbf{T}}$$

 $\rho_{\chi} \dot{U}_{2}...\dot{U}_{2T} = \frac{1}{2T+1} f \dot{W} ; \dot{U}_{2}...\dot{U}_{2T}$

The identity (34) together with (35), (36) may be easily verified with the help of (31). Substituting now (34) and (36), into eq. (33) we obtain, after some manipulations

$$\begin{array}{c} \overset{\partial}{}_{A} & \overset{\gamma}{\mathbf{y}}; \dot{\mathbf{y}}_{1} \dots \dot{\mathbf{y}}_{2T} & \stackrel{=-\rho \partial}{}_{A} \underline{\mathbf{y}}_{1} & \overset{\chi}{\mathbf{y}}_{2} \dots \underline{\mathbf{y}}_{2T} & \stackrel{+2T(\partial_{A} \underline{\mathbf{y}}_{1}^{\rho}) \times \underline{\mathbf{y}}_{2} \dots \underline{\mathbf{y}}_{2T} & (37) \\ \text{Acting on both sides of eq. (37) by } \partial^{A} \text{ we get (using } \\ \partial^{A} \underline{\mathbf{x}} & \partial_{A} & \stackrel{\mathbf{y}}{=} -\frac{1}{2} \delta_{\underline{\mathbf{x}}} & \overset{\mathbf{y}}{=} \end{array}$$

$$\frac{1}{2} \Box \eta_{\dot{\mathbf{x}}; \dot{\mathbf{U}}_{1} \dots \dot{\mathbf{U}}_{2\mathbf{T}}} = \partial^{\mathbf{A}} \dot{\mathbf{x}}^{\left[\rho \partial_{\mathbf{A}} \dot{\underline{\mathbf{U}}}_{1}^{\chi} \mathbf{\underline{U}}_{2} \dots \dot{\underline{\mathbf{U}}}_{2\mathbf{T}}} - \frac{2\mathbf{T}(\partial_{\mathbf{A}} \underline{\underline{\mathbf{U}}}_{1}^{\rho) \chi} \dot{\mathbf{\mathbf{U}}}_{2\mathbf{U}} \dots \dot{\underline{\mathbf{U}}}_{2\mathbf{T}}}{\mathbf{A} \underline{\underline{\mathbf{U}}}_{1}^{\rho) \chi} \dot{\mathbf{\mathbf{U}}}_{2\mathbf{U}} \dots \dot{\underline{\mathbf{U}}}_{2\mathbf{T}}}]. (38)$$

Let us denote the r.h.s. of eq. (35) by $\mathbf{R} \cdot \mathbf{x}; \mathbf{v}_1 \dots \mathbf{v}_{2T}$. The symmetrization of the R yields

$${}^{R}_{\dot{x}; \dot{U}_{1} \dots \dot{U}_{2T}} = {}^{S}_{\dot{x}; \dot{U}_{1} \dots \dot{U}_{2T}} + {}^{\epsilon}_{\dot{\underline{U}}_{1} \dot{x}} {}^{D}_{\dot{\underline{U}}_{2} \dots \dot{\underline{U}}_{2T}} , \qquad (39)$$

where

$${}^{\mathbf{S}}\dot{\mathbf{x}};\dot{\mathbf{v}}_{1}...\dot{\mathbf{v}}_{2\mathbf{T}} = {}^{\mathbf{Q}}\dot{\mathbf{x}};\dot{\underline{\mathbf{v}}}_{1}...\underline{\mathbf{v}}_{2\mathbf{T}}$$
(40)

with

$$\mathbf{Q}_{\dot{\mathbf{X}}; \dot{\mathbf{U}}_{1} \dots \dot{\mathbf{U}}_{2\mathbf{T}}} = (\partial^{\mathbf{A}}_{\mathbf{X}} \dot{\boldsymbol{\mu}}_{1}) (\partial_{\mathbf{A}} \underline{\dot{\mathbf{U}}}_{1}^{\mathbf{X}} \boldsymbol{\underline{\mathbf{U}}}_{2\mathbf{Z}} \dots \underline{\dot{\mathbf{U}}}_{2\mathbf{T}})$$
(41)
and

$${}^{\mathrm{D}}\dot{\mathrm{U}}_{2}\ldots\dot{\mathrm{U}}_{2\mathrm{T}} = \frac{1}{2}\rho_{\,\mathrm{D}}\,\chi\,\dot{\mathrm{U}}_{2}\ldots\dot{\mathrm{U}}_{2\mathrm{T}} - (\partial_{\,\underline{\mathrm{U}}_{2}}^{\mathrm{A}}\rho)(\partial_{\,\mathrm{A}}\dot{\mathrm{Y}}\,\chi\overset{\dot{\mathrm{Y}}}{\underline{\mathrm{U}}_{3}}\ldots\underline{\mathrm{U}}_{2\mathrm{T}}). \tag{42}$$

Since both the l.h.s. of eq. (38) and the S in (39) are entirely symmetric, the expressions (42) must vanish

$$\frac{1}{2}\rho \Box \chi_{\vec{U}_{2}...\vec{U}_{2T}} - (\partial^{A}_{\underline{U}_{2}} \rho)(\partial_{A}\dot{Y} \chi^{Y}_{\underline{U}_{3}}...\underline{U}_{2T}) = 0.$$
(43)

Eq: (43) coincides with eq. (25) for V=0, which we have already solved for T =1/2, 1. We have thus obtained a compatibility condition for eq. (37); if (43) were not satisfied, eq. (37) would not have solution. Let $\chi \dot{u}_{2}...\dot{u}_{2T}$ be a solution of eq. (43). Denote the r.h.s. of eq. (37) by $\Re_{A};\dot{u}_{1}...\dot{u}_{2}\uparrow$.

 $\eta_{\dot{X};\dot{U}_{1}...\dot{U}_{2T}} = -2\partial^{A}_{\dot{X}} (\Box^{-1} \Re_{A;\dot{U}_{1}...\dot{U}_{2T}}); \qquad (44)$ to verify this, one has only to employ the relation $\partial_{A}^{\dot{X}} \partial_{\dot{X}}^{B} =$ = $-\frac{1}{2} \partial_{A}^{B} \Box$. However, (cf. (38), (39))

$$\overset{\mathcal{L}}{\partial}^{\mathbf{A}} \dot{\mathbf{x}} \overset{\mathcal{R}}{\mathbf{A}}; \dot{\mathbf{U}}_{1} \dots \dot{\mathbf{U}}_{2\mathbf{T}} \overset{=}{\mathbf{R}} \overset{\mathbf{R}}{\mathbf{x}}; \dot{\mathbf{U}}_{1} \dots \dot{\mathbf{U}}_{2\mathbf{T}}$$

and hence, in view of (43)

$$\dot{x}; \dot{v}_{1}...\dot{v}_{2T} = 2c^{-1} S \dot{x}; \dot{v}_{1}...\dot{v}_{2T}$$

The general solution of eq. (37) is the sum of (45) and an arbitrary solution $\eta^{(0)}$ of the homogeneous equation corresponding to (37), i.e.,

$$\partial_{A} \overset{\dot{x}}{\eta} \overset{(0)}{\dot{x}}; \dot{v}_{1} ... \dot{v}_{2T} = 0.$$
 (46)

Concluding this section we may write the explicit solutions of cq. (30) (up to an arbitrary solution of (46)) for T = 1/2and 1. For T = 1/2 we have, according to (32), (34), (40), (41), (43) and (45)

$$\psi_{\dot{X}; \dot{U}_{1}} = \rho^{-\frac{1}{2}} \{ 2_{\Box}^{-1} [(\partial^{A}_{\dot{X}} \rho)(\partial_{A \dot{U}_{1}} \chi)] + \epsilon_{\dot{X} \dot{U}_{1}} \rho_{\chi} \}$$
(47)

with $\Box \chi = 0$.

η

(35)

(36)

The expression (47) is equivalent to the result obtained in ref.²⁷ by direct methods. For T=1 we may write two types of solutions

(i)

$$\mathbf{\dot{v}}_{\dot{\mathbf{x}}; \dot{\mathbf{U}}_{1} \dot{\mathbf{U}}_{2}} = \rho^{-1} \left[2 \Box^{-1} \mathbf{S}_{\dot{\mathbf{x}}; \dot{\mathbf{U}}_{1} \dot{\mathbf{U}}_{2}} \left(\rho, {}^{(i)} \chi \right) + \epsilon \dot{\mathbf{x}} \dot{\mathbf{U}}_{1} \rho^{(i)} \chi \dot{\mathbf{U}}_{2} \right],$$

where $\mathbf{i} = 1, 2$ the $\mathbf{S}(\rho, \chi)$ is given by (40), (41) and

ACKNOWLEDGEMENTS

One of the authors (V.N.P.) is indebted to Ya.A.Smorodinskii and A.V.Efremov for useful discussions. The second author

9

(45)

(J.H.), thanks Prof. V.A.Meshcheryakov for hospitality offered at the Laboratory of Theoretical Physics of JINR, Dubna.

REFERENCES

- 1. Pervushin V.N. Theor.Math.Phys., 1981, 45, p. 1145. Первушин В.Н. ТМФ, 1980, с.343.
- 2. Первушин В.Н. ЯФ, 1982, 35, в.6.
- 3. Newman E., Penrose R. J.Math.Phys., 1962, 3, p. 566. 4. Pirani F.A.E. Proceedings of Brandeis Summer Institute in Theoretical Physics, 1964, vol.I., Lectures on General Relativity, p. 305. Фролов В.П. Труды ФИАН, 1977, 96, с.72.
- 5. Jackiw R., Rebbi C. Phys.Rev., 1977, D16, p. 1052.
- 6. Yang C.N. Phys.Rev.Lett., 1977, 38, p. 1977.
- 7. Prasad M.K. Physica 1980, D1, p. 167.
- 8. Actor A. Rev. Mod. Phys., 1979, 51, p. 461.

Received by Publishing Department on May 3 1982.

WILL YOU FILL BLANK SPACES IN YOUR LIBRARY?

You can receive by post the books listed below. Prices - in US \$,

including the packing and registered postage

D13-11807	Proceedings of the III International Meeting on Proportional and Drift Chambers. Dubna, 1978.	14.00
	Proceedings of the VI All-Union Conference on Charged Particle Accelerators. Dubna, 1978. 2 volumes.	25.00
D1,2-12450	Proceedings of the XII International School on High Energy Physics for Young Scientists. Bulgaria, Primorsko, 1978.	18.00
D-12965	The Proceedings of the International School on the Problems of Charged Particle Accelerators for Young Scientists. Minsk, 1979.	8.00
D11-80-13	The Proceedings of the International Conference on Systems and Techniques of Analytical Comput- ing and Their Applications in Theoretical Physics. Dubna, 1979.	8.00
D4-80-271	The Proceedings of the International Symposium on Few Particle Problems in Nuclear Physics. Dubna, 1979.	8.50
D4-80-385	The Proceedings of the International School on Nuclear Structure. Alushta, 1980.	10.00
	Proceedings of the VII All-Union Conference on Charged Particle Accelerators. Dubna, 1980. 2 volumes.	25.00
D4-80-572	N.N.Kolesnikov et al. "The Energies and Half-Lives for the a - and β -Decays of Transfermium Elements"	10.00
D2-81-543	Proceedings of the VI International Conference on the Problems of Quantum Field Theory. Alushta, 1981	9.50
10,11-81-622	Proceedings of the International Meeting on Problems of Mathematical Simulation in Nuclear Physics Researches. Dubna, 1980	9.00
D1,2-81-728	Proceedings of the VI International Seminar on High Energy Physics Problems. Dubna, 1981.	9.50
D17-81-758	Proceedings of the II International Symposium on Selected Problems in Statistical Mechanics. Dubna, 1981.	15.50
D1,2-82-27	Proceedings of the International Symposium on Polarization Phenomena in High Energy Physics. Dubna, 1981.	9.00

Orders for the above-mentioned books can be sent at the address: Publishing Department, JINR Head Post Office, P.O.Box 79 101000 Moscow, USSR

SUBJECT CATEGORIES **OF THE JINR PUBLICATIONS**

Index Subject
1. High energy experimental physics
2. High energy theoretical physics
3. Low energy experimental physics
4. Low energy theoretical physics
5. Mathematics
6. Nuclear spectroscopy and radiochemistry
7. Heavy ion physics
8. Cryogenics
9. Accelerators
10. Automatization of data processing
11. Computing mathematics and technique
12. Chemistry
13. Experimental techniques and methods
14. Solid state physics. Liquids
15. Experimental physics of nuclear reactions at low energies
16. Health physics. Shieldings
17. Theory of condenced matter
18. Applied researches
19. Biophysics

E2-82-316 Первушин В.Н., Горжейши И. Спинорный анализ теории Янга-Миллса в пространстве Минковского

Спинорные методы применяются к решению уравнений самодуальности для полей Янга-Миллса и уравнения Дирака с внешним самодуальным полем в пространстве Минковского. Рассматривается случай калибровочной группы SU(2). Показано, что в рамках спинорного формализма естественно получается аналог конструкции Янга для самодуальных полей. Получены решения уравнения Дирака для безмассового фермиона с произвольным изоспином, взаимодействующего с внешним самодуальным или антисамодуальным полем. В качестве внешнего поля берется аналог евклидовой подстановки Хоофта. Показано, что для изоспина 1/2 и 1 решения уравнения Дирака выражаются через решения уравнения Даламбера. Полученные решения можно использовать в рамках недавно предложенного подхода к калибровочным теориям, основанного на аналогии с теорией сверхтекучести; в таком подходе самодуальные решения уравнений Янга-Миллса играют роль вакуумных полей.

Работа выполнена в Лаборатории теоретической физики ОИЯИ. Препринт Объединенного института ядерных исследований. Дубна 1982

Pervushin V.N., Hoteiši J. Spinor Analysis of Yang-Mills Theory in the Minkowski Space E2-82-316

Spinorial methods are applied to the solution of self-duality equations for Yang-Mills field and the Dirac equation with an external self-dual field in the Minkowski space. Gauge group SU(2) is considered. It is shown that in the spinorial formalism an analog of the Yang construction of self-dual fields emerges naturally. Solutions of the Dirac equation for massless fermion with an arbitrary isospin, interacting with an external self-dual or anti-self-dual field are obtained. The external field is chosen to be the Minkowskian analog of the Euclidean 't Hooft Ansatz. It is shown that for the isospin 1/2 and 1 the solutions of the Dirac equation may be expressed in terms of the solutions of d'Alembert equation. The solutions obtained in this paper may be employed in the approach to gauge theories proposed recently, which is based on an analogy with the superfluidity theory; in such an approach the self-dual solutions of the Yang-Mills equations represent the vacuum. Preprint of the Joint Institute for Nuclear Research. Dubna 1982