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V.I.Inozemtsev

ON THREE-PARTICLE STATES  
IN TWO-DIMENSIONAL STATIC  
MIT BAG MODEL

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## 1. INTRODUCTION

Recent success in the description of static properties of hadrons within the MIT bag model<sup>/1/</sup> has raised considerable interest in the physical and mathematical structure of the model. One can distinguish two main ways of development of the model<sup>/2/</sup>: the description of light hadronic states composed of fermion fields in a spherical bag<sup>/2,3/</sup> and studies of the interaction between heavy quarks (considered as localized color charges) and color (electrostatic) gauge fields confined in the bag<sup>/4-7/</sup>. Along the second way a good description was obtained for the quark interaction in heavy mesons at long distances in the adiabatic approximation: quarks were treated as fixed sources of color (effectively Abelian with a small coupling constant) gauge fields. Boundary conditions of the model define both the bag surface (far from being spherical) and the field distribution inside the bag. The energy of configuration "bag + field" is then used as an effective potential in the Schrödinger equation for heavy quarks.

The basic mathematical problem in this approach is to find the bag surface on which color electric fields should obey the conditions

$$\vec{n} \vec{E}_j = 0, \quad (1a)$$

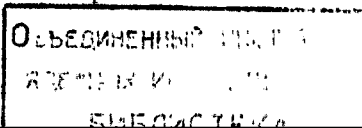
$$\frac{1}{2} \sum_j \vec{E}_j^2 = B, \quad (1b)$$

where  $B$  is the universal constant of the MIT bag model,  $\vec{n}$  is the unit normal to the surface. Inside the bag fields are solutions to the equations

$$\vec{\nabla} \vec{E}_j = \sum_a q_a^{(j)} \delta(\vec{r} - \vec{r}_a), \quad \vec{\nabla} \times \vec{E}_j = 0, \quad (2)$$

where  $q_a^{(j)}$  are static color charges and  $\vec{r}_a$  are their positions. The system "quark + antiquark" is well described by one gauge field<sup>/4/</sup>.

The problem is much complicated in the case of three-quark states ("heavy baryons") which require two effective Abelian fields  $\vec{E}_1, \vec{E}_2$ . The interaction of fields with color quarks forming a singlet state is defined by the color hypercharge and third projection of the color isospin<sup>/7/</sup>:



$$q_a^{(1)} = \{2q, -q, -q\}; \quad q_a^{(2)} = \{0, \sqrt{3}q, -\sqrt{3}q\}.$$

In the three-dimensional space the free-boundary problem (1a-b) can be solved only by a variational method. Results obtained for three-quark states by this approach<sup>6,7</sup> satisfactorily describe the dependence of an effective potential on adiabatic variables, separations between color sources, in two limiting cases. At small separations the bag shape was taken spherical and at large separations the bag shape was approximated by a system of finite cylinders. Considering only the two simplest forms, one obtains a tripod (Y) or a triangle ( $\Delta$ ) shape. It turned out however<sup>6,7</sup> that within the variational approach one has no way of distinguishing between the various configurations, i.e., of solving the question of the spatial structure of heavy baryons in the model.

In two space dimensions the problem (1a-b, 2) is essentially simplified as in this case fields  $E_j$  can be treated as analytic functions of the complex variable  $z = x + iy$  ( $x, y$  are space coordinates) with pole singularities at the positions of charges. An exact solution has been found for two sources<sup>4</sup> by conformal mappings. Apparently, exact solutions may be obtained for all problems of the type (1a-b, 2). The construction of such solutions is extremely nontrivial, however, it seems important for understanding the bag structure in three dimensions.

Here we analyse the three-particle problem in a two-dimensional bag. Particles are assumed to be sources of a gauge field.

The particle charges  $q_a$  are taken the same as  $q_a^{(1)}$  in the case of three quarks:  $q_a = \{2q, -q, -q\}$ . The study of this simplest three-particle model can be regarded as a first step towards the construction of exact solutions to an interesting but more complicated problem of three quarks interacting with two effective gauge fields. Our results may also help to gain an idea on the bag shape for some 4-quark states.

We shall consider 3-charges configurations symmetric relatively to, say, the  $x$ -axis, in the  $(x, y)$  plane. Throughout the paper we measure distances in units  $q(2B)^{-1/2}$  and energies in units  $q^2$ ; in these units the boundary conditions (1a-b) are

$$\vec{n} \cdot \vec{E} = 0, \quad (3a)$$

$$\vec{E}^2 = 1 \quad (3b)$$

$$\text{and } q_a = \{2, -1, -1\}.$$

A method for constructing solutions to the three-body problem is expounded in Sec. II. In Sec. III we present results and conclusions.

## II. METHOD OF SOLUTION IN TWO SPACE DIMENSIONS

The essential observation by R. Giles<sup>4</sup>, which allows an exact solution of the problem in the two-dimensional model, is that the two boundary conditions (3a-b) can be represented in the form

$$E e^{i\theta} = 1, \quad (4)$$

where  $E = E_x - iE_y$  is an analytic function of variable  $z = x + iy$ ,  $e^{i\theta}$  is the unit tangent to the surface with angle  $\theta$  defined relative to the  $x$ -axis. The function  $E(z)$  has simple poles at

the charges  $z_{q_1}, z_{q_2}, z_{q_3}$  ( $\sum_{a=1}^3 z_{q_a} = 0$ ) with residues

$$\text{res } E(z) \Big|_{z=\{z_{q_1}, z_{q_2}, z_{q_3}\}} = (2\pi)^{-1} \times \{2, -1, -1\}, \quad (5)$$

We assume that  $|E(z)| \geq 1$  inside the bag (this will be confirmed later). By eq. (4) the conformal mapping

$$z_1 = \frac{1}{E(z)} \quad (6)$$

maps the bag boundary  $G$  onto the unit-circle boundary; all the three points of location of charges are mapped into the center of the circle  $z_1 = 0$ ; the bag interior is mapped into cut unit circle; the cut being to range along the real axis from  $z_1 = -a_1$  to  $z_1 = 1$ ,  $0 < a_1 < 1$ .

Our aim is to find an inversion of the mapping (6)

$$z = \tilde{F}(z_1). \quad (7)$$

This transformation maps the unit circle in the  $z_1$ -plane onto the bag boundary in the  $z$  plane. The center of the circle should be mapped into three points in the  $z$  plane:  $z_{q_1}, z_{q_2}, z_{q_3}$ . Therefore,  $\tilde{F}(z_1)$  should have three branches, each being an analytic function in the cut unit circle in the  $z_1$ -plane. These three branches compose one function  $\tilde{F}(z_1)$  analytic on the three-sheeted Riemann surface. From (4,6) it follows<sup>4</sup> that on the boundary  $z_1 = e^{i\theta}$  the condition

$$\text{Re} \frac{d\tilde{F}}{dz_1} = 0 \quad (8)$$

should hold.

Owing to symmetry of the charge configuration relative to the  $x$ -axis, at least one of the branches  $\tilde{F}(z_1)$  should obey the condition

$$\operatorname{Im} \tilde{F}(z_1) = 0 \quad (9)$$

at real  $z_1$ .

Appropriate functions  $\tilde{F}(z_1)$  may be found by making one more transformation that maps the unit circle into the half-plane

$$z_2 = \frac{1+z_1}{1-z_1} \quad (10)$$

This maps the bag boundary to the axis  $\operatorname{Re} z_2 = 0$ , the circle center to the point  $z_2 = 1$ , the cut in the  $z_1$ -plane to the cut  $\{a \leq \operatorname{Re} z_2 < \infty, \operatorname{Im} z_2 = 0\}$ ,  $0 < a < 1$ .

Now we shall present the conditions to be satisfied by the function  $\tilde{F}(z_1(z_2)) \equiv F(z_2)$ . With the notation

$$\frac{dF}{dz_2} = \frac{\phi(z_2)}{\pi(1+z_2)^2} \quad (11)$$

the condition (8) reads

$$\operatorname{Re} \phi(z_2) = 0 \quad \text{at} \quad \operatorname{Re} z_2 = 0. \quad (8a)$$

In view of (9), at least one of the branches  $\phi^{(a)}(z_2)$  ( $a = 1, 2, 3$ ) of the function  $\phi(z_2)$  should obey the condition

$$\operatorname{Im} \phi^{(a)}(z_2) = 0 \quad \text{at} \quad \operatorname{Im} z_2 = 0. \quad (9a)$$

The residues of  $E(z) = \frac{z_2+1}{z_2-1}$  at points  $z_{q_1}, z_{q_2}, z_{q_3}$  are calculated by formula (5); calculating them with the use of (7, 10-11), we find that at  $z_2 = 1$  the branches of  $\phi(z_2)$  should satisfy the conditions

$$\phi^{(1)}(1) = 2, \quad \phi^{(2)}(1) = \phi^{(3)}(1) = -1. \quad (5a)$$

As  $F(z_2)$  is finite in the half-plane,  $|F(z_2)| < \text{const}$ ,  $\phi(z_2)$  has no poles:

$$\begin{aligned} |\phi(z_2)| &< |z_2|^{1-\beta}, \quad \beta > 0, \quad |z_2| \rightarrow \infty \\ |\phi(z_2)| &< |z_2 - z_2^{(0)}|^{\gamma-1}, \quad \gamma > 0 \end{aligned} \quad (12)$$

around any point of the half plane  $z_2^{(0)}$ .

By transformation  $z = F(z_2)$  the cut in the  $z_2$ -plane should not be mapped onto the bag boundary  $G$  in the  $z$ -plane. For this condition to be fulfilled it suffices that the point  $z_2 = a$  (the cut start) be the only branch point of  $\phi(z_2)$  at  $\operatorname{Re} z_2 \geq 0$ .

It is natural to look for the functions  $\phi(z_2)$  obeying the above conditions in the class of algebraic functions<sup>18/</sup>

$$\Phi(z_2, \phi) = P_0(z_2)\phi^3 + P_1(z_2)\phi^2 + P_2(z_2)\phi + P_3(z_2) = 0, \quad (13)$$

where  $P_a(z_2)$  are polynomials in  $z_2$ . All functions of this class have three branches, and branch points belong to a set of zeros of  $P_0(z_2)$  and discriminant  $\Delta(z_2)$ , a polynomial resulting from the elimination of  $\phi$  out of the equations

$$\Phi(z_2, \phi) = 0, \quad \frac{\partial \Phi}{\partial \phi}(z_2, \phi) = 0.$$

In accordance with (8a) and (12)  $P_1(z_2) \equiv 0$ ,  $P_2 P_0^{-1}$  is an even, and  $P_3 P_0^{-1}$  is an odd function of  $z_2$ . As  $z_2 = a$  is the only branch point of  $\phi(z_2)$  at  $\operatorname{Re} z_2 \geq 0$ , the polynomial  $P_0(z_2)$  has only one zero at  $\operatorname{Re} z_2 \geq 0$ :  $P_0(z_2) = (z_2^2 - a^2)^n$ ,  $n = 1, 2$ . All the three branches of  $\phi(z_2)$  should change into each other while going around the point  $z_2 = a$ ; for this reason and in view of (5a, 8a),  $n = 1$  is not possible, and the condition  $P_2(a) = 0$  takes place.

So, formula (13) can be rewritten as

$$\phi + 3p(z_2)\phi - 2q(z_2) = 0, \quad (14)$$

$$p(z_2) = \frac{s(z_2^2 - \lambda^2)}{z_2^2 - a^2}; \quad q(z_2) = \frac{r z_2(z_2^2 - \mu)}{(z_2^2 - a^2)^2}.$$

Here  $s, \lambda, r, \mu$  are real numbers; factors 3, -2 are introduced for convenience. For any different choice of polynomials  $P_2(z_2), P_3(z_2)$  in (13) one of the conditions (8a, 12) does not hold.

The coefficients  $s, \lambda, r, \mu, a$  are to be determined from (5a, 8-9a, 12). According to (5a)

$$p(1) = -1, \quad q(1) = 1. \quad (5b)$$

Note that (9a) is fulfilled automatically: at least one root of the cubic equation with real coefficients is real.

All roots of eq. (14) are imaginary on the axis  $z = ir$ ,  $\operatorname{Im} r = 0$  if  $p(ir) > 0$ , i.e.,

$$s > 0 \quad (15)$$

and the cubic equation discriminant  $\tilde{\Delta}(z_2) = p^3(z_2) + q^2(z_2)$  is non-negative at  $z_2 = ir$ . Possible branch point of  $\phi(z_2)$ , different from  $z_2 = a$ , belong to a set of zeros  $z_2^{(\beta)}$  of the polynomial  $\Delta(z_2)$ :

$$\Delta(z_2) = (z_2^2 - a^2)^4 \tilde{\Delta}(z_2) = s^3 (z_2^2 - \lambda^2)^3 (z_2^2 - a^2) + r^2 (z_2^2 - \mu)^2 z_2^2$$

(note that  $z_2 = 1$ , in view of (5b), belongs to the set  $\{z_2^{(\beta)}\}$ ). If in the vicinity of  $z_2^{(\beta)}$  the polynomial  $\Delta(z_2)$  can be represented as  $\Delta(z_2) = \text{const} (z_2 - z_2^{(\beta)})^{2n}$ , with integer  $n$  and

$|p(z_2^{(\beta)})| + |q(z_2^{(\beta)})| > 0$ , then at  $z_2 = z_2^{(\beta)}$  there is no branching of solutions to eq. (14). Hence,  $z = a$  is the only branch point of  $\phi(z_2)$  in the half-plane  $\text{Re } z_2 \geq 0$ , provided

$$\Delta(z_2) = \text{const } (z_2^2 - 1)^2 (z_2^2 - \gamma)^2 \quad \lambda^2 \neq \mu \quad (16)$$

with some real  $\gamma$ .

Conditions (5b, 15-16) determine the one-parameter family of functions  $\phi(z_2)$ :

$$s = \frac{(\lambda^2 - 1)^2}{\lambda^2(\lambda^2 + 3)^2}; \quad r = \frac{(\lambda^2 - 1)^2(3\lambda^4 + 6\lambda^2 - 1)}{\lambda^4(\lambda^2 + 3)^4} \quad (17)$$

$$a = \frac{1 + 3\lambda^2}{\lambda(\lambda^2 + 3)}; \quad \mu = \frac{\lambda^2(3 + 6\lambda^2 - \lambda^4)}{3\lambda^4 + 6\lambda^2 - 1}, \quad \lambda > 1$$

The branches of  $\phi(z_2)$  are standard solutions of cubic eq. (14) with parameters (17). The branches  $\phi^{(\alpha)}(z_2)$  on axes  $\text{Re } z_2 = 0$  and  $\text{Im } z_2 = 0$  are given in the Table with the notation used:

$$p(r) = \frac{(\lambda^2 - 1)^2(r^2 + \lambda^2)}{\lambda^2(\lambda^2 + 3)^2 r^2 + (1 + 3\lambda^2)^2}; \quad \cos \alpha = \frac{r[3\lambda^4 + 6\lambda^2 - 1] + \lambda^2(3 + 6\lambda^2 - \lambda^4)}{\{(r^2 + \lambda^2)^3[\lambda^2(\lambda^2 + 3)^2 r^2 + (1 + 3\lambda^2)^2]\}^{1/2}} \quad (18)$$

$$A_{\pm}(t) = \frac{(\lambda^2 - 1)(\lambda \pm t)}{\lambda(\lambda^2 + 3)t \pm (1 + 3\lambda^2)} \left[ \frac{\lambda(\lambda^2 + 3)t \pm (1 + 3\lambda^2)}{\lambda(\lambda^2 + 3)t \pm (1 + 3\lambda^2)} \right]^{2/3}$$

Cubic roots in (18) should be taken real.

Table

Branches of  $\phi(z_2)$  at  $z_2 = t$ ,  $\text{Im } t = 0$ , and  $z_2 = ir$ ,  $\text{Im } r = 0$ .

| $z_2$                        | $\phi^{(1)}$                                           | $\phi^{(2)}$                                           | $\phi^{(3)}$                                           |
|------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|
| $t + i\epsilon$ ,<br>$t > a$ | $A_+ + A_-$                                            | $A_+ e^{\frac{2\pi i}{3}} + A_- e^{-\frac{2\pi i}{3}}$ | $A_+ e^{-\frac{2\pi i}{3}} + A_- e^{\frac{2\pi i}{3}}$ |
| $t - i\epsilon$ ,<br>$t > a$ | $A_+ e^{\frac{2\pi i}{3}} + A_- e^{-\frac{2\pi i}{3}}$ | $A_+ e^{-\frac{2\pi i}{3}} + A_- e^{\frac{2\pi i}{3}}$ | $A_+ + A_-$                                            |
| $0 \leq t < a$               | $A_+ e^{-\frac{2\pi i}{3}} + A_- e^{\frac{2\pi i}{3}}$ | $A_+ + A_-$                                            | $A_+ e^{\frac{2\pi i}{3}} + A_- e^{-\frac{2\pi i}{3}}$ |
| $ir$                         | $2i\sqrt{p} \cos\left(\frac{\alpha + 2\pi}{3}\right)$  | $2i\sqrt{p} \cos\left(\frac{\alpha - 2\pi}{3}\right)$  | $2i\sqrt{p} \cos \frac{\alpha}{3}$                     |

The Riemann surface of  $\phi(z_2)$  is mapped into the bag interior in the  $z$ -plane by the transformation

$$z = F(z_2) = \frac{1}{\pi} \int_a^{z_2} \frac{dv \phi(v)}{(1+v)^2} \quad (19)$$

The field strength  $E(z) = \frac{z_2 + 1}{z_2 - 1}$  throughout the interior of the bag obeys the condition  $|E(z)| \geq 1$  and approaches  $\infty$  at points of charge positions

$$z_{q_a} = x_{q_a} + iy_{q_a} = \frac{1}{\pi} \int_a^1 \frac{dv \phi^{(\alpha)}(v)}{(1+v)^2}, \quad a = 1, 2, 3. \quad (20)$$

The bag boundary  $G$  is defined by eq. (19) at  $\text{Re } z_2 = 0$ .

### III. RESULTS AND CONCLUSIONS

The equation for boundary  $G$  can be written as

$$x_G(\tau) = \text{Re } F(i\tau), \quad y_G(\tau) = \text{Im } F(i\tau), \quad -\infty < \tau < \infty.$$

Integration in (19) can be performed numerically. The bag shape at  $\lambda = 30$  is shown in Fig. 1. The boundary  $G$  has four cusps at points, where  $\frac{dx_G}{d\tau}$  and  $\frac{dy_G}{d\tau}$  vanish simultaneously.

The positions of charges (Fig. 1) are calculated by (20). As was to be expected, the bag is symmetric relative to the  $x$ -axis. At points  $A_4, A_5, A_6$   $\frac{dx_G}{d\tau} = 0$ ,  $\frac{dy_G}{d\tau} \neq 0$ ; at point  $A_3$   $\frac{dy_G}{d\tau} = 0$ ,  $\frac{dx_G}{d\tau} \neq 0$ .

As  $\lambda \rightarrow \infty$  the bag "stretches" along the  $x$ -axis, with its size along the  $y$ -axis being finite; the positions of all charges and extremal points of the boundary  $A_1 \dots A_7$  can be calculated analytically up to the terms exponentially decreasing with growing  $x_{q_1} = R$ . An idea of the bag asymptotic shape can be gained from the following relations as  $\lambda \rightarrow \infty$ :

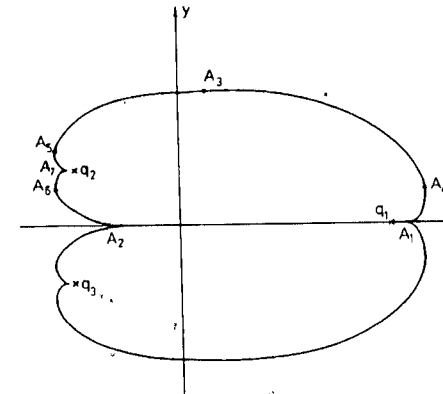


Fig. 1. The bag shape at  $\lambda = 30$ .  $A_1, \dots, A_7$  are points at which at least one of the derivatives  $\frac{dx_G}{d\tau}$ ,  $\frac{dy_G}{d\tau}$  vanishes;  $x$  denotes the charge positions.

$$\begin{aligned}
x_{q_1} &= R \rightarrow \frac{2}{\pi} \left( \log \frac{\sqrt{3}\lambda}{8} - \frac{1}{2} \right); & x_{q_2} &= -x_{q_3} = -\frac{R}{2}; \\
y_{q_1} &= 0, & y_{q_2} &= -y_{q_3} \rightarrow 1/2; \\
x_{A_1} &\rightarrow R + (2 \log 2 - 1)/\pi; & x_{A_2} &\rightarrow -\frac{2}{\pi} \log 2 \\
x_{A_3} &\rightarrow \frac{\log 2}{\pi}, & y_{A_3} &\rightarrow 1; & x_{A_4} &\rightarrow R + \frac{3 \log 2 - 1}{2\pi}, & y_{A_4} &\rightarrow \frac{1}{2} - \frac{1}{\pi}; \\
x_{A_5} &= x_{A_6} \rightarrow -\frac{R}{2} - \frac{3 \log 2 - 1}{4\pi}, & y_{A_5} &= y_{q_2} = y_{q_3} & -y_{A_6} &\rightarrow \frac{1}{4} - \frac{1}{2\pi}; \\
x_{A_7} &\rightarrow -\frac{R}{2} - \frac{2 \log 2 - 1}{2\pi}, & y_{A_7} &\rightarrow y_{q_2} = \frac{1}{2}.
\end{aligned} \tag{21}$$

Corrections to coordinates in (21) are of the order  $O(\exp(-\frac{\pi R}{2}))$ .

From (21) it follows that at large separations between the charge  $q_1$  and charges  $q_{2,3}$  the bag is a set of three strips of width 2, 1, 1, respectively, with the point of junction  $x=y=0$ .

The total energy of the 3-particle state in the bag is a sum of energies of the field,  $U_{el}$ , and bag  $U_B$ :

$$U = U_{el} + U_B,$$

where  $U_B = \frac{1}{2} S_B$ ,  $S_B$  is the bag area;  $U_{el} = \frac{1}{2} (2\Phi(z_{q_1}) - \Phi(z_{q_2}) - \Phi(z_{q_3}))$ ,

$\Phi$  is the nonsingular part of the electrostatic potential. The energy  $U$  can also be calculated numerically; by asymptotic estimations,  $U \approx 3R$  as  $R \rightarrow \infty$ .

With decreasing  $\lambda$  all points of the charge positions approach the origin of coordinates. At  $\lambda=3,4$  two parts of the boundary  $G$  corresponding to the integration of branches  $\phi^{(1)}$ ,  $\phi^{(3)}$  in (19) osculate at a point  $(x_0, 0)$ ,  $x_0 < 0$  (Fig. 2). With further decreasing  $\lambda$  the boundary  $G$  gets self-intersections; such "two-sheeted" solutions, physically, are not acceptable.

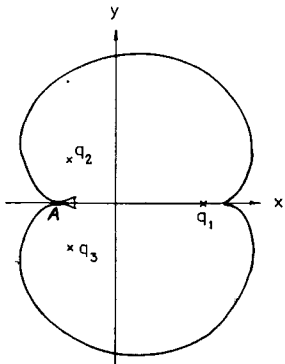


Fig. 2. The bag shape at  $\lambda=3.4$ . Two parts of the boundary osculate at point A. The scale is twice that of Fig. 1.

The fact of appearance of self-intersections points to a possibility of other solutions for such configurations of charges. Note that analogous self-intersections of the bag boundary were observed in paper<sup>5</sup> for some states of four particles placed on a straight line.

So, for  $3.4 \leq \lambda < \infty$  we have found physically acceptable solutions for symmetric three-particle states in the static bag, described by the relations (14, 17, 19-20). These solutions depend on one parameter  $\lambda$  connected with coordinates of particles by (20), and these, naturally, do not cover the whole (two-parameter) set of symmetric configurations. Complete description of all configurations of such a type, in our view, can be achieved on the basis of the class of functions (13) with a more complicated structure of cuts on the complex  $z_2$ -plane.

It seems that in the two-dimensional model exact solutions can be found also for three particles interacting with two effective Abelian gauge fields (color quarks in a heavy baryon). In fact, the two gauge fields are subject to the following boundary conditions

$$E_1 e^{i\theta} = \cos \chi, \quad E_2 e^{i\theta} = \sin \chi, \quad \text{Im } \chi = 0$$

and the transformation  $z_1 = \frac{1}{E_1^2(z) + E_2^2(z)}$  maps the bag boundary onto the unit circle. The choice of appropriate analytic functions  $E_1(z), E_2(z)$  represents an interesting but much more complex problem than in the case of one gauge field we have considered.

The exact solutions we have obtained are the first ones in the static bag model, which describe the states of particles not lying on the same straight line. These solutions allow us to understand what is the bag shape for configurations of two pairs "quark-antiquark" of one color, in which both the quarks are very close to each other (and form an "effective charge"  $2q$ ) and the positions of antiquarks are symmetric with respect to  $x$ -axis (charges  $q_2, q_3$ ). An exact solution of the four-quark problem seems to be possible on the basis of the class of functions like (13), where  $\Phi(z_2, \phi)$  is a polynomial of the fourth power in  $\phi$ .

Further study of multiparticle states in the two-dimensional bag model is, in our opinion, of a considerable interest from the mathematical and physical viewpoint; it will allow a deeper understanding of the structure of the model in a more realistic case of three space dimensions.

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Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1982

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The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1982