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NONINVARIANCE OF REGULARIZATION BY DIMENSIONAL REDUCTION: AN EXPLICIT EXAMPLE OF SUPERSYMMETRY BREAKING

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Regularization by dimensional reduction (RDR) or supersymmetric dimensional regularization [1], inconsistent in the superfield language [2], has been formulated unambiguously in terms of component fields [3]. Being consistent this formulation is however noninvariant. The supersymmetry Ward identities (SWI) can be violated in higher orders by additional symmetry-breaking terms. Although in ref. [3] the order has been estimated, to which the SWI do not receive these additional contributions, no explicit example of the violation has been found for practically important identities. Another way of studying the invariance of the regularization is to check whether the coupling constants of different interactions in the theory have identical renormalizations as prescribed by supersymmetry. The identical renormalization of the Yukawa and gauge coupling in the $N=4$ supersymmetric Yang-Mills theory (SYM) using the RDR has been established up to two loops [4]. In the present paper the similar three-loop test of the RDR is carried out for the $N=1,2,4$ SYM. The renormalization group β-function of the Yakawa coupling is calculated in the three-loop approximation and compared with the result [5] for the gauge coupling. The discovered difference indicates noninvariance of the RDR.

It is convenient to perform calculations in the $N=1,2,4$ SYM simultaneously [5], making use of the fact that these theories can be obtained uniformly by dimensional reduction [6] from $\mathfrak{D}=4,6,10$ to $d=4$ (using the RDR, to $d=4-2\epsilon$) dimensions applied to the action

$$S = \int dx\,[-\frac{1}{4}(F^a_{\mu\nu})^2 + i\bar\lambda^a \Gamma_\mu D_\mu \lambda^a]\,. \qquad (1)$$

The vector A^a_μ and spinor λ^a fields are in the adjoint representation of the gauge group. After the reduction the space-time coordinates become d-dimensional and the $\mathfrak{D}$-component field A^a_μ splits ($A^a_\mu = V^a_{\hat\mu} + S^a_{\tilde\mu}$) into a d-dimensional vector $V^a_{\hat\mu}$ and $\mathfrak{D}-d$ (pseudo) scalars $S^a_{\tilde\mu}$. The gauge-breaking and Faddeev-Popov ghost terms include $V^a_{\hat\mu}$ alone,

$$S_G = \int dx\,[-\frac{1}{2\alpha}(\partial_\mu V^a_{\hat\mu})^2 - (\partial_\mu \bar\eta^a + g f^{abc} V^b_{\hat\mu}\bar\eta^c)\,\partial_\mu \eta^a]\,. \qquad (2)$$

All the following calculations are performed in the Feynman gauge $\alpha=1$.

In the tree approximation the gauge, Yukawa and scalar quartic interactions, which appear in (1) after the splitting of A^a_μ, are governed by a common coupling constant g, according to supersymmetry. In principle, renormalizations may break this correlation. But all the Yukawa couplings remain equal to each other because the RDR does not influence the invariance of the action (1)+(2) under the following linear global transformations /6/

$$\delta V^a_{\hat{\mu}} = 0\,, \quad \delta S^a_{\tilde{\mu}} = \tilde{\Lambda}_{\mu\nu} S^a_{\tilde{\nu}}\,, \qquad\qquad (3)$$

$$\delta \lambda^a = \frac{1}{4} \tilde{\Lambda}_{\mu\nu} \Gamma_\mu \Gamma_\nu \lambda^a\,, \quad \delta \bar{\lambda}^a = -\frac{1}{4} \bar{\lambda}^a \Gamma_\mu \Gamma_\nu \tilde{\Lambda}_{\mu\nu}\,,$$

where $\tilde{\Lambda}_{\mu\nu} = -\tilde{\Lambda}_{\nu\mu}$ has $(\mathfrak{D}-d)$-dimensional indices.

To calculate the three-loop renormalization group functions the method described in refs. /7,8/ is employed. The β-function of the Yukawa coupling is given by the formula

$$\beta^Y(h) = h\,[\,2\gamma^{FFS}(h) - 2\gamma^F(h) - \gamma^S(h)\,]\,, \qquad\qquad (4)$$

where γ^{FFS}, γ^F and γ^S are the anomalous dimensions of the fermion-fermion-scalar vertex, fermion and scalar propagators, respectively, and $h = g^2/(4\pi)^2$. In the minimal subtraction scheme /9/ these anomalous dimensions can be expressed through the coefficients of the $1/\epsilon$ terms in the corresponding renormalization constants. The evaluation of the diagrams is performed using the computer system "SCHOONSCHIP" /10/.

It is convenient to operate with the $\mathfrak{D}$-component field $A^a_\mu(x)$ depending on the d-dimensional x's rather than with the vector and scalar fields explicitly. This allows one to compute the Yukawa vertex simultaneously with the fermion-fermion-vector one. It has been used to check the calculations: The β-function of the gauge coupling, obtained from the fermion-fermion-vector vertex

$$\beta^G(h) = h\,[\,2\gamma^{FFV}(h) - 2\gamma^F(h) - \gamma^V(h)\,]\,,$$

reproduces the result of ref. /5/,

$$\beta^G_{3\,\mathrm{loops}}(h) = \frac{1}{2}(\mathfrak{D}-10)Ch^2[\,1-(\mathfrak{D}-6)Ch + \frac{7}{4}(\mathfrak{D}-6)^2C^2h^2\,]\,, \qquad (5)$$

where C is a group invariant defined by $f^{abc}f^{dbc} = C\delta^{ad}$. However, formula (4) leads to a different β-function,

$$\beta^Y_{3\,\mathrm{loops}}(h) = \frac{1}{2}(\mathfrak{D}-10)Ch^2 - \frac{1}{2}(\mathfrak{D}-10)(\mathfrak{D}-6)C^2h^3 +$$
$$+ \{\,(\frac{\mathfrak{D}^3}{8} - \frac{13}{2}\mathfrak{D}^2 + 66\mathfrak{D} - 207)C^3 + \qquad\qquad (6)$$
$$+ 2(\mathfrak{D}-10)(\mathfrak{D}-4)[\,\mathfrak{D}-3-12(\mathfrak{D}-\frac{15}{2})\zeta(3)]F\,\}\,h^4\,.$$

The three-loop coefficient of (6) does not coincide with that of (5) at any $\mathfrak{D}$. The expression (6) includes $\zeta(3) = \sum_{n=1}^{\infty} 1/n^3$ and a group invariant F, $f^{ajt}f^{ijk}f^{bmk}f^{lmn}f^{crn}f^{irs}f^{lst} = F f^{abc}$. For the SU(n) group $F = \frac{3}{2}n$ (see ref. /8/). This invariant cannot appear in the three-loop β-function of the gauge coupling /8/.

Consider possible explanations of the difference between (6) and (5). The estimates of ref. /3/ can be extended to all the SWI which contain the Green functions essential for the renormalization structure of the N = 1,2,4 SYM. These identities are derived through the change

$$\delta A^a_\mu = i\,(\,\bar{\xi}\Gamma_\mu\lambda^a - \bar{\lambda}^a\Gamma_\mu\xi\,)\,, \qquad\qquad (7)$$

$$\delta\lambda^a = -\frac{1}{2}F^a_{\mu\nu}\Gamma_\mu\Gamma_\nu\xi\,, \quad \delta\bar{\lambda}^a = -\frac{1}{2}\bar{\xi}\Gamma_\mu\Gamma_\nu F^a_{\mu\nu}$$

of the path integral variables in the generating functional followed by differentiating it with respect to a constant spinor ξ, once with respect to a source of λ^a and $n = 1, 2$ or 3 times to that of A^a_μ. At $n = 1$ one gets the propagator-type SWI studied in ref. /11/. At $n = 2$ and $n = 3$ the identities for the triple and quartic vertices are obtained. In the Table the minimal number of loops in the diagrams is pointed out, at which the symmetry-breaking terms, resulting from the variation of the action (1)

$$\delta S = \int dx\, g f^{abc}(\bar{\xi}\Gamma_\mu\lambda^a - \bar{\lambda}^a\Gamma_\mu\xi)\,\bar{\lambda}^b\Gamma_\mu\lambda^c \qquad\qquad (8)$$

under the supersymmetry transformations (7), can display themselves in the corresponding SWI written for the N = 1,2,4 SYM. Also the level is indicated, from which divergent parts of these terms could survive.

Table

Possibility of the contributions from (8) to the SWI (minimal number of loops)

SYM	$\mathfrak{D}$	violations			divergences		
		n=1	n=2	n=3	n=1	n=2	n=3
N=1	4	4	2	1	4	2*	2*
N=2	6	6	4	2	6	4	4
N=4	10	10	8	6	10	8	8

It should be mentioned that the estimates of the Table take into account power counting only, and, e.g., a detailed consideration of the two-loop symmetry-breaking diagrams shows that their divergent parts cancel. This fact is indicated in the Table by stars. But even these minimal estimates allow one to conclude that the contribution of (8) can explain the difference between (6) and (5) (it concerns just the divergent parts of the diagrams) for $\mathfrak{D}=4$ (N=1 SYM) only. For the N=2 and N=4 SYM the three-loop SWI are true. But in the component-field formulation auxiliary fields of supersymmetry are eliminated by fixing Wess-Zumino gauge (WZG) and using equations of motion, therefore the supersymmetry transformations (7) are nonlinear and S_G (2) is noninvariant under (7). This implies the SWI include also Green functions of composite-field operators and therefore appear insufficient to prove the identical renormalization of the different couplings. It is these terms that cause the anomaly in the N=2 and N=4 SYM.

There are two possible reasons for it: Either extended supersymmetry does not survive in the WZG at all or the RDR is noninvariant, and the invariance requires additional conditions but the absence of the contributions to the SWI in the WZG from the variation (8). To show the second is true, let us use the formulation of the N=1,2,4 SYM in terms of N=1 superfields [12,13,14],

$$S_{N=1} = \frac{1}{64g^2} \mathrm{Tr} \int dx\, d^2\theta\, W^\alpha W_\alpha \,, \tag{9}$$

$$S_{N=2} \doteq S_{N=1} + \mathrm{Tr} \int dx\, d^4\theta\, e^{-gV} \bar{\Phi}\, e^{gV} \Phi \,, \tag{10}$$

$$S_{N=4} = S_{N=1} + \mathrm{Tr}\,[\int dx\, d^4\theta\, e^{-gV}\, \bar{\Phi}_i\, e^{gV}\Phi_i + \qquad (11)$$
$$+ (\frac{ig}{3!} \int dx\, d^2\theta\, \epsilon_{ijk}\, \Phi_i\, [\Phi_j, \Phi_k] + \mathrm{h.c.})],$$

where V is a Hermitian gauge (vector) superfield, Φ and Φ_i ($i = 1,2,3$) are chiral superfields in the adjoint representation of the gauge group, $W_\alpha = \bar{D}^2 (e^{-gV} D_\alpha e^{gV})$. The gauge-breaking term can be chosen in the $N = 1$ supersymmetric form:

$$S_{G.B.} = - \frac{1}{16} \mathrm{Tr} \int dx\, d^4\theta\, (D^2 V)(\bar{D}^2 V). \qquad (12)$$

Now the $N=1$ supersymmetry transformations are linear and no composite-field operators are involved in the corresponding SWI. Therefore, if an invariant regularization is used, only one coupling constant appears in (9) and (10). Based on ref /15/ one can conclude that in the minimal subtraction scheme the β-function does not depend on the way of gauge fixing at all. Thus, in the WZG one necessarily obtains just one β-function for the $N=1$ and $N=2$ SYM if the regularization preserves $N=1$ supersymmetry. In the case of the $N=4$ SYM the Φ^3 coupling in (11) could generally differ from the gauge one. But in the WZG the Yukawa couplings originating from these two interactions are equal as a consequence of the linear SU(4) symmetry (3) of the quantum action. Hence, due to gauge independence of the β-function, the theory has only one coupling constant /16/ both in the $N=1$ supersymmetric (12) and WZG, although the existence of a manifestly $N=4$ invariant formulation is not supposed.

Thus, the RDR is noninvariant. To ensure invariance it is necessary that the SWI in the gauge (12) (without any Green functions of composite-field operators*) were true. To check them, the RDR recipe used above should be completed by introducing auxiliary fields of $N=1$ sypersymmetry. Then the spinors will appear asymmetrically: one of them in the vector superfield, the others in the chiral superfields. Therefore, considering $N=1$ supersymmetry breaking, one has to use the estimates for $\mathfrak{D}=4$, or maybe, still lower ones (the explicit form of δS is rather difficult to write down, but the terms which lead to the $\mathfrak{D}=4$ estimates will surely be present). These estimates explain the violation discovered at the three-loop level.

Noninvariance of the RDR makes the significance of the three-loop calculations /4,5,16,17/ doubtful. But the superfield computations in the $N=4$ SYM /16,17/ demonstrate the finiteness of the Green functions in the gauge (12) and therefore are reliable requiring no regularization. For the $N=4$

SYM the β-function (5) of the gauge coupling in the WZG corresponds to these calculations. It allows one to believe that the results of (5) for the N = 1 and N = 2 SYM are also reliable.

Although, in principle, the problem of invariant regularization for N = 1 supersymmetric gauge models has been solved/18/, a convenient calculational recipe with a sufficiently wide region of applicability is not available up to now.

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