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E2-82-194

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ASYMPTOTIC BEHAVIOUR OF MULTIPARTICLE DISTRIBUTION AND FRAGMENTATION FUNCTIONS IN QUANTUM CHROMODYNAMICS

1982

The methods of perturbative QCD are widely used to successfully describe the hard processes[1]. All perturbative calculations are carried out for free quarks and gluons, and the conversion of these colour objects into hadrons is now described phenomenologically using the soft bleaching hypothesis. The factorization of soft and hard stages (physics of short and long distances) and the independence of the kind of a processes makes it possible to use the universal distribution and fragmentation functions which play a decisive role to determine the features of hard processes. In ref.[2], we obtained the equations describing Q^2-evolution of multiparton distribution and fragmentation functions utilizing the parton interpretation of perturbative QCD theory diagrams in the leading logarithm approximation[3]. The equations obtained are not identical, but the solutions are the same under definite initial conditions and coincide with the jet calculus rules[4]. The standard conjectures about hadronization enable us to generalize these equations for describing the process of parton fragmentation into hadrons. And it was shown in[2] that the solutions of these equations contain as initial conditions the phenomenological multihadron fragmentation functions, in the same way as it is necessary to set initial distribution of partons in a hadron to have multiparton distribution in a hadron.

In the present note we investigate the dependence of the asymptotic ($Q^2 \to \infty$) behaviour of distribution and fragmentation functions on the initial conditions (which generally speaking, are arbitrary and must be extracted from experimental data). It is shown that multihadron fragmentation functions "remember" the initial correlations in the region of finite x, unlike the distribution functions which "forget" the initial conditions. But new phenomenological functions are unnecessary to define correlation of an average hadron number in a jet, whereas it is necessary to know the initial distribution of partons in a hadron for defining their average number.

Attempting to make a complete interpretation here we give the equations for one-and two-particle distribution and fragmentation functions. Then we consider the basic asymptotic properties of multiparticle distribution and fragmentation functions obtained as solutions to corresponding equations by exploiting an example of the model with one sort of partons. Eventually we present the result of investigation of these properties in quantum chromodynamics.

The one-parton distribution function $D_h^j(x,t)$ defines the probability to find a parton of sort j (j means a quark, an antiquark, a gluon) with fraction x of the longitudinal momentum in a hadron h and with any virtuality up to Q^2. These functions satisfy the equations [3,4,5]

$$\frac{\partial D_h^j(x,t)}{\partial t} = \sum_{j'} \int_x^1 \frac{dx'}{x'} D_h^{j'}(x,t) P_{j'\to j}\left(\frac{x}{x'}\right), \qquad (1)$$

where

$$t = \frac{1}{2\pi b} \ln\left[1 + \frac{g^2(\mu^2)}{4\pi} b \ln\frac{|Q^2|}{\mu^2}\right]; \qquad b = \frac{33 - 2n_f}{12\pi}, \qquad (2)$$

n_f is the number of quark flavours and μ^2 is the distinctive value of virtuality for which the running coupling constant is perfectly small to use the perturbation theory. The explicit form of the kernels $P(x)$ and their moments

$$P_{j'\to j}(n) = \int_0^1 x^n P_{j'\to j}(x)\,dx \qquad (3)$$

is well known and may be borrowed, for example, from papers [1,4,5].

The two-parton distribution functions $D_h^{j_1 j_2}(x_1, x_2, t)$ define the probability of finding in a hadron h partons of type j_1, j_2 with fractions x_1, x_2 of the longitudinal momentum and with any virtuality up to Q^2. The equations for these functions were obtained in ref. [2] by Lipatov's method [3]. Here we write down the equations for the moments only

$$M_h^{j_1 j_2}(n_1, n_2, t) = \int_0^1 x_1^{n_1} x_2^{n_2}\, \theta(1 - x_1 - x_2) D_h^{j_1 j_2}(x_1, x_2, t)\, dx_1 dx_2, \qquad (4)$$

$$\frac{\partial M_h^{j_1 j_2}(n_1,n_2,t)}{\partial t} = \sum_{j_1'} M_h^{j_1' j_2}(n_1, n_2, t) P_{j_1'\to j_1}(n_1) +$$
$$+ \sum_{j_2'} M_h^{j_1 j_2'}(n_1,n_2,t) P_{j_2'\to j_2}(n_2) + \sum_j M_h^j(n_1 + n_2, t) P_{j\to j_1 j_2}(n_1, n_2), \qquad (5)$$

where we introduced

$$M_h^j(n,t) = \int_0^1 x^n D_h^j(x,t)\,dx, \qquad (6)$$

and the explicit form of the kernel $P_{j\to j_1 j_2}(n_1, n_2)$ is defined with the probability that a parton of type j with fraction x of the longitudinal momentum decays into two partons of type j_1, j_2 one of which has fraction x_1 of the longitudinal momentum and may be found by the method of ref. [3].

The single particle fragmentation function $\bar{D}_i^h(x,t)$ defines the probability that the parton i with virtuality up to Q^2 fragments into a hadron h with fraction x of the parton longitudinal momentum. The moments of these functions

$$\bar{M}_i^h(n,t) = \int_0^1 x^n \bar{D}_i^h(x,t)\,dx \qquad (7)$$

satisfy the equations [4,6]

$$\frac{\partial \bar{M}_i^h(n,t)}{\partial t} = \sum_{i'} \bar{M}_{i'}^h(n,t) P_{i\to i'}(n). \qquad (8)$$

The two-particle fragmentation functions $\bar{D}_i^{h_1 h_2}(x_1, x_2, t)$ define the probability that a parton of type i with virtuality up to Q^2 fragments into two hadrons h_1, h_2 with fractions x_1, x_2 of the longitudinal momentum. The equations for the moments of these functions

$$\bar{M}_i^{h_1 h_2}(n_1, n_2, t) = \int_0^1 x_1^{n_1} x_2^{n_2}\, \theta(1 - x_1 - x_2) \bar{D}_i^{h_1 h_2}(x_1, x_2, t)\, dx_1 dx_2 \qquad (9)$$

are of the following form [2,6,7,8]

$$\frac{\partial \bar{M}_i^{h_1 h_2}(n_1,n_2,t)}{\partial t} = \sum_{i'} \bar{M}_{i'}^{h_1 h_2}(n_1, n_2, t) P_{i\to i'}(n_1 + n_2) +$$
$$+ \sum_{i_1 i_2} \bar{M}_{i_1}^{h_1}(n_1, t) \bar{M}_{i_2}^{h_2}(n_2, t) P_{i\to i_1 i_2}(n_1, n_2). \qquad (10)$$

In Ref.[2], we found that the solutions of Eqs. (5) and (10) may be put down in the forms

$$M_h^{j_1j_2}(n_1,n_2,t)=\sum_{j_1'j_2'} M_h^{j_1'j_2'}(n_1,n_2,0)M_{j_1'}^{j_1}(n_1,t)M_{j_2'}^{j_2}(n_2,t)+$$

$$+\sum_{jj_1'j_2'}\int_0^t dt' M_h^{j}(n_1+n_2,t')P_{j\to j_1'j_2'}(n_1,n_2)M_{j_1'}^{j_1}(n_1t-t')M_{j_2'}^{j_2}(n_2t-t'), \qquad (11)$$

$$\bar M_i^{h_1h_2}(n_1,n_2,t)=\sum_j \bar M_j^{h_1h_2}(n_1,n_2,0)\bar M_i^{j}(n_1+n_2,t)+$$

$$+\sum_{j,j_1,j_2}\int_0^t dt'\bar M_i^{j}(n_1+n_2,t-t')P_{j\to j_1j_2}(n_1,n_2)\bar M_{j_1}^{h_1}(n_1,t')\bar M_{j_2}^{h_2}(n_2,t'), \qquad (11')$$

where $M_h^{j_1j_2}(n_1,n_2,0)$ is the initial condition for distribution functions, $\bar M_i^{h_1h_2}(n_1,n_2,0)$ is the initial condition for fragmentation functions and $M_i^j(n,t)$, $\bar M_i^j(n,t)$ are the moments of distribution and fragmentation function at the parton level for which the following initial conditions are valid

$$M_i^j(n,t=0)=\bar M_i^j(n,t=0)=\delta_{ij}\ . \qquad (12)$$

It may be seen already from (11) and (11') that the initial conditions for distribution and fragmentation functions are provided in the solutions with the different t - dependences. We shall now demonstrate the character of this dependence at first in a toy model with one type of partons only (for instance, ϕ^3-model in 6-dimensional space or QCD without fermions). In this case the expressions (11) and (11') become simpler

$$M_h^2(n_1,n_2,t)=\frac{P(n_1,n_2)M_h^1(n_1+n_2,0)}{P(n_1+n_2)-P(n_1)-P(n_2)}\left[e^{P(n_1+n_2)t}-\right.$$

$$\left.-e^{[P(n_1)+P(n_2)]t}\right]+M_h^2(n_1,n_2,0)e^{[P(n_1)+P(n_2)]t}, \qquad (13)$$

$$\bar M^{h_1h_2}(n_1,n_2,t)=\frac{P(n_1,n_2)\bar M^{h_1}(n_1,0)\bar M^{h_2}(n_2,0)}{P(n_1+n_2)-P(n_1)-P(n_2)}\left[e^{P(n_1+n_2)t}-\right.$$

$$\left.-e^{[P(n_1)+P(n_2)]t}\right]+\bar M^{h_1h_2}(n_1,n_2,0)e^{P(n_1+n_2)t}. \qquad (13')$$

The asymptotic behaviours of distribution and fragmentation functions will be defined by the smaller values of $P(n_1)+P(n_2)$ or $P(n_1+n_2)$ and thus we have two different asymptotic regimes.

1) $P(n_1)+P(n_2)>P(n_1+n_2)$, in this case have

$$M_h^2(n_1,n_2,t)|_{t\to\infty}=\left[M_h^2(n_1,n_2,0)+\frac{P(n_1,n_2)M_h^1(n_1+n_2,0)}{P(n_1)+P(n_2)-P(n_1+n_2)}\right]e^{[P(n_1)+P(n_2)]t}, \qquad (14)$$

$$\bar M^{h_1h_2}(n_1,n_2,t)|_{t\to\infty}=\frac{P(n_1,n_2)\bar M^{h_1}(n_1,0)\bar M^{h_2}(n_2,0)}{P(n_1)+P(n_2)-P(n_1+n_2)}e^{[P(n_1)+P(n_2)]t}. \qquad (14')$$

We can see that the asymptotic behaviour of the fragmentation functions is independent of the initial conditions $\bar M^{h_1h_2}(n_1,n_2,0)$, unlike the asymptotic behaviour of distribution functions which contains the initial distribution of protons $M_h^2(n_1,n_2,0)$.

2) $P(n_1)+P(n_2)<P(n_1+n_2)$, on the contrary, in this regime the distribution functions are independent of the initial conditions, but the fragmentation functions "remember" their initial conditions

$$M_h^2(n_1,n_2,t)|_{t\to\infty}=\frac{P(n_1,n_2)M_h^1(n_1+n_2,0)}{P(n_1+n_2)-P(n_1)-P(n_2)}e^{P(n_1+n_2)t}, \qquad (15)$$

$$\bar M^{h_1h_2}(n_1,n_2,t)|_{t\to\infty}=\left[M^{h_1h_2}(n_1,n_2,0)+\frac{P(n_1,n_2)\bar M^{h_1}(n_1,0)\bar M^{h_2}(n_2,0)}{P(n_1+n_2)-P(n_1)-P(n_2)}\right]\times$$

$$\times e^{P(n_1+n_2)t} \qquad (15')$$

Using the explicit form of the kernels $P(n)$ in ϕ^3-model[3]

$$P(n)=\frac{1}{(n+2)(n+3)}-\frac{1}{12} \qquad (16)$$

we have that $P(0)+P(0)>P(0+0)$, but $P(n_1)+P(n_2)<P(n_1+n_2)$ for $n_1,n_2\geq 2$, it means that both asymptotic regimes are realized. Therefore, the behaviour of distribution functions and fragmentation functions even at large t depends on the initial conditions if we are dealing with these in the whole range of x_1,x_2. However, if we are interested in finite x_1 and x_2 , that is, the region defined by large moments n_1 and n_2 where only one of the asymptotic regimes is realized, we have the dependence of the fragmentation functions on the initial conditions and the independences of these of the distribution functions. Recalling that the zero moments define the average number of hadrons in a jet for the fragmentation function and the average of partons in a hadron for the distribution function, we get the opposite picture for these quantities, namely, the correlation of an average parton number in a hadron depends on the initial correlations whereas such a dependence for the correlations of an average hadron number in a jet is absent in the region of large t .

It is easy to understand that the asymptotic regime of distribution and fragmentation functions will be determined with an analogous relation between the maximum eigenvalues of $\Lambda(n_1+n_2)$ and $\Lambda(n_1)+\Lambda(n_2)$, which are found from the equations for single-particle distribution and fragmentation functions when we have several types of partons. For this purpose it is enough to express the single-particle functions through the eigenvalues of appropriate equation to substitute them into (11) and (11'), and to take into account the leading contributions only. Now we get two asymptotic regimes of the behaviour again.

1) $\Lambda(n_1)+\Lambda(n_2)>\Lambda(n_1+n_2)$, the asymptotic behaviour is thus independent of the initial data for the fragmentation function.

2) $\Lambda(n_1)+\Lambda(n_2)<\Lambda(n_1+n_2)$, we have the asymptotic behaviour independence of initial data for the distribution functions.

Using the explicit form of the kernels $P(n)$ and the expressions for the eigenvalues $\Lambda(n)$ in QCD [1,7,8] , we have the realization of both regimes of asymptotic behaviour, as in the model example. Therefore, the prediction of the distribution and fragmentation functions in the whole region of x_1 and x_2 changing even at large t demands setting the initial conditions which are extracted from experimental data. But as in the ϕ^3-model, we have the relations $\Lambda(n_1+n_2)>\Lambda(n_1)+\Lambda(n_2)$ (at large n it will be $\Lambda(n)\sim -\ln n$) in the region of large moments, consequently for finite x_1,x_2 the distribution functions "forget" the initial conditions, whereas the fragmentation functions do not. The same result for the

fragmentation functions was discovered in ref [7] . The interesting point is the relation between $\Lambda(0)+\Lambda(0)$ and $\Lambda(0+0)$ that defines the influence of the initial conditions on the asymptotic behaviour of the correlations between the average parton and hadron numbers. Unfortunetely, in QCD the quantity $\Lambda(0)$ is infrared divergent, so it is impossible to state strictly, as in the ϕ^3-model, the dependence on (or independence of) the initial conditions. However, if we use the simplest regularization of the dimensional type when $n\to 0$, we get $\Lambda(n)+\Lambda(n)>\Lambda(2n)$, i.e., the asymptotic regime is available where the fragmentation functions "forget" the initial data. We may predict in this case asymptotic ratio for the correlations of the average hadron number in quark and gluon jets.

$$\left.\frac{M_q^{h_1h_2}(n,n,t)}{M_q^{h_1}(n,t)M_q^{h_2}(n,t)}\right|_{\substack{t\to\infty\\ n\to 0}} = \frac{3C_F+C_A}{3C_F} = \frac{7}{4} ,$$

$$\left.\frac{M_g^{h_1h_2}(n,n,t)}{M_g^{h_1}(n,t)M_g^{h_2}(n,t)}\right|_{\substack{t\to\infty\\ n\to 0}} = \frac{4}{3} , \qquad (17)$$

where $C_F=(N^2-1)/2N$, $C_A=N$, N is the colour number. The result is independent of the hadron type and coincides with a similar ratio for the fragmentation of partons into partons [4].

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Received by Publishing Department
on Marth 16 1982.

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Шелест В.П., Снигирев А.М., Зиновьев Г.М. E2-82-194

Асимптотическое поведение многочастичных функций распределения и фрагментации в квантовой хромодинамике

Исследовано влияние начальных условий на асимптотическое поведение двухчастичных функций распределения и фрагментации, получаемых как решения соответствующих уравнений. Показано, что в области конечных x функции распределения не зависят от начальных данных, в отличие от функций фрагментации, которые зависят от начальных корреляций, а для среднего числа адронов в струе зависимость от начальных данных исчезает, в то время как среднее число партонов в адроне зависит от их начального распределения.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1982

Shelest V.P., Snigirev A.M., Zinovjev G.M. E2-82-194

Asymptotic Behaviour of Multiparticle Distribution and Fragmentation Functions in Quantum Chromodynamics

The initial conditions dependence of asymptotic behaviour of two-particle distribution and fragmentation functions found as the solutions of appropriate equations is investigated. It is shown that in the region of finite x the distribution functions are independent of the initial data, unlike the fragmentation functions depending on the initial correlations. But the average hadron number in a jet is independent of the initial data, whereas the average parton number in a hadron depends on their initial distribution.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1982