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E2-82-126

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PION FORM FACTOR AND QCD SUM RULES

Submitted to "Письма в ЖЭТФ"

1982

The discovery that perturbative QCD methods can be applied to study the elastic high momentum transfer processes, made at the close of 70's, was an important step in the development of perturbative QCD of hard processes (see, e.g., ref.[1] and reviews[2,3]). In particular, asymptotic freedom enables one to easily reproduce in the asymptotic region the well-known quark counting rules for electromagnetic form factors of hadrons[4]. However, for experimentally accessible momentum transfers Q^2 the agreement between the existing theory[1-3] and experimental data for pion and proton form factors is very poor. This observation, nevertheless, should not be treated as an evidence against QCD itself because the perturbative QCD approach[1-3] is applicable only for asymptotically large Q^2, and the extrapolation of the asymptotic QCD formulas into the region of moderately large Q^2 is not justified. For pion, e.g., the main contribution to the asymptotic $Q^2 \to \infty$ region is due to the hard rescattering process (fig.1a). However, as was established in ref.[5], a straightforward use of the asymptotic formalism in the $Q^2 \lesssim 20$ GeV^2 region leads to that the mean virtualness of the gluon (see fig.1a) is smaller than (300 MeV)^2. In such a situation it is, of course, misleading to rely on perturbation theory because this region of momenta is dominated by nonperturbative effects. Earlier one of the

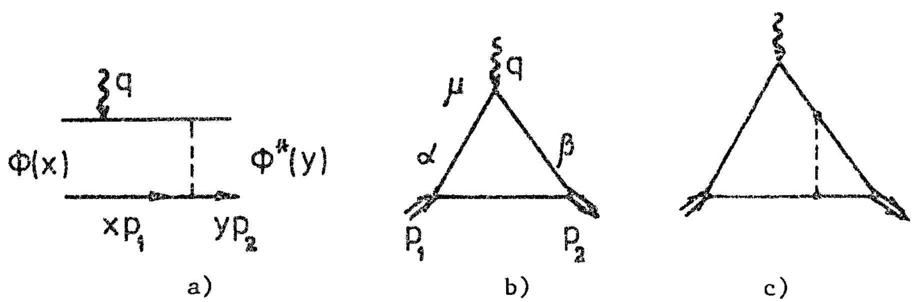

Fig.1. Diagrams relevant to calculation of the pion form factor in QCD: a) asymptotic perturbative QCD diagram; b) lowest-order diagram of the QCD sum rule approach; c) one of 2-loop diagrams of the QCD sum rule approach.

БИБЛИОТЕКА

authors (A.R.) attempted to take these effects into account within the framework of a QCD inspired model [6]. In the present paper we describe the main points of a new approach to the investigation of exclusive processes in QCD based on the systematic use of the QCD sum rule approach [7], in which the nonperturbative effects are incorporated into analysis by introducing nonzero vacuum averages of quark and gluon operators. As the simplest example we consider here the pion form factor.

To generalize the QCD sum rule approach onto the pion form factor problem, we start with the 3-point amplitude

$$T^{\mu\alpha\beta}(p_1,p_2)=i^2\int d^4x\, d^4y\, e^{-ip_1x+ip_2y}\langle 0|T\{j^{\beta}(y)\, J^{\mu}(0)\, j^{+\alpha}(x)\}|0\rangle \qquad (1)$$

(fig.1b), where J^μ is the electromagnetic current and $j^\alpha = \bar{d}\gamma_5\gamma^\alpha u$ is the axial current. The latter satisfies the necessary condition that it should have nonzero projection onto the pion state $|P\rangle$

$$\langle 0|j_\alpha(0)|P\rangle = if_\pi P_\alpha,$$

where f_π = 133 MeV is the pion decay constant.

The amplitude $T^{\mu\alpha\beta}(p_1,p_2)$ is the sum of various structures, and the corresponding invariant amplitudes T_i depend on 3 variables: p_1^2, p_2^2, $q^2=(p_1-p_2)^2$. Owing to asymptotic freedom, one may calculate $T_i(p_1^2, p_2^2, q^2)$ in the Euclidean region $p_1^2, p_2^2, q^2 < -\mu_0^2 \sim -(1\ \text{GeV})^2$. To extract the desired information about the form factors of the physical states, we use the double dispersion relation

$$T_i(p_1^2,p_2^2,q^2)=\frac{1}{\pi^2}\int_0^\infty ds_1\int_0^\infty ds_2\,\frac{\rho_i(s_1,s_2,q^2)}{(s_1-p_1^2)(s_2-p_2^2)}+\dots . \qquad (2)$$

We have omitted in eq. (2) the terms that are polynomials in p_1^2 and/or p_2^2, because they disappear after one applies to eq. (2) the Borel procedure described in ref. [7]:

$$\Phi_i(M_1,M_2;Q^2)=\frac{1}{\pi^2}\int_0^\infty\frac{ds_1}{M_1^2}\int_0^\infty\frac{ds_2}{M_2^2}\,\rho_i(s_1,s_2,q^2)\exp\{-\frac{s_1}{M_1^2}-\frac{s_2}{M_2^2}\}, \qquad (3)$$

where $Q^2 = -q^2$ and Φ_i is the (double) Borel transform of T_i. To treat initial and final state on equal footing we take henceforth $M_1=M_2=M$. After this has been done, it makes sense to rewrite eq. (3) as an integral over $s=s_1+s_2$ and $\xi=s_1/s$.

For the pion form factor analysis the most important is the amplitude (hereafter referred to as Φ) related to the structure $P^\mu P^\alpha P^\beta$, where $P=p_1+p_2$. The most simple way to extract it is to multiply $T^{\mu\alpha\beta}$ by $n_\alpha n_\beta n_\mu$, where n is the light-like vector characterized by $n^2=0$, $(nq)=0$, $(np_1)=(np_2)\neq 0$. Neglecting quark masses ($m_{u,d} \lesssim 10$ MeV) we calculated the contribution of the diagram 1b into the Φ-amplitude:

$$\Phi^{(1b)}(M^2,Q^2)=\frac{3}{4\pi^2M^2}\int_0^1 x(1-x)\exp\{-\frac{Q^2}{2M^2}\,\frac{x}{1-x}\}dx = \qquad (4a)$$

$$=\frac{1}{2\pi^2M^4}\int_0^\infty\frac{s^2(2s+3Q^2)}{(2s+Q^2)^3}\,e^{-s/M^2}\,ds. \qquad (4b)$$

The x-variable in eq. (4a) is in fact the fraction of the total pion momentum carried by the passive quark in the infinite momentum frame. The representation (4a) is most convenient to study the asymptotic behaviour of Φ as $Q^2\to\infty$ or $Q^2\to 0$. On the other hand, eq. (4b) looks like the spectral representation (3) provided that one has performed the integration over $\xi=s_1/s$ in eq. (3).

For the spectral density $\rho(s_1,s_2,q^2)$ we use the standard ansatz (cf. ref. [7]) that ρ is a sum of the resonance (pion) contribution and the "background" one. Moreover, starting from some $s=s_0$ the latter coincides with the free-quark value that can be easily extracted from eq. (4b)*

$$\rho(s_1,s_2,q^2)=\pi^2 f_\pi^2 F_\pi(Q^2)\delta(s_1-m_\pi^2)\delta(s_2-m_\pi^2) + \theta(s-s_0)\,\frac{s(2s+3Q^2)}{2(2s+Q^2)^3}. \qquad (5)$$

One of the most important results of the QCD sum rule approach is that the duality interval s_0 is not a free parameter. Rather, it is determined by the power $(1/M^2)^N$ corrections to eq. (4). Taking into account the contributions proportional to $\frac{\alpha_s}{\pi}\langle G^a_{\mu\nu}G^a_{\mu\nu}\rangle$ and $\alpha_s\langle\bar{q}q\rangle^2$ and using eqs. (3), (5) we arrive at the following representation for the form factor

$$f_\pi^2F_\pi(Q^2)=\frac{3M^2}{4\pi^2}\int_0^{2s_0/(2s_0+Q^2)} x(1-x)\exp\{-\frac{Q^2}{2M^2}\,\frac{x}{1-x}\} - \frac{M^2s_0^2(2s_0+3Q^2)}{2\pi^2(2s_0+Q^2)^3}\exp\{-\frac{s_0}{M^2}\}+\frac{\alpha_s\langle G^a_{\mu\nu}G^a_{\mu\nu}\rangle}{12\pi M^2}+\frac{176\pi\alpha_s\langle\bar{q}q\rangle^2}{81M^4}\left(1-\frac{2}{11}\,\frac{Q^2}{M^2}\right). \qquad (6)$$

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* We are grateful to B.L.Ioffe who suggested to us this choice.

In numerical estimates we use the values $(\alpha_s/\pi)\langle GG\rangle \simeq 0.012\ \mathrm{GeV}^4$ and $\alpha_s\langle\bar{q}q\rangle^2 \simeq 1.83\cdot 10^{-4}\ \mathrm{GeV}^6$ taken from ref. /7/. Note that the physical quantity like $F_\pi(Q^2)$ should not depend on the arbitrary ("unphysical") parameter M^2 which can be changed by our convenience. It is easy to establish that for sufficiently large M^2 the M^2-dependence of the r.h.s. of eq. (6) is in fact very weak, but the onset of the asymptotic regime depends on s_0. It is natural to define that the "true" s_0 is that for which the region, where the r.h.s. of eq. (6) is insensitive to variations of M^2, is the most broad one. For $Q^2 = 2\ \mathrm{GeV}^2$ this criterion gives the value $s_0 = 1.0\ \mathrm{GeV}^2$ which is in excellent agreement with the value $s_0 = 1.05\ \mathrm{GeV}^2$ obtained from the requirement that the square of our "duality triangle" (which equals $s_0^2/2$, according to eq. (5)) should coincide with $(s_0^{SVZ})^2$, where $s_0^{SVZ} = 0.75\ \mathrm{GeV}^2$ is the duality interval for the 2-point function calculated by SVZ in ref. /7/.

Choosing M^2 one should take into account that increasing M^2 results in the decrease of power corrections whereas the background contribution increases (and vice versa, if M^2 decreases, power corrections get larger whereas the background gets smaller). Taking in mind these observations we choose the minimal M^2 for which power corrections (at $Q^2 = 2\ \mathrm{GeV}^2$) are smaller than 30%. This gives $M^2 = 1.8\ \mathrm{GeV}^2$.

Eq. (6) is applicable only in a restricted Q^2-range. First, in the region $Q^2 \lesssim m_\rho^2 = 0.6\ \mathrm{GeV}^2$ there may exist potentially dangerous $1/Q^2$ power corrections. Thus, $Q^2_{min} \simeq 0.6\ \mathrm{GeV}^2$. Second, the main contribution into $\Phi^{(1b)}$ for large Q^2 gives the region $x \sim M^2/Q^2$, where the passive quark has the virtuallness $k^2 \sim M^4/Q^2$. Hence, for $Q^2 > M^4/m_\rho^2$ (i.e., for $Q^2 \gtrsim 5\ \mathrm{GeV}^2$) one should expect that there appear large corrections of Q^2/M^2-type. Really, the $\Phi^{(1b)}$-term behaves like $1/Q^4$ for large Q^2, whereas the power corrections contain the Q^2-independent terms (see. eq. (6)). As a result, power corrections in eq. (6) (which were settled for $Q^2 = 2\ \mathrm{GeV}^2$ to give the 30%-contribution) reach almost 100% for $Q^2 = 6\ \mathrm{GeV}^2$. Hence, for $Q^2 > 4\ \mathrm{GeV}^2$ one should take into account higher (and, maybe, all) $(1/M^2)^N$ power corrections. Thus, the theoretical curve for $F_\pi(Q^2)$ corresponding to eq. (6) is reliable only in the $1 < Q^2 < 4\ \mathrm{GeV}^2$ region. The comparison between this curve and the existing experimental data /8/ is shown in fig.2.

In principle, apart from the power correction, one must take into account also higher perturbative corrections due to diagrams like that shown in fig.1c. Although at present we are unable to calculate these 2-loop diagrams, our estimates show that their contribution is damped by the α_s/π -factor with

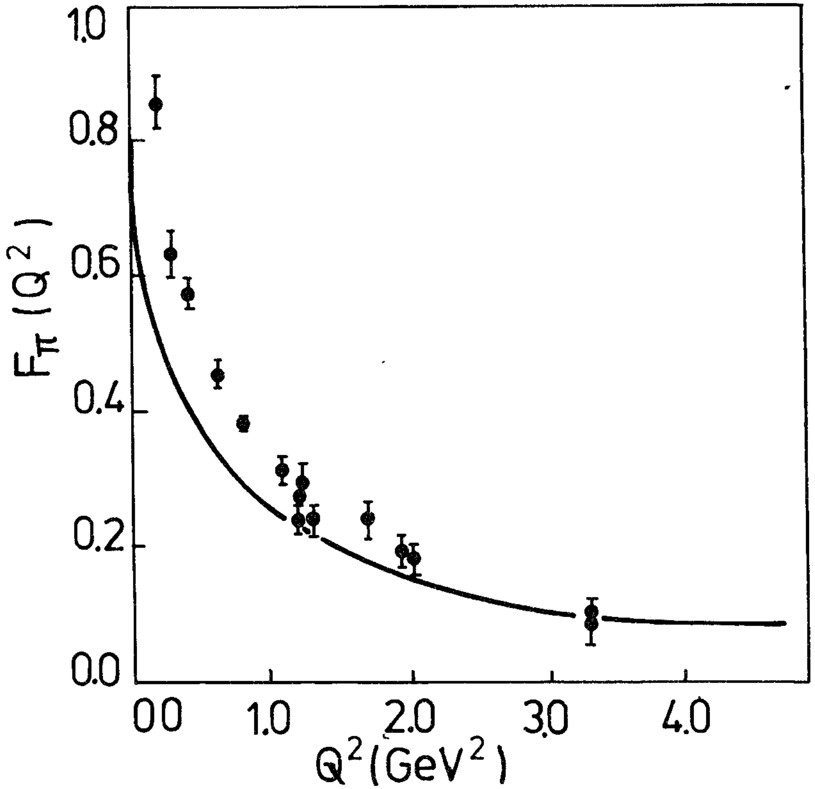

Fig.2. Comparison between our theoretical curve (eq. (6), $s_0 = 1\ \mathrm{GeV}^2$, $M^2 = 1.8\ \mathrm{GeV}^2$) and experimental data taken from ref. /8/.

respect to that of fig.1b and for $Q^2 \lesssim 10\ \mathrm{GeV}^2$ they produce only the 10% (positive) correction to $F_\pi(Q^2)$.

It should be emphasized, however, that in the asymptotic $Q^2 \to \infty$ region the diagram 1c is much more important than the diagram 1b because for very large Q^2 the former gives $1/Q^2$-contribution (corresponding to the quark counting rules /4/ and asymptotic QCD analysis /1-3/) compared to $1/Q^4$ -contribution of the latter.

ACKNOWLEDGEMENTS

We would like to express our gratitude to A.V.Efremov for stimulating discussions and to B.M.Barbashov and V.A.Meshcheryakov for inretest in this work and support. In the course of our investigations we have learned that a similar approach to the pion form factor in QCD was independently proposed by B.L.Ioffe and A.V.Smilga. We are thankful to them and acknowledge their fruitful discussions.

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Received by Publishing Department
on February 16 1982.

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Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1982

Nesterenko V.A., Radyushkin A.V. E2-82-126

Pion Form Factor and QCD Sum Rules

We propose a new approach to the investigation of the pion electromagnetic form factor in QCD based on the systematic use of the QCD sum rules technique. The resulting theoretical curve for $F_{\pi}(Q^2)$ is in good agreement with existing experimental data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1982