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ANOMALOUS DIMENSIONAL QUARK COUNTING OF HARD PROCESSES IN QCD

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1. INTRODUCTION

Usually the hadron-hadron reactions at large transverse momentum can be explained assuming that the large p_T jets arise from hard scattering between constituent particles (partons) in the incoming hadrons [1].

The first application of asymptotically free gauge theory [2] in this direction has been found in ref. [3]. The observation of scale breaking effects in ep, μp and νp-scattering being in agreement with the QCD predictions suggests us to parametrize the constituent distributions in a manner consistent with QCD and to include it in large p_T production calculations. Such attempts are made in refs. [4-6].

In the recent note [7] concerning scale breaking phenomena in the large transverse momentum hadron production, the anomalous dimension quark counting rules have been proposed. It turned out that scaling violation rate of the corresponding invariant cross-sections is related to the number of valence quarks participating in the hard scattering.

Note, that these results were derived in the leading order of QCD restricting to the quark scattering subprocess and the so-called nonsignlet part of quark distributions, which dominate in hadron scattering at large x_T.

On the other hand, it is commonly known from QCD the relative importance of the flavour singlet sector of quark and gluon densities, especially at low x. Because the Q^2-dependence is different for different parton distributions, the singlet contribution can play an important role in derivation of the counting rules for inclusive reactions.

The purpose of this work is to extend the method, proposed in ref. [7] to cover the singlet part of the structure functions and all allowed parton subprocesses as well.

In the next section we reconsider the Q^2-evolution of the DIS structure functions in the parametrization needed for our purposes. Then, we repeat shortly the results: for the cross sections asymptotics in nonsinglet case [7]. Contributions of the singlet parton densities are presented in Sec.4.

Section 5 deals with the higher order perturbative corrections to the one-particle (jet) invariant cross sections. Finally, in the last section we discuss the problem of the asymptotic p_T^{-4} law in hadron collisions in terms of the obtained quark counting rules.

2. Q^2-DEVELOPMENT OF DEEP INELASTIC STRUCTURE FUNCTIONS

The structure functions of deep inelastic lepton scattering in parton model satisfy approximate Bjorken scaling, QCD then predicts the deviation from exact scaling, which manifests itself in the Q^2-dependence of different parton distributions. This behaviour can be easily understood in terms of the so-called evolution equations for the quark and gluon densities[8].

$$\frac{\partial q_i(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\left[P_{qq}\left(\frac{x}{y}\right)q_i(y,Q^2) + P_{qG}\left(\frac{x}{y}\right)G(y,Q^2)\right]$$

$$\frac{\partial G(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\left[P_{qq}\left(\frac{x}{y}\right)\sum_{j=1}^{2f} q_j(y,Q^2) + P_{GG}\left(\frac{x}{y}\right)G(y,Q^2)\right]. \tag{2.1}$$

These eqs. account the effects of scaling violation in parton structure functions arising in perturbation expansion of renormalizable gauge theory.

Here the sum runs over the quark flavors, and $P_{ij}(z), i,j=q,G$ are probabilities of the corresponding $q \to Gq$ transitions. Eqs. (2.1) are valid strictly for the massless quarks.

Let us consider an arbitrary structure function $F(x,Q^2) = \sum x q(x,Q^2)$. It can be split into a singlet and nonsinglet pieces. By making separation between valence and "sea" quark densities (for $SU(f)$ symmetric sea)

$$q(x) = q_v(x) + q_s(x), \qquad q_v = u,d \tag{2.2}$$

$$q_s(x) = u,\bar{u},d,\bar{d},s,\bar{s},\ldots,$$

eqs. (2.1) reduce to the three integrodifferential equations for the $q(x)$, $q_s(x)$ and $G(x)$, respectively. The first one is determined by the operators, nonsinglets in flavor symmetry group and is decoupled from two others. The latter eqs. are satisfied by the q_s and G densities, which do not carry flavor quantum numbers and are determined by mixing between flavor singlet and nonsinglet parts of the corresponding operators.

It is important to note, that solutions of eqs. (2.1) require the knowledge of the definite boundary conditions. If we know the functions $q(x)$ and $G(x)$ for some $Q^2 = Q_0^2$ ($\alpha_s(Q_0^2)/2\pi \ll 1$) then the latter eqs. enable us to obtain the distributions $q(x,Q^2)$ and $G(x,Q^2)$ for arbitrary values of $Q^2 > Q_0^2$.

Following the prescription of ref.[7] we choose the form of x-dependence of the structure functions at large x and fixed $Q^2 = Q_0^2$ as dictated by the dimensional quark counting (spectator rule)* [9]

$$xF(x, Q^2 = Q_0^2) \sim (1-x)^{2n-3}, \quad x \to 1, \tag{2.3}$$

where n is minimal number of hard constituents in hadron:

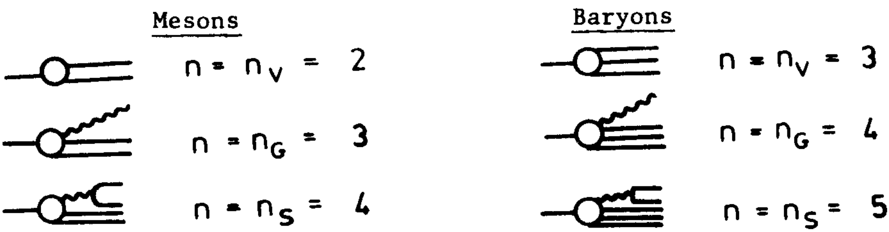

Fig. 1

and n_V, n_S, n_G are the power law exponents in (2.3) for the valence, sea-quark and gluon distributions, respectively.

Solutions of the evolution equations (2.1) for (2.2) with the boundary conditions (2.3) now can be easily obtained, e.g., by Mellin transformation of the corresponding quark and gluon densities

$$F_i(n, Q^2) = \int_0^1 x^{n-2} dx F_i(x, Q^2), \quad i = V, S, G. \tag{2.4}$$

We write down these solutions in Appendix A*. For the Q^2-development of different parton densities, contributing to the $x \sim 1$ region, we have:

$$x\vec{F}(x, Q^2) = K(\xi)(1-x)^{2n_V - 3 + r\xi} \quad \vec{H}(x, \xi) \tag{2.5}$$

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* Up to $2\Delta h = 2(h_1 - h_2)$, where h_1 and h_2 are hadron and quark helicities, respectively.

** See also the results of ref.[10]. In addition in the present paper contributions of the nonleading terms are included, dominating at low values of x.

$$\vec{F}(x,Q^2) = \begin{pmatrix} q_V(x), \\ G(x) \\ q_S(x) \end{pmatrix}, \quad \vec{H}(x,\xi) = \begin{pmatrix} \frac{1}{\Gamma(2n_V-2+r\xi)}, \\ \frac{2}{5}\left(\frac{(1-x)}{\Gamma(2n_V-1+r\xi)[\ln\frac{1}{1-x}+\Psi(2n_V-1+r\xi)+C]} - \right. \\ \left. -\frac{(1-x)^{1+5/4r\xi}}{\Gamma(2n_V-1+\frac{9}{4}r\xi)[\ln\frac{1}{1-x}+\Psi(2n_V-1+\frac{9}{4}r\xi)+C]}\right), \\ \frac{3/40\, r\xi(1-x)^2}{\Gamma(2n_V+r\xi)[\ln\frac{1}{1-x}+\Psi(2n_V+r\xi)+C]} \end{pmatrix}$$

where $\xi = \ln[\alpha_s(Q_0^2)/\alpha_s(Q^2)]$, $r = 16/(33-2f)$, f - quark flavor number, $c = \gamma_E - \frac{21-2f}{20}$,

$$K(\xi) = [\alpha_s(Q_0^2)/\alpha_s(Q^2)]^{r(3/4-\gamma_E)} \cdot \Gamma(2n_V-2),$$

and

$$\gamma_E = 0.5772\ldots; \quad \Psi(z) = d\ln\Gamma(z)/dz$$

are Euler's number and digamma function, respectively.

Note, that in the framework of leading order QCD perturbation theory, we can consider the Q^2-development of the parton fragmentation functions on the equal footing. The difference between $F(x,Q^2)$ and $D(z,Q^2)$ which arises beyond the leading order corresponds to the breakdown of the Gribov-Lipatov reciprocity relation/11/.

3. ANOMALOUS DIMENSION QUARK COUNTING. NONSINGLET CASE

In this section we repeat the arguments of ref./7/ and give a general formula called in the following the anomalous dimension quark counting rule. It determines the leading log corrections to the canonical point-like asymptotics/9/ of an arbitrary particle reaction at large transferred momentum in terms of active and passive quarks of the hadrons participating in the relevant reaction.

The invariant cross section for the large p_T hadron (jet) production in two hadron collisions $AB \to C(J) + \ldots$ reads

$$\sigma\begin{pmatrix} AB \to CX \\ AB \to \mathrm{jet}\,X \end{pmatrix} \backsim \sum_{a,b,c} \int_{x_a^{min}}^{1} dx_a \int_{x_b^{min}}^{1} dx_b F_{a/A}(x_a) F_{b/B}(x_b) \begin{Bmatrix} D_{C/c}(x_c)/x_c^2 \\ \delta(x_c-1) \end{Bmatrix} dx_c \cdot \tag{3.1}$$

$$\sigma \equiv E\, d\sigma/d^3p, \quad \cdot\, \hat{s}/\pi\, \delta(\hat{s}+\hat{t}+\hat{u})(d\hat{\sigma}/d\hat{t})_{ab},$$

where $(d\hat{\sigma}/d\hat{t})_{ab}$ is the hard parton scattering cross-section $a,b=q,\bar{q},G$ and in the jet kinematic case

$$\hat{s}=(p_a+p_b)^2\simeq x_a x_b s\,;\quad \hat{t}=(p_a-p_c)^2\simeq x_a t\,;\quad \hat{u}=(p_b-p_c)^2\simeq x_b u\,;$$

$$x_a^{min}=x_1/(1-x_2),\quad x_b^{min}=x_a x_2/(x_a-x_1);\quad x_1=-\frac{u}{s},\quad x_2=-\frac{t}{s}.$$

In the leading log approximation the initial valence quark distribution function (nonsinglet part of (2.5)) in the $x\to 1$ region has the following form

$$x_a F_{a/A}(x_a,Q^2)=\frac{c_A K_A(\xi)}{\Gamma(\bar{A})}(1-x_a)^{\bar{A}-1},\quad F(x)\equiv F_2^{NS}(x), \tag{3.2}$$

where

$$\bar{A}=A+r\xi=2(n_V^A-1)+r\xi,\qquad \xi=-\log\alpha_s(Q^2)$$

$$K_A(\xi)=\exp\{r\xi(3/4-\gamma_E)\}\cdot\Gamma(A),$$

and $n_V^A\equiv n_A$ is the valence quark number in hadron A. Inserting this expression (and similarly the $F_{b/B}$ -function) in hard scattering Ansatz (3.1), we find in the jet production case:

$$\sigma(AB\to \text{jet}\,X)=\left(\frac{\alpha_s}{p_T^2}\right)\frac{\exp\{2r\xi(3/4-\gamma_E)\}}{\Gamma(\bar{A})\Gamma(\bar{B})}c_A c_B\Gamma(A)\Gamma(B)(x_1x_2)^2 J\,,$$

$$J=\frac{(1-x_1-x_2)^{\bar{A}+\bar{B}-1}}{(1-x_1)^{\bar{A}}(1-x_2)^{\bar{B}}}\int_0^1\!\!\int_0^1\frac{du\,dv\,u^{\bar{A}-1}v^{\bar{B}-1}\,\delta\left[(1-u-v)+\frac{1-x_1-x_2}{(1-x_1)(1-x_2)}uv\right]}{\left[1-\frac{1-x_1-x_2}{1-x_2}u\right]^2\left[1-\frac{1-x_1-x_2}{1-x_1}v\right]^2} \tag{3.3}$$

$$=\frac{(1-x_1-x_2)^{\bar{A}+\bar{B}-1}}{(1-x_2)^{\bar{A}}(1-x_1)^{\bar{B}}}B(\bar{A},\bar{B})\left(\frac{1-x_2}{x_1}\right)^2F_1\left(\bar{B},\bar{A}-2,4,\bar{A}+\bar{B};\frac{1-x_1-x_2}{(1-x_1)(1-x_2)},\frac{1-x_1-x_2}{1-x_1}\right),$$

where $B(\alpha,\beta)$ is Euler's Beta function and $F_1(\alpha,\beta,\beta',\gamma',x,y)$ is hypergeometric function of two variables [12] (see Appendix B).

In the case of $\Theta=90°$, $x_1=x_2=x_T/2$

$$\sigma(AB\to \text{jet})=\left(\frac{\alpha_s}{p_T^2}\right)^2\frac{c_A c_B\Gamma(A)\Gamma(B)}{4\Gamma(A+B)}\epsilon'^{\,A+B-1}\{[\alpha_s(p_T^2)]^{d(A+B)-\frac{r}{A+B}-r\ln\epsilon'}\}\Phi,$$

where

$$\Phi=x_T^2(1-x_T/2)F_1\,,\qquad \epsilon'=(1-x_T)/(1-x_T/2)$$

and

$$F_1 \equiv F_1(\bar{B}, \bar{A}-2, 4, \bar{A}+\bar{B};\ \ \epsilon'/(1-x_T/2), \epsilon'). \tag{3.4}$$

Thus, in the limit $x_T \to 1$ we have

$$\sigma(AB \to \text{jet}) = \sigma_0 \cdot \frac{c_A c_B \Gamma(A)\Gamma(B)}{4\Gamma(A+B)} \epsilon^{A+B-1} [\alpha_s(p_T^2)]^{2D(A+B)}, \tag{3.5}$$

where

$$\sigma_0 = \left(\frac{\alpha_s}{p_T^2}\right)^2 2^{A+B} \left(\frac{x_T^2}{4}\right)^2 (\alpha_s)^{-2r\ln 2},$$

$$\epsilon = 1 - x_T = 1 - 2p_T/\sqrt{s}, \quad A(B) = 2(n_V^{A(B)} - 1),$$

$$D(n) = d(n) - r(1/n + \log \epsilon),$$

and

$$d(n) \equiv d_n^{NS} = r\left(1/4 - \frac{1}{2n(n+1)} + \sum_{k=2}^{n} 1/k\right)$$

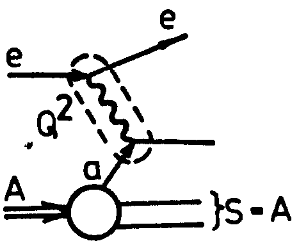
is the one-loop anomalous dimension of the nonsinglet quark distribution function.

For the arbitrary hard process asymptotic the following general formula is valid [7]

$$\frac{E d\sigma}{d^3p}(AB \to CX) \sim \left(\frac{\alpha_s}{p_T^2}\right)^2 \frac{\epsilon^{s_p - 1}}{\Gamma(s_p)} [\alpha_s(p_T^2)]^{HD(s_p) - 2r\ln 2}. \tag{3.6}$$

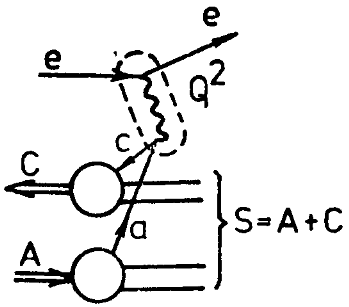
Here H is the total number of active quarks, which coincides with the total number of hadrons in reaction and $s_p = 2n_{\text{passive}} = \sum_{i-\text{hadrons}} 2(n_i - 1)$ is twice the total number of passive quarks (spectators) belonging to the hadrons which participate in the reaction.

We present some illustrative examples of this formula:



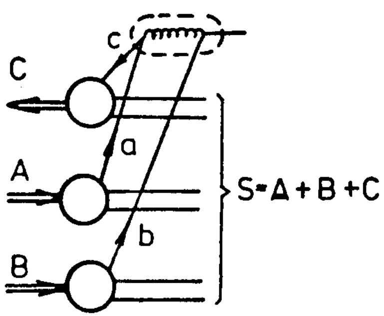
$$\frac{E d\sigma}{d^3k} \sim \left(\frac{\alpha}{Q^2}\right)^2 \frac{(1-x)^{A-1}}{\Gamma(A)} [\log Q^2]^{-1 \cdot D(A)}$$

Fig.2. Deeep inelastic scattering $eA \to e + \dots$; $s_p = A = 2(n_A - 1)$, $H = 1$.



$$\frac{E\,d\sigma}{d^3k} \backsim \left(\frac{\alpha}{Q^2}\right)^2 \frac{(1-x)^{A+C-1}}{\Gamma(A+C)} (\log Q^2)^{-2D(A+C)}$$

Fig.3. Deep inelastic semi-inclusive hadron electroproduction $eA \to e + C + \ldots$; $s_p = A+C = 2(n_A+n_C-2)$; $H=2$.



$$\frac{E\,d\sigma}{d^3p} \backsim \left(\frac{\alpha_s}{p_T^2}\right)^2 \frac{(1-x_T)^{A+B+C-1}}{\Gamma(A+B+C)} [\ \alpha_s(p_T^2)]^{+3D(A+B+C)}$$

Fig.4. Inclusive hadron production at large p_T in $A+B \to C+\ldots$; $s_p = A+B+C = 2(n_A+n_B+n_C-3)$; $H=3$

Finally, we stress that the logarithmic exponents in (3.6) are determined by the anomalous dimensions of the moments of nonsinglet quark distribution and decay functions of the participating hadrons. The numbers of the corresponding moments (indices of d_n^{NS}) are related to the numbers of valence quarks constituting these hadrons (see Table 1).

4. QUARK COUNTING FOR SINGLET DISTRIBUTIONS

Now we proceed with the calculation of the more general case of jet and single particle production in hadron collision at large p_T.

In what follows we shall consider the contribution of the singlet parton distribution as well as examine all the fundamental parton subprocesses

$$q_i q_j \to q_i q_j\ , \quad q_i G \to q_i G\ , \quad GG \to q_i \bar{q}_i$$
$$q_i \bar{q}_j \to q_i \bar{q}_j\ , \quad q_i \bar{q}_i \to GG\ , \quad GG \to GG$$

which can contribute in the leading order in α_s to the invariant cross-section (3.1). Now instead of the (F_a, F_b, D_c) we

Table 1

Anomalous dimensions $d(n) = d_n^{NS}$ of the nonsinglet quark operators, corresponding to the different hard processes (different spectator numbers s_p)

K	PROCESS $\ell = e^{\pm}, \mu^{\pm}, \nu^{(\pm)}$ A + B → C + ...;	d(Sp) Sp=2K	H·d(Sp)
1	$\ell + \pi \to \ell + \ldots$ $\ell + \bar{\ell} \to \pi + \ldots$	d[2(2−1)] d[2(2−1)]	1·d(2) 1·d(2)
2	$\ell + p \to \ell + \ldots$ $\ell + \pi \to \ell + \pi + \ldots$	d[2(3−1)] d[2(4−2)]	1·d(4) 2·d(4)
3	$\ell + p \to \ell + \pi + \ldots$ $\pi + p \to$ jet, ℓ, $\ell\bar{\ell}$ + ...	d[2(5−2)] d[2(5−2)]	2·d(6) 2·d(6)
4	$\ell + p \to \ell + p + \ldots$ $p + p \to$ jet, ℓ, $\ell\bar{\ell}$ + ... $\pi + p \to \pi + \ldots$	d[2(6−2)] d[2(6−2)] d[2(7−3)]	2·d(8) 2·d(8) 3·d(8)
5	$p + p \to \pi + \ldots$ $\pi + p \to \pi + \pi + \ldots$	d[2(8−3)] d[2(9−4)]	3·d(10) 4·d(10)
6	$p + p \to p + \ldots$ $\pi + p \to \pi + p + \ldots$ $p + p \to \pi + \pi + \ldots$	d[2(9−3)] d[2(10−4)] d[2(10−4)]	3·d(12) 4·d(12) 4·d(12)

substitute the functions $\sum_{ijk} F_i\ F_j\ D_k$, where the indices i,j,k run over the f flavor of quarks (antiquarks) and gluons

$F_{i/H}(x,Q^2) \sim x q_i(x,Q^2)$, $i = V,S,G$

and the final jet consists of the possible combinations of $q(\bar{q})$ and G in accordance with the initial j -state. The derivation of the valence quark VV contribution to the $\sigma(AB \to C(j)+X)$ was performed in preceding section. We proceed now with the analysis of the nondiagonal terms VG, VS , etc., in leading order of perturbation theory.

Inserting in (3.1) the corresponding singlet components of the parton distributions (2.5) and carrying out the integration in the large $p_T(x_T)$ region and $\theta = 90°$, we obtain final results for the processes of jet production and single-particle generation.

Jet Production $A+B \to J+X$

$$\sigma(AB\to J) = \sigma_0 \sum_{ij} \frac{\epsilon^{s_p-1}}{\Gamma(s_p)} a_{ij}(AB)\Phi_i^A \Phi_j^B , \tag{4.1}$$

where

$$\sigma_0 = (\alpha_s/p_T^2)^2 2^{A+B} (x_T^2/2)^2 (\alpha_s)^{-2r\ln 2} .$$

The sum runs over the fundamental subprocesses V,S,G.

Three-component vectors Φ_i^H entering eq. (4.1) contain information on the scale breaking of parton distributions and lead to the logarithmic deviation from the canonical p_T^{-4} law

$$\vec{\Phi}^H = c_H \Gamma(H)[\alpha_s(p_T^2)]^{D(s_p)} \vec{E}(\xi), \quad H=A,B$$

$$\vec{\Phi}^H = \begin{pmatrix} \Phi_V^H, \\ \Phi_G^H \\ \Phi_S^H \end{pmatrix}, \ \vec{E}(\xi) = \begin{pmatrix} 1, \\ \frac{2}{5}\left(\frac{1}{\Psi(H+r\xi+1)+\ell} - \frac{[\alpha_s]^{D^s(s_p)-D(s_p)}}{\Psi(H+\frac{9}{4}r\xi+1)+\ell}\right), \\ \frac{3/40\, r\xi}{\Psi(H+r\xi+2)+\ell} \end{pmatrix} \tag{4.2}$$

where c_H are the normalization constants, and the exponents $D(s_p)$, $D^s(s_p)$ are determined by the corresponding anomalous dimensions $d(d^s)$ of the nonsinglet and singlet operators

$$D(n) = d(n) - r(1/n + \log\epsilon), \qquad \epsilon = 1 - x_T$$

$$\begin{aligned} D^s(n) &= d^s(n) - \frac{9}{4} r(1/n + \log\epsilon) \\ d(n) &\equiv d_n^{NS} = -r(3/4 - \sum_1^n 1/k) + O(1/n^2) \\ d^s(n) &\equiv d_n^s = -r\left(\frac{33-2f}{16} - \frac{9}{4}\sum_1^n 1/k\right) + O(1/n^2). \end{aligned} \tag{4.3}$$

The components of the $\hat{a}$ matrix which enter eq. (4.1) correspond to the definite combinations of the partonic subprocess cross sections [13]

$$(d\hat{\sigma}/d\hat{t})_{ij} = \pi\alpha_s^2/\hat{s}^2 \cdot \Sigma_{ij}(\Theta) \qquad \text{for} \ \Theta = 90^\circ.$$

$$a_{ij}(AB) = \begin{pmatrix} V_A V_B, & V_A S_B, & V_A G_B \\ S_A V_B, & S_A S_B, & S_A G_B \\ G_A V_B, & G_A S_B, & G_A G_B \end{pmatrix}, \tag{4.4}$$

where the matrix elements VV, GG,... correspond to the subprocesses $qq \to qq$, $GG \to GG$,..., etc., and are in general process dependent. For example:

i) quark jet trigger $AB \to q(\bar{q}) + X$

$$\hat{a}(AB) = \begin{pmatrix} VV, & 2n_A p, & 55n_A/9 \\ 2n_B \cdot p, & 4f \cdot p, & 110f/9 \\ 55n_A/9, & 110f/o, & 7f/24 \end{pmatrix} \qquad (4.5)$$

$$p = 32/27 + 14f/3 = 2/27[16 + 63f].$$

Note, that singlet components of the parton distributions contribute in this case only few percent to the invariant cross section at large p_T when compared to the dominant valence quark contribution and increase for the small x_T values.

ii) gluon jet trigger $AB \to G + X$

$$\hat{a}(AB) = \begin{pmatrix} VV, & 56n_A/27, & 55n_A/9, \\ 56n_B/27, & 112f/27, & 110f/9 \\ 55n_B/9, & 110f/9, & 243/4 \end{pmatrix} \cdot \qquad (4.6)$$

iii) quark-gluon jet trigger $AB \to q(\bar{q})G + X$

$$\hat{a}(AB) = \begin{pmatrix} VV, & 2n_A \cdot k, & 110n_A/9, \\ 2n_B k, & 4f \cdot k, & 220f/9, \\ 110n_B/9, & 220f/9, & 243/4 + 7f/24 \end{pmatrix} \qquad (4.7)$$

$$k = 2/9 \cdot [10 + 21f]$$

where, the valence VV quark scattering contributions $qq \to qq$ distinguish for the different initial particles VV: ($f = 4$)

	i) $q(\bar{q})$jet	ii) G get	iii) $q(\bar{q})$ G jet
$pp \to jX$	1360/27	0	1360/27
$p\bar{p} \to jX$	1360/27	280/27	1640/27
$\pi^- p \to jX$	296/9	112/27	1000/27
$\pi^+ p \to jX$	476/27	56/27	532/27

(4.8)

In contrast with the case of quark jet production, the gluon trigger component in hadron collisions dominates.

Single Hadron Production $A+B \to C+X$

$$\sigma(AB \to C) = \sigma_0 \sum_{ijk} \frac{\epsilon^{s_p - 1}}{\Gamma(s_p)} a_{ijk}(ABC)\,\Phi_i^A \Phi_j^B \Phi_k^C \tag{4.9}$$

All quantities entering eq. (4.9)* are defined by (4.1)-(4.3), and elements of the three-dimensional matrix $a_{ijk}(ABC)$ have the following form:

$$\begin{pmatrix} VVV, & VVS, & VVG \\ VSV, & VSS, & VSG \\ VGV, & VGS, & VGG \end{pmatrix} = \begin{pmatrix} VVV, & VVS, & VVG \\ VSV, & 2n_A \cdot p, & 56\,n_A/27 \\ VGV, & 55n_A/9, & 55n_A/9 \end{pmatrix} \tag{4.10}$$

$$\begin{pmatrix} SVV, & SVS, & SVG \\ SSV, & SSS, & SSG \\ SGV, & SGS, & SGG \end{pmatrix} = \begin{pmatrix} SVV, & 2n_B p, & 56n_B/27 \\ 2n_G p, & 4f \cdot p, & 112f/27 \\ 55n_C/9, & 110f/9, & 110f/9 \end{pmatrix} \tag{4.11}$$

and

$$\begin{pmatrix} GVV, & GVS, & GVG \\ GSV, & GSS, & GSG \\ GGV, & GGS, & GGG \end{pmatrix} = \begin{pmatrix} GVV, & 55n_B/9, & 55n_B/9 \\ 55n_C/9, & 110f/g, & 110f/9 \\ 7n_C/48, & 14f/48, & 243/4 \end{pmatrix}, \tag{4.12}$$

where the elements VVV,...,etc., label the corresponding elementary subprocess, for example

$$\begin{aligned} VVV:&\quad q+q \to q+X \\ VGV:&\quad q+G \to q+X \\ VGG:&\quad q+G \to G+X,\ldots \end{aligned} \tag{4.13}$$

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* Assuming the validity of the reciprocity relation /11/.

Note, that elements VVV, SVV,..., etc., depend in general on the type of hadrons A, B, C participating in the inclusive reaction $A+B \to C+X$ and on the quark flavors. In particular, for the case of two proton collisions, and for $f=4$:

$pp \to$	π^+	π^-	π^0	p	$\bar{p}$
VVV	$\frac{944}{27}$	$\frac{416}{27}$	$\frac{680}{27}$	$\frac{256}{9}$	0
SVV = VSV	$\frac{1480}{27}$	$\frac{944}{27}$	$\frac{404}{9}$	122	$\frac{614}{27}$
VGV = GVV	$\frac{110}{9}$	$\frac{55}{9}$	$\frac{55}{6}$	$\frac{275}{9}$	0

and independently on the final states

$$VVG(pp) = 0, \qquad VVS(pp) = 1360/27 .$$

The matrix elements a_{ijk} for the $\pi^{\pm}p$ and $p\bar{p}$-scattering are given in Table 2.

Note, that spectator number s_p (twice of the passive constituent number in the nonsinglet case) becomes

$$s_p = 2\left(\sum_{i=1}^{H} n_i - H\right) + \sum_{i=1}^{H} \Delta s_p , \tag{4.14}$$

here, H is the number of hadrons in process, n_i is the valence quark number in the i-th hadron ($n=2$ for mesons; $n=3$ for baryons), Δs_p is additional passive quark from the nonvalence states, $\Delta s_p = 1(2)$ in the case of gluon ("sea" quark), respectively.

Thus, s_p runs over the following values

$A+B \to \text{jet}+X$		$A+B \to C+X$		
VV	s_p	VVV	s_p	
VG	s_p+1	VGV	s_p+1	
VS, GG	s_p+2	VSV, VGG	s_p+2	(4.15)
GS,	s_p+3	GSV, GGG	s_p+3	
SS	s_p+4	SSV, GGS	s_p+4	
		SSG	s_p+5	
		SSS	s_p+6	

Table 2

The elements of matrices (4.10)-(4.12) for the $p\bar{p}$- and $\pi^{\pm}p$-scattering

$p\bar{p} \to$	π^+	π^-	π^0	$\bar{p}$	p
VVV	$\frac{620}{27}$	$\frac{620}{27}$	$\frac{620}{27}$	38	38
SVV	$\frac{944}{27}$	$\frac{2480}{27}$	$\frac{404}{9}$	122	$\frac{614}{27}$
GVV	$\frac{55}{9}$	$\frac{110}{9}$	$\frac{55}{6}$	$\frac{275}{9}$	0

$VVS = \frac{1360}{27}$, $VVG = \frac{280}{27}$, $VSV = VSV(pp)$, $VGV = VGV(pp)$

$\pi^- p \to$	π^+	π^-	π^0	$\bar{p}$	p
VVV	$\frac{272}{27}$	$\frac{568}{27}$	$\frac{140}{9}$	$\frac{412}{27}$	$\frac{296}{9}$
VSV	$\frac{272}{27}$	$\frac{448}{9}$	$\frac{808}{27}$	$\frac{1480}{27}$	$\frac{944}{27}$
VGV	0	$\frac{110}{9}$	$\frac{55}{9}$	$\frac{110}{9}$	$\frac{55}{9}$

$VVS = \frac{296}{9}$, $VVG = \frac{112}{27}$, $SVV = SVV(pp)$, $GVV = GVV(pp)$

$\pi^+ p \to$	π^+	π^-	π^0	$\bar{p}$	p
VVV	$\frac{728}{27}$	$\frac{136}{27}$	32	$\frac{202}{27}$	$\frac{134}{3}$
VSV	$\frac{448}{9}$	$\frac{272}{27}$	$\frac{808}{27}$	$\frac{944}{27}$	$\frac{1480}{27}$
VGV	$\frac{110}{9}$	0	$\frac{55}{9}$	$\frac{55}{9}$	$\frac{110}{9}$

$VVS = \frac{476}{27}$, $VVG = \frac{56}{27}$, $SVV = SVV(pp)$, $GVV = GVV(pp)$

Thus, in the leading order of QCD perturbation theory taking into account the nonvalence hadron constituents modifies the hard scattering cross section and leads at large $p_T(x_T)$ to the power series expansion in $\epsilon = 1-x_T$

$$\sigma\begin{pmatrix} AB\to jet \\ AB\to C \end{pmatrix} \simeq \frac{(\alpha_s)^{2-2r\ln 2+HD(s_p)}\,\epsilon^{s_p-1}}{p_T^4} \sum_{k=0}^{2H} c_k \epsilon^k (\alpha_s)^{\Delta(s_p,k)}, \quad (4.16)$$

where $s_p = A+B$, and $H=2$, for $AB\to jet+X$, $A(B)=2n_V^{A(B)}-2$, $s_p = A+B+C$, and $H=3$, for $AB\to C+X$, and $c_0 = 1$, $\Delta(s_p, k=0)=0$, $\Delta(s_p,k=1) = 2[D(s_p+1)-D(s_p)]$, etc.

Like the nonsinglet case/7/, exponents of $\alpha_s(p_T^2)$ characterizing the deviation of the inclusive cross sections from the scaling form (p_T^{-4}) are determined by the anomalous dimensions of the structure function moments. Furthermore, the moments number (indices of anomalous dimension) becomes the physical meaning of spectator number. According to eq. (4.16) the effective exponent Δn, which corresponds to the scaling deviation of inclusive cross sections: p_T^{-n} $(n = 4+\Delta n)$ increases with s_p, i.e., together with the complexity of hadrons participating in reaction (number of nonvalence passive constituents). Note, that this analysis becomes more significant for the lower x_T values, and/or for the gluon initiated processes.

5. ANOMALOUS DIMENSION QUARK COUNTING BEYOND THE LEADING ORDER

In this section we shall discuss the next to leading order corrections to inclusive jet and single particle production cross sections at large transverse momentum. A lot has been learned the last time about higher order QCD corrections to deep-inelastic scattering/14/. These corrections turn out to improve agreement of the theory with the experimental data. There are however still many problems to be solved. In particular, the open questions remain, especially the uncertainties in the form and magnitude of higher order corrections in analysis of inclusive hadron initiated processes in QCD/14-17/.

It was shown in preceding sections that the asymptotics of the hard hadronic cross-sections are related with the anomalous $\log p_T^2$ counting, controlled in some universal way in leading order QCD perturbation theory by the number of active and passive constituents in hadrons.

Here, we shall put particular emphasis on the higher order corrections in structure functions of DIS and the validity of proposed anomalous dimension quark counting rules beyond the leading order. In what follows we restrict ourselves to the $\Theta=90^\circ$ case and neglect the next-loop corrections to the hard scattering cross sections, which do not affect the universality of the latter.

Let us consider now the two-loop corrections to the evolution of the quark distribution and decay functions [18-19]. For the nonsinglet moments (2.4) on the two-loop level, we have

$$F_2^{NS(2)}(n,Q^2)=F_2^{NS}(n,Q_0^2)\left[\frac{a_s(Q^2)}{a_s(Q_0^2)}\right]^{d_n}\left[1+\frac{a_s(Q^2)}{4\pi}R_2(n)\right], \qquad (5.1)$$

where $d_n=\gamma_n^{(0)}/2\beta_0$ is the anomalous one-loop dimension ($\beta_0=$ $=11-2/3f$), and the two-loop correction have the form *

$$\frac{1}{4\pi}R_2(n)\equiv\frac{\gamma_n^{(0)}}{2\beta_0}-\frac{\beta_1\gamma_n^{(1)}}{2\beta_0^2}+c_n^{(1)}=c_0+c_1\log n+8/3\log^2 n\,, \qquad (5.2)$$

where $\beta_1=102-\frac{38}{3}f$, $c_0=-1.18$, $c_1=0.66$.

Note, that the running coupling constant $a_s(Q^2)$ is defined now as [20]

$$a_s^{(2)}(Q^2)=a_s(Q^2)=\frac{4\pi}{\beta_0\log(Q^2/\Lambda^2)}\left[1-\frac{\beta_1}{\beta_0^2}\frac{\log\log(Q^2/\Lambda^2)}{\log(Q^2/\Lambda^2)}\right]. \qquad (5.3)$$

Notice, that the two-loop calculations depend in general on the choice of the renormalization scheme and violate the reciprocity relation of ref. [11]. We must take into account a difference arising in higher orders between the functions $F_2^{(2)}(x,Q^2)$ and $D^{(2)}(z,Q^2)$.

Taking into account the boundary conditions (2.3) and inverting the moments** (5.1), (5.2) we receive for the valence quark distribution in hadron

$$xF_2^{(2)}(x,Q^2)=\frac{c_A K_A(\xi)}{\Gamma(\tilde{A})}(1-x)^{\tilde{A}-1}\exp\{a_s\cdot a'(A)\}\left[1+\frac{2x_s}{3\pi}\log^2(1-x)\right], \qquad (5.4)$$

where

$$a'(A)=c_0-2/3\pi\cdot[\Psi^2(\bar{A})+\Psi'(\bar{A})],$$

$$\tilde{A}=\bar{A}-a_s(Q^2)\cdot b(\bar{A})=2(n_V^A-1)+r\,\xi-a_s\cdot b(\bar{A}),$$

$$b(\bar{A})=c_1+4/3\pi\cdot\Psi(\bar{A}),\quad \xi=-\log a_s(Q^2);$$

---

* Here and through the paper in scheme $\overline{MS}$ and $f=4$.

** See also ref. [21].

Factor $K_A(\xi)$ is defined by (3.2), but the constants $c_0(c_1)$ are given in $\overline{\text{MS}}$ scheme by eq. (5.2), and $\Psi(z) = d\ln\Gamma(z)/dz$ and $\Psi'(z)$ are Euler's digamma function and its derivative, respectively.

Note, that evolution development up to $O(\alpha_s^2)$ of the structure function can be represented in the form

$$\frac{\partial\tilde{A}}{\partial\log Q^2} \simeq \alpha_s \cdot 4r - \alpha_s^2 (a(\bar{A}) + \frac{4}{3\pi}\Psi(\bar{A})), \tag{5.5}$$

with "boundary" condition

$$\tilde{A}|_{Q^2=Q_0^2} = A = 2(n_A - 1),$$

where

$$a(\bar{A}) = c_0 + c_1\Psi(\bar{A}) + \frac{2}{3\pi}[\Psi^2(\bar{A}) - \Psi'(\bar{A})],$$

that guarantees in the final result square dependence on the anomalous dimension d_n^{NS}.

To calculate the jet production cross section (3.1) in the second order, we rewrite now (5.4) in the following form:

$$F^{(2)}_{a/A}(x_a, Q^2) = F_{a/A}(x_a, Q^2)\cdot e^{\alpha_s(Q^2)[a(\bar{A}) - b(\bar{A})\log(1-x_a)]} \times$$
$$\times[1 + \frac{2\alpha_s}{3\pi}\log^2(1-x_a)] \tag{5.6}$$

and $F_{a/A}(x_a, Q^2)$ is the usual one-loops result (3.2).

Inserting this expression into eq. (3.1) and cyrraing out the integrations, we obtain for $x_T \sim 1$ and $\Theta = 90°$:

$$\sigma^{(2)}(AB\to JX) = \sigma^{(1)}(AB\to JX)\{1 + 2\alpha_s(p_T^2)R(\bar{A},\bar{B}) + O(\alpha_s^2)\}, \tag{5.7}$$

where $\sigma^{(1)} \equiv E\,d\sigma/d^3p(AB\to JX)$ is given by (3.4) and

$$R(\bar{A},\bar{B}) \equiv c_0 - \frac{1}{3\pi}[\Psi^2(\bar{A}) + \Psi^2(\bar{B}) + \Psi'(\bar{A}) + \Psi'(\bar{B})] +$$
$$+\Psi(\bar{A}+\bar{B})[c_1 + \frac{2}{3\pi}\Psi(\bar{A}) + \frac{2}{3\pi}\Psi(\bar{B})] -$$
$$-[c_1 + \frac{4}{3\pi}\Psi(\bar{A}+\bar{B})]\log\epsilon' + \frac{2}{3\pi}[\log^2\epsilon' + I(\bar{A},\bar{B})] \tag{5.8}$$
$$\epsilon' = (1-x_T)/(1-x_T/2)$$

and the function $I(\bar{A},\bar{B})$:

$$I(\bar{A},\bar{B}) = \frac{1/2}{B(\bar{A},\bar{B})F_1} \cdot \int_0^1 \frac{dV\, V^{\bar{B}-1}(1-V)^{\bar{A}-1}}{\left[1-\frac{\epsilon' V}{1-x_T/2}\right]^2 [1-\epsilon' V]^2} \left[\log^2 V + \right.$$

$$\left. + \log^2 \frac{1-V}{1-\frac{\epsilon' V}{1-x_T/2}}\right],$$

reduces to

$$2I(\bar{A},\bar{B}) = [\Psi(\bar{A}) - \Psi(\bar{A}+\bar{B})]^2 + [\Psi(\bar{B}) - \Psi(\bar{A}+\bar{B})]^2 + \\ + \Psi'(A) + \Psi'(B) - 2\Psi'(A+B) + O(\alpha_s^2). \tag{5.9}$$

Collecting the factors (5.8) and (5.9), we obtain

$$\sigma^{(2)}(AB\to JX) = \sigma^{(1)}(AB\to JX)\cdot\left[1 + 2\frac{2\alpha_s(p_T^2)}{3\pi}\log^2 2\epsilon\right] \times \\ \times \exp\{2\alpha_s(p_T^2)[a(\bar{A}+\bar{B}) - b(\bar{A}+\bar{B})\log 2\epsilon]\}. \tag{5.10}$$

Performing similar calculations in the case of single particle cross section, we can formulate the general rule:

$$\sigma^{(2)}(AB\to CX) = \sigma^{(1)}(AB\to CX)\cdot\exp\{\alpha_s A(\bar{s}_p)\}\cdot\left[1 + \frac{2\alpha_s}{3\pi} H \log^2 2\epsilon\right], \\ A(\bar{s}_p) = [a(\bar{s}_p) - b(\bar{s}_p)\log 2\epsilon] + \frac{2\pi}{3} H_T \ , \tag{5.11}$$

and

$$\sigma^{(1)}(AB\to CX) \simeq \frac{(\alpha_s)^{2-2r\ln 2}}{p_T^4} \cdot \frac{\epsilon^{s_p-1}}{\Gamma(s_p)} \cdot [\alpha_s(p_T^2)]^{HD(s_p)},$$

where $\bar{s}_p = s_p + H\cdot r\,\xi$, and all other quantities in these expressions are defined above and factor $\exp[\frac{2\pi}{3}\alpha_s H_T]$ arises due to the difference in higher order between the quark distribution and fragmentation functions. This fact is related with the transition from the spacelike to timelike values of variable Q^2 /22/, and H_T is the total number of active quarks in subprocesses, where $q^2 = -Q^2 > 0$*.

---

* Formula (5.10) represents the particular case of eq.(5.11) for $s_p = A+B$, $H=2$ and $H_T = 0$.

Thus, taking into account the next to leading QCD corrections (at least to the nonsinglet quark distribution and fragmentation functions) does not violate the universal character of the anomalous dimension counting rules/7/.

Note, that addition to these calculations of the second loop corresponds to the expansion of the effective α_s exponent in anomalous dimension series $(\alpha_s)^{\mathfrak{D}}$,

$$\mathfrak{D} = D(s_p) + c\,\alpha_s D^2(s_p) \pm \ldots .$$

6. EFFECTIVE POWER AND p_T^{-4} PROBLEM

The a.d.q.c. rules give the algorithm for calculation of the hadron hard scattering asymptotics up to one-loop logarithmic QCD corrections to the canonical point-like p_T^{-4} law.

Let us consider an arbitrary cross section for the large p_T jet (single hadron) production

$$\sigma(AB \to C) \sim p_T^{-4}\,\Phi(x_T)\cdot\alpha_s^m(p_T^2), \tag{6.1}$$

where, $m = 2 - 2r\ln 2 + HD(s_p)$ controls the magnitude of QCD scale violation in hadron collisions.

Calculation of the effective power

$$\sigma(AB \to C) \sim p_T^{-n_{eff}}, \tag{6.2}$$

is rather simple because the invariant cross section (6.1) has the form of the structure function moments (2.4), satisfying the evolution equation

$$\frac{\partial \log[p_T^4\sigma(n, p_T^2)]}{\partial \log p_T^2} = \frac{\alpha_s(p_T^2)}{2\pi} d(n), \tag{6.3}$$

and thus, effective power (6.2) can be expressed in terms of the hadron structure corrections to the hard scattering of quarks in QCD.

For the phenomenological purposes we exploite the following definition of the n_{eff} at fixed values of x_T variable:

$$n_{eff}\ (x_T = \text{fixed}) = \log\frac{\sigma(s_1, p_T^{(1)})}{\sigma(s_2, p_T^{(2)})} \Big/ \log\frac{\sqrt{s_1}}{\sqrt{s_2}},$$
$$x_T = \frac{2p_T^{(1)}}{\sqrt{s_1}} = \frac{2p_T^{(2)}}{\sqrt{s_2}} \tag{6.4}$$

which corresponds to the two different measurements at energy-momentum s_1, $p_T^{(1)}$ and $s_2, p_T^{(2)}$ respectively. According to this definition the total effective power at large x_T values is

$$n_{eff}(x_T) = 4 - 2[2 - 2r\log 2 + HD(s_p)] \cdot R(\frac{s_1}{s_2}), \tag{6.5}$$

where

$$D(s_p) = d(s_p) - r(\frac{1}{s_p} + \log(1 - x_T)),$$

and quark resolution function at energies s_1 , s_2 is

$$R(\frac{s_1}{s_2}) = \frac{\log(\alpha_s(s_1)/\alpha_s(s_2))}{\log(s_1/s_2)}. \tag{6.6}$$

Thus, we obtain a parameter-free solution of the p_T^{-4} problem in hadron collisions*. (The only parameter in eqs.(6.5)-(6.6) is the running constant scale $\Lambda_{\overline{MS}}$, which is fixed in DIS). Note, that this solution is expressed in terms of the perturbative logarithmic corrections to the dimensional power asymptotics. The magnitude of these short distance corrections is governed by the quark content of the hadrons in reaction. The deviation from the p_T^{-4} point-like behaviour is larger for a larger total number of passive constituents of the hadrons. See for illustration Table 3 and Figs.5-6**.

Notice, the interesting consequence of these results adopting to the different cross section ratios. In the case when the hadrons, participating in hard scattering reactions differ by some definite number of quarks, it is useful to formulate the following quark interval rule,e.g., for the one-quark differentiated processes:

$H=2$,	$s_p=$;	$H=3$,		s_p =
$\pi\pi \to$ jet,	4	$\pi\pi \to \pi$		6
$\pi p \to$ jet,	6	$\pi p \to \pi$		8
$pp \to$ jet,	8	$pp \to \pi$		10
		$pp \to p$,	etc.	12

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*Note, we do not consider here the transverse momentum smearing effects, which have a little effect on the results, obtained in the large p_T and x_T region.

** Note, that we use the $Q^2 = -\hat{t}$ variable.

Table 3

The p_T power n_{eff} defined for fixed x_T by $E\,d\sigma/d^3p$ $(pp \to \pi^o, jet+x) \sim p_T^{-n_{eff}}$ (see eqs. (6.5) and (6.6)). The two values of n_{eff} at each energy correspond to $\Lambda = 0.1$ GeV/c and $\Lambda = 0.5$ GeV/c (lower value), respectively

$pp \to \pi^o + X$

S (GeV²) \ x_T	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
10^2	5.23 5.97	5.29 6.05	5.35 6.15	5.41 6.26	5.50 6.39	5.59 6.55	5.72 6.75	5.90 7.04	6.21 7.53
10^3	4.99 5.41	5.03 5.47	5.08 5.53	5.13 5.61	5.20 5.71	5.28 5.82	5.38 5.96	5.52 6.17	5.77 6.52
10^4	4.81 5.08	4.85 5.12	4.89 5.18	4.93 5.24	4.98 5.31	5.05 5.39	5.13 5.51	5.25 5.66	5.46 5.93
10^5	4.69 3.88	4.72 4.91	5.76 4.96	4.79 5.01	4.84 5.06	4.90 5.14	4.97 5.23	5.07 5.36	5.24 5.58

$pp \to jet + X$

S (GeV²) \ x_T	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
10^2	5.14 6.34	5.07 5.99	5.06 5.88	5.07 5.84	5.10 5.86	5.15 5.90	5.22 5.99	5.32 6.14	5.51 6.43
10^3	4.85 5.37	4.82 5.27	4.82 5.24	4.83 5.24	4.87 5.27	4.91 5.32	4.96 5.39	5.05 5.51	5.20 5.72
10^4	4.67 4.96	4.66 4.91	4.66 4.91	4.68 4.92	4.70 4.95	4.74 4.99	4.79 5.06	4.86 5.15	4.99 5.32
10^5	4.54 4.74	4.55 4.72	4.56 4.72	4.57 4.74	4.60 4.76	4.63 4.80	4.67 4.85	4.73 4.93	4.84 5.07

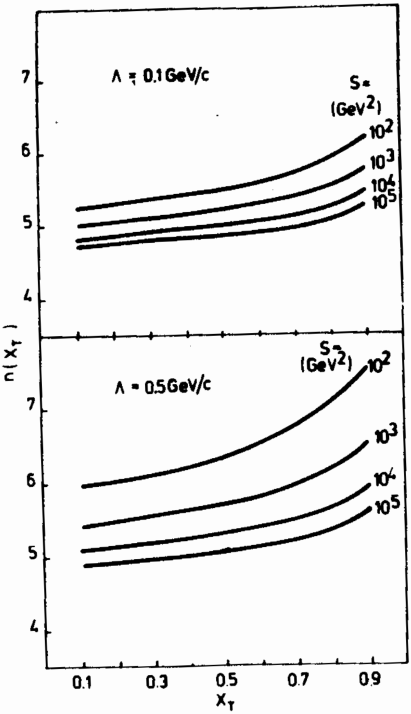

Fig.5. Effective p_T power $n_{eff}(x_T)$ (see eqs. (6.5) and (6.6)) of the invariant cross-section for π° -meson production in pp-collision as a function of x_T, calculated for two QCD scale $\Lambda=0.1$ GeV/c and $\Lambda=0.5$ GeV/c, respectively

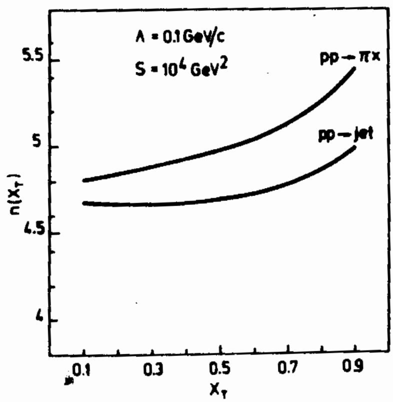

Fig.6. Comparative curves, as in Fig.5, for jet and single π-production in pp-collision.

we have

$$R\left[\frac{\sigma(s_p+2)}{\sigma(s_p)}\right]=\frac{\epsilon^2}{(s_p+1)(s_p+2)}[\alpha_s(p_T^2)]\,,\,{}^{\Delta D(s_p)\cdot H}_{x_T-\text{fixed}} \tag{6.7}$$

where

$$\Delta D(s_p)=D(s_p+2)-D(s_p)=$$

$$=r\left[\frac{2s_p+3}{(s_p+1)(s_p+2)}++\frac{2}{s_p(s_p+2)}\right]\simeq\frac{2r}{s_p}=\frac{8}{s_p}\gamma_F \tag{6.8}$$

and $\gamma_F = 4/25$ is the quark anomalous dimension (Feynman gauge). As an illustration of this rule consider here the following ratios:

$$\begin{array}{ll}
\text{beam ratio} & \left(\frac{pp\to\pi}{\pi p\to\pi}\right) = \frac{(1-x_T)^2}{9\cdot 10}\,[\alpha_s(p_T^2)]^{3\gamma_F} \\[2ex]
\text{particle ratio} & \left(\frac{pp\to p}{pp\to\pi}\right) = \frac{(1-x_T)^2}{11\cdot 12}[\alpha_s(p_T^2)]^{\frac{12}{5}\gamma_F} \\[2ex]
\text{particle/jet ratio} & \left(\frac{pp\to\pi}{pp\to \text{jet}}\right) = \frac{(1-x_T)^{2+4\gamma_F\xi}}{9\cdot 10}[\alpha_s(p_T^2)]^{11\gamma_F}
\end{array} \qquad (6.9)$$

These ratios can be tested experimentally in measurements at different energies and transverse momentum. Due to the parameterless nature of these predictions, a full phenomenological analysis of experimental data will be very desirable.

7. CONCLUDING REMARKS

Starting from the QCD evolution of the quark and gluon distribution (fragmentation) functions and exploiting the spectator counting rules for the initial x-dependence of $F(x,Q_0^2)$ and $D(z,Q_0^2)$ at $Q^2 = Q_0^2$ we obtain in leading log approximation the a.d.q.c. rules for any arbitrary inclusive hadron reactions at large p_T involving all the possible QCD hard scattering subprocesses and singlet distributions.

Analysis of the next to the leading order QCD corrections to the quark evolution does not destroy the universality of these rules. On this base, QCD gives a unique parameter-free predictions ($p_T^{-n_{eff}}$ -solution) for a wide class of hard processes.

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APPENDIX I

Let us assume that the initial valence, "sea" quark and gluon distributions at $Q^2 = Q_0^2$ at large x have the form:

$$x q_V(x,Q_0^2) = c_V(1-x)^{2n_V-3}$$

$$x G(x,Q_0^2) = c_G(1-x)^{2n_V-1} \qquad \text{(AI.1)}$$

$$x q_s(x,Q_0^2) = c_{q_s}(1-x)^{2n_V+1}.$$

The solutions of the evolution equations for an arbitrary Q^2 receive then the following form:

1. Valence quarks

$$x q_V(x,Q^2) = c_V(\xi)\frac{(1-x)^{2n_V-3+r\xi}}{\Gamma(2n_V-2+r\xi)}$$

$$c_V(\xi) = c_V \exp\{r\xi(3/4-\gamma_E)\}\cdot\Gamma(2n_V-2) \qquad \text{(AI.2)}$$

and

$$r = \frac{16}{33-2f}, \quad \xi = \ln[\alpha_s(Q_0^2)/\alpha_s(Q^2)], \quad \lambda = \gamma_E - \frac{21-2f}{20};$$

2. "Sea" quarks

$$x q_s(x,Q^2) = F_{q_sV} + F_{q_sG} + F_{q_sq_s}$$

$$\text{i) } F_{q_sV} = c_{q_sV}(\xi)\frac{(1-x)^{2n_V-1+r\xi}}{\Gamma(2n_V+r\xi)[\ln\frac{1}{1-x}+\Psi(2n_V+r\xi)+\lambda]} \qquad \text{(AI.3)}$$

$$c_{q_sV}(\xi) = c_V\, 3/40\, r\xi \exp\{r\xi(\tfrac{3}{4}-\gamma_E)\}\cdot\Gamma(2n_V-2)$$

$$\text{ii) } F_{q_sG} = c_{q_sG}(\xi)\frac{(1-x)^{2n_V+r\xi}}{\Gamma(2n_V+1+r\xi)[\ln\frac{1}{1-x}+\Psi(2n_V+1+r\xi)+\lambda]} \qquad \text{(AI.4)}$$

$$-\bar{c}_{q_sG}(\xi)\frac{(1-x)^{2n_V+9/4\,r\xi}}{\Gamma(2n_V+1+\frac{9}{4}r\xi)[\ln\frac{1}{1-x}+\Psi(2n_V+1+\frac{9}{4}r\xi)+\lambda]},$$

$$c_{q_sG}(\xi) = \frac{3}{20}c_G \exp\{r\xi(\tfrac{3}{4}-\gamma_E)\}\cdot\Gamma(2n_V)$$

$$\bar{c}_{q_sG}(\xi) = \frac{3}{20}c_G \exp\{r\xi(\tfrac{1}{r}-\gamma_E)\}\Gamma(2n_V)$$

$$\text{iii) } F_{q_sq_s} = c_{q_sq_s}(\xi)\frac{(1-x)^{2n_V+1+r\xi}}{\Gamma(2n_V+2+r\xi)}, \qquad \text{(AI.5)}$$

$$c_{q_sq_s}(\xi) = c_{q_s}\exp\{r\xi(3/4-\gamma_E)\}\Gamma(2n_V+2)$$

3. Gluons

$$xG(x,Q^2) = F_{GV} + F_{Gq_s} + F_{GG}$$

$$\text{i) } F_{GV} = c_{GV}(\xi)\frac{(1-x)^{2n_V-2+r\xi}}{\Gamma(2n_V-1+r\xi)[\ln\frac{1}{1-x}+\Psi(2n_V-1+r\xi)+\lambda]} - \tag{AI.6}$$

$$-\bar{c}_{GV}(\xi)\frac{(1-x)^{2n_V-2+9/4r\xi}}{\Gamma(2n_V-1+\frac{9}{4}4r\xi)[\ln\frac{1}{1-x}+\Psi(2n_V-1+\frac{9}{4}r\xi)+\lambda]},$$

$$c_{GV}(\xi) = \frac{2}{5}c_v \exp\{r\xi(3/4-\gamma_E)\}\cdot\Gamma(2n_V-2),$$

$$\bar{c}_{GV}(\xi) = \frac{2}{5}c_v \exp\{r\xi(\frac{1}{r}-\frac{9}{4}\gamma_E)\}\cdot\Gamma(2n_V-2)$$

$$\text{ii) } F_{Gq_s} = c_{Gq_s}(\xi)\frac{(1-x)^{2n_V+2+r\xi}}{\Gamma(2n_V+3+r\xi)[\ln\frac{1}{1-x}+\Psi(2n_V+3+r\xi)+\lambda]} - \tag{AI.7}$$

$$-\bar{c}_{Gq_s}(\xi)\frac{(1-x)^{2n_V+2+9/4r\xi}}{\Gamma(2n_V+3+\frac{9}{4}r\xi)[\ln\frac{1}{1-x}+\Psi(2n_V+3+\frac{9}{4}r\xi)+\lambda]},$$

$$c_{Gq_s}(\xi) = \frac{4f}{5}c_{q_s}\exp\{r\xi(\frac{3}{4}-\gamma_E)\}\cdot\Gamma(2n_V+2),$$

$$\bar{c}_{Gq_s}(\xi) = \frac{4f}{5}c_{q_s}\exp\{r\xi(\frac{1}{r}-\frac{9}{4}\gamma_E)\}\cdot\Gamma(2n_V+2), \tag{AI.8}$$

$$\text{iii) } F_{GG} = c_{GG}(\xi)\frac{(1-x)^{2n_V-1+9/4r\xi}}{\Gamma(2n_V+\frac{9}{4}r\xi)},$$

$$c_{GG}(\xi) = c_G\exp\{r\xi(\frac{1}{r}-\frac{9}{4}\gamma_E)\}\cdot\Gamma(2n_V)$$

Neglecting the non-leading (at $x\to 1$) terms in (AI.1-8) we receive the set (2.5).

APPENDIX II

i) Hypergeometric function of two variables

$$F_1(\alpha,\beta,\beta',\gamma;x,y) = \frac{\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\beta')\Gamma(\gamma-\beta-\beta')}\iint\frac{u^{\beta-1}v^{\beta'-1}du\,dv}{(1-u-v)^{\beta+\beta'-\gamma+1}(1-ux-vy)^{\alpha}} =$$

$$= \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\gamma-\alpha)} \int_0^1 \frac{du\, u^{\alpha-1} (1-u)^{\beta-1}}{(1-ux)\,(1-uy)^{\beta'}} =$$

$$= \sum_{m,n} \frac{(\alpha)_{m+n}\,(\beta)_m\,(\beta')_n}{(\gamma)_{m+n}\, m!\, n!} x^m y^n, \quad (a)_n = \frac{\Gamma(a+n)}{\Gamma(a)} \tag{AII.1}$$

ii) $\int_0^1 dz\, z^{\alpha-1} \log z (1-z)^{\beta-1} = B(\alpha,\beta)[\Psi(\alpha)-\Psi(\alpha+\beta)]$,

$$\text{iii)} \int_0^1 dz \log^2 z\, z^{\alpha-1} (1-z)^{\beta-1} = B(\alpha,\beta)\{[\Psi(\alpha)-\Psi(\alpha+\beta)]^2 + \Psi'(\alpha)-\Psi'(\alpha+\beta)\} \tag{AII.2}$$

where $B(\alpha,\beta)$ is the Euler's Beta function, and

$$\Psi(z) = \frac{d \log \Gamma(z)}{dz}, \quad \Psi'(z) = \frac{d^2 \log \Gamma(z)}{dz^2};$$

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