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COVARIANT THREE-DIMENSIONAL EQUATION FOR THE WAVE FUNCTION OF π-MESON IN THE COMPOSITE MODEL OF SPINOR QUARKS

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1. INTRODUCTION

According to the quark model a pion is the bound state of quark and antiquark. The discovered Ψ and γ-particles and their radial excitations were successfully described on the basis of the Schrödinger equation. These mesons were considered to be compounds of heavy charmed and beauty quarks. After this there appeared an opinion that the quarks which compose a pion should be light and their relative motion in the pion should have a relativistic nature.

That is why the relativistic equations for two particle bound states should be used for the description of the quarks motion inside the pion. There are well-known relativistic two-particle equations of Bethe-Salpeter [1] and three-dimensional equations that appear in the single-time approach of Logunov and Tavkhelidze [2,3] and in the diagram technique of Kadyshevsky [4,5].

In the present paper we shall apply the covariant single-time equal for the description of the asymptotics of a pion wave function (WF) when the interaction between quarks is chosen to be mediated by one gluon exchange. The interest in this problem stems from the interest to the problem of studing the asymptotic behaviour of the elastic pion form factor in QCD.

It is natural that since in QCD there do not exist up to now methods of describing the interactions at large distances, the behaviour of the wave function (WF) can be studied only at large values of relative momentum in the framework of this theory.

In the paper to study the WF asymptotics we apply the formalism of the covariant single-time equations for the system composed of two particles with spins 1/2.

The covariant single-time WF (denoted by ~), which describes the relative motion in quark-antiquark system, is defined through the Bethe-Salpeter WF as follows [2,6,7]:

$$\tilde{\Psi}^{\sigma_1 \sigma_2}_{MK}(p_1, p_2) = -\frac{(2\pi)^4}{4m^2} \cdot \bar{u}_{q\alpha}(\Lambda^{-1}_{\lambda \mathcal{P}} p_1, \sigma_1) \cdot u_{\bar{q}\beta}(\Lambda^{-1}_{\lambda \mathcal{P}} p_2, \sigma_2) \times \tag{1.1}$$

$$\times \int d^4x \, e^{i(p_1 - p_2)x} \, \delta[\lambda_{\mathcal{P}} x] \langle 0 | T\{\Psi_\alpha(\frac{x}{2}) \bar{\Psi}_\beta(-\frac{x}{2})\} | M\vec{K}; S, \sigma \rangle ,$$

where $\Psi(x)$ and $\tilde{\Psi}(x)$ are the quark and antiquark field operators, $\mathcal{P} = p_1+p_2$, $x = x_1-x_2$, the vector $|M\vec{K};\ S\sigma>$ characterizes the compound system as a whole with the total mass M, spin S, its projection σ and moving as a whole with momentum $\vec{K}$. The presence of the invariant $\delta[\lambda\mathcal{P}x]$ -function, $(\lambda^{\mu}_{\mathcal{P}} = \mathcal{P}^{\mu}/\sqrt{\mathcal{P}^2})$ under the integrand in (1.1) leads in the c.m.s. $\vec{\lambda}_{\mathcal{P}} = \vec{\mathcal{P}}/\sqrt{\mathcal{P}^2} = 0$ to the coincidences of individual times $x^{\circ}_1 = x^{\circ}_2$ of quarks.

To take into account the QCD effect of the asymptotically free behaviour of the runnung coupling constant, we shall use here a modification of the amplitude of the one-gluon exchange in the region of small values of the momentum transfer Q^2. This modification has been suggested in ref.[10] and provides the most simple Coulomb-like form of the corresponding quasipotential in the relativistic configurational representation.

The relativistic single-time equation for spin WF (1.1) has the form [6,3,7-9,11]:

$$2\Delta^{\circ}_{p,m\lambda_{\mathcal{P}}}[M-2\Delta^{\circ}_{p,m\lambda_{\mathcal{P}}}]\tilde{\Psi}^{\sigma_1\sigma_2}_{MK}(\vec{\Delta}_{p,m\lambda_{\mathcal{P}}}) = \tag{1.2}$$

$$= \frac{1}{(2\pi)^3}\sum_{\sigma_1'\sigma_2'}\int\frac{d^3\vec{\Delta}_{k,m\lambda_{\mathcal{P}}}}{2\Delta^{\circ}_{k,m\lambda_{\mathcal{P}}}}\cdot V^{\sigma_1\sigma_2}_{\sigma_1'\sigma_2'}(\vec{\Delta}_{p,m\lambda_{\mathcal{P}}};\vec{\Delta}_{k,m\lambda_{\mathcal{P}}};\mathcal{P}^2)\tilde{\Psi}^{\sigma_1'\sigma_2'}_{M,K}(\vec{\Delta}_{k,m\lambda_{\mathcal{P}}}).$$

In (1.2) the vectors $\vec{\Delta}_{p,m\lambda_{\mathcal{P}}}$ and $\vec{\Delta}_{k,m\lambda_{\mathcal{P}}}$ are the covariant generalizations of the particles momenta in the c.m.s. before the scattering $\vec{p}_1=-\vec{p}_2=\vec{p}$ and after: $\vec{k}_1=-\vec{k}_2=\vec{k}$. They are defined according to refs.[12,7] by the relations:

$$\vec{\Delta}_{p,m\lambda_{\mathcal{P}}} \equiv \overset{\circ}{\vec{p}} = \overrightarrow{(\Lambda^{-1}_{\lambda_{\mathcal{P}}}p_1)} = -\overrightarrow{(\Lambda^{-1}_{\lambda_{\mathcal{P}}}p_2)} = -\vec{\Delta}_{p_2,m\lambda_{\mathcal{P}}},$$

$$\vec{\Delta}_{k,m\lambda_{\mathcal{P}}} \equiv \overset{\circ}{\vec{k}} = \overrightarrow{(\Lambda^{-1}_{\lambda_{\mathcal{P}}}k_1)} = -\overrightarrow{(\Lambda^{-1}_{\lambda_{\mathcal{P}}}k_2)} = -\vec{\Delta}_{k_2,m\lambda_{\mathcal{P}}}, \tag{1.3}$$

where $\Lambda_{\lambda_{\mathcal{P}}}$ is the matrix of the Lorentz boost in the rest frame of the compound particle, moving with the 4-velocity $\lambda^{\mu}_{\mathcal{P}} = \mathcal{P}^{\mu}/\sqrt{\mathcal{P}^2}$, so that $\Lambda_{\lambda_{\mathcal{P}}}(M,\vec{0}) = (\mathcal{P}_0,\ \vec{\mathcal{P}})$. The time components of $\Delta^{\mu}_{p,m\lambda_{\mathcal{P}}}$ and $\Delta^{\mu}_{k,m\lambda_{\mathcal{P}}}$ are defined by the relations:

$$\Delta^{\circ}_{p,m\lambda_{\mathcal{P}}} \equiv \overset{\circ}{p}_0 = \sqrt{m^2+\vec{\Delta}^2_{p,m\lambda_{\mathcal{P}}}}\ ;\quad \Delta^{\circ}_{k,m\lambda_{\mathcal{P}}} \equiv \overset{\circ}{k}_0 = \sqrt{m^2+\vec{\Delta}^2_{k,m\lambda_{\mathcal{P}}}}. \tag{1.4}$$

In equation (1.2) the momenta of all the particles belong to the mass hyperboloid

$$(\Delta^{\circ}_{p,m\lambda_{\mathcal{P}}})^2 - (\vec{\Delta}_{p,m\lambda_{\mathcal{P}}})^2 = m^2\ ;\quad p^2_{i0} - \vec{p}^{\,2}_i = m^2 \quad (i=1,2)$$

$$(\Delta^{\circ}_{k,m\lambda_{\mathcal{P}}})^2 - (\vec{\Delta}_{k,m\lambda_{\mathcal{P}}})^2 = m^2\ ;\quad k^2_{i0} - \vec{k}^{\,2}_{i0} = m^2, \tag{1.5}$$

but their time components are over the "energy shell" $\overset{\circ}{p}_0 \neq \overset{\circ}{k}_0$.

As has been shown in refs.[3,13] and [9,11] the equation for a single-time WF of the system of two particles with 1/2 spins derived by the Logunov-Tavkhelidze method [2] coincides in the form with that obtained on the basis of the diagram technique of Kadyshevsky [4,5,8]. The quasipotential $V^{\sigma_1\sigma_2}_{\sigma_1'\sigma_2'}(\vec{\Delta}_{p,m\lambda_{\mathcal{P}}};\vec{\Delta}_{k,m\lambda_{\mathcal{P}}};\mathcal{P}^2)$ is constructed of the invariant matrix elements of the relativistic scattering amplitude. The values $\Delta^{\circ}_{p,m\lambda_{\mathcal{P}}}\ (\equiv \overset{\circ}{p}_0)$ that enter the free Green function of equation (1.2) are the relativistic invariants as well as the volume element $d^3\vec{\Delta}_{k,m\lambda_{\mathcal{P}}}/2\Delta^{\circ}_{k,m\lambda_{\mathcal{P}}}$. The covariance of the whole equation (1.2) has been proved in refs.[5-9].

The aim of the present paper is to consider in a consistent way the spin degrees of freedom of the quark-antiquark system in a pion and to account their influence on the structure of the interaction kernel as well as their influence on the asymptotics of the wave function at large values of the relative momentum.

2. EQUATION FOR THE RELATIVE MOTION OF QUARK AND ANTIQUARK IN A PION

We would use in equation (1.2) the quasipotential, which is built of the amplitude of the massles vector particle (gluon) exchange:

$$V^{\sigma_1\sigma_2}_{(2)\nu_1\nu_2}(\vec{\Delta}_{p,m\lambda_{\mathcal{P}}};\vec{\Delta}_{k,m\lambda_{\mathcal{P}}}) = \tag{2.1}$$

$$= \{\bar{u}^{\sigma_1}_q(p_1)\gamma_{\mu}u^{\nu_1}_{\bar{q}}(k_1)\}\cdot g_{\mu\nu}\cdot\{\bar{u}^{\sigma_2}_{\bar{q}}(p_2)\gamma^{\nu}\bar{u}^{\nu_2}(k_2)\}\cdot V_0(q^2),$$

where

$$V_0(q^2) = \frac{-g^2}{(p_1-k_1)^2};\quad q = p_1-k_1 \tag{2.2}$$

and the coupling constant g can depend on the Q^2 in the case of QCD. As has been mentioned in refs.[9] the polarization indices σ_i and ν_i of the quasipotential (2.1) are "sitting" (by terminology of the authors of refs. [12,14]) each on its momentum. It is convenient to pass from (2.1) to the quasipotential, the polarization indices of which would "sit" on one and the same momentum, the $\overset{\circ}{\vec{p}}\ (\equiv\vec{\Delta}_{p,m\lambda_{\mathcal{P}}})$, for example. This can be acheived with the help of the transformation [9]

$$V^{\sigma_1\sigma_2}_{\nu_1\nu_2}(\mathring{\vec p},\mathring{\vec k}) = \sum_{\sigma_i,\nu_i} D^{+1/2}_{\sigma_1\sigma_{1\mathring p}}\{V^{-1}(\Lambda_{\lambda\mathcal{P}},\mathring p_1)\}\cdot D^{+1/2}_{\sigma_2\sigma_{2\mathring p}}\{V^{-1}(\Lambda_{\lambda\mathcal{P}},\mathring p_2)\}\times$$
$$\times V^{\sigma_{1\mathring p}\sigma_{2\mathring p}}_{\nu_{1\mathring p}\nu_{2\mathring p}}(\mathring{\vec k}(-)\mathring{\vec p};\mathring{\vec p})\times D^{1/2}_{\nu_{1\mathring p},\nu_{1\mathring k}}\{V^{-1}(\Lambda_{\mathring k_1},\mathring p_1)\}\times \tag{2.3}$$
$$\times D^{1/2}_{\nu_{1\mathring k},\nu_1}\{V^{-1}(\Lambda_{\lambda\mathcal{P}},k_1)\}\cdot D^{1/2}_{\nu_{2\mathring p},\nu_{2\mathring k}}\{V^{-1}(\Lambda_{\mathring k_2},\mathring p_2)\}\cdot D^{1/2}_{\nu_{2\mathring k},\nu_2}\{V^{-1}(\Lambda_{\lambda\mathcal{P}},k_2)\},$$

where the summation is performed over the repeated indices $\sigma_{i\mathring p}$ and $\nu_{i\mathring k}$, $\nu_{i\mathring p}$ $(i=1,2)$. The matrices $D^{1/2}\{V^{-1}(\Lambda_p,k)\}$ describe the Wigner rotations of spins 1/2 $R\{V^{-1}(\Lambda_p,k)\} = (\Lambda_{\Lambda_p^{-1}k})^{-1}\Lambda_p^{-1}\Lambda_k$, where Λ_p are the matrices of the corresponding Lorentz boost: $\Lambda_p(m,\vec o) = (p_0,\vec p)$. Each polarization index in (2.3) is accompanied by the momentum, on which this index is "sitting" (see for details ref. [9]). As a result of a set of "removes" of polarization indices, we come to the amplitude $V^{\sigma_{1\mathring p},\sigma_{2\mathring p}}_{\nu_{1\mathring p},\nu_{2\mathring p}}$ whose all spin indices would "sit" on one and the same momentum $\mathring{\vec p}$.

In the second approximation in the coupling constant, the amplitude $V^{\sigma_{1\mathring p},\sigma_{2\mathring p}}_{(2)\nu_{1\mathring p},\nu_{2\mathring p}}(\mathring{\vec k}(-)\mathring{\vec p};\mathring{\vec p})$ can be represented in the form:

$$V^{\sigma_{1\mathring p}\sigma_{2\mathring p}}_{\nu_{1\mathring p},\nu_{2\mathring p}}(\mathring{\vec k}(-)\mathring{\vec p};\mathring{\vec p}) = \xi^{*\sigma_{1\mathring p}}\xi^{*\nu_{2\mathring p}}\,V_{(2)}(\mathring{\vec k}(-)\mathring{\vec p};\mathring{\vec p})\,\xi_{\nu_{1\mathring p}}\,\xi_{\sigma_{2\mathring p}}, \tag{2.4}$$

where ξ^σ are the two component Pauli spinors, and the operator $\hat V_{(2)}$, according to ref. [9], can be expressed through the variable $\vec\Delta = \mathring{\vec k}(-)\mathring{\vec p}$, the momentum transfer in the Lobachevsky space [15], in the following way:

$$\hat V_{(2)}(\mathring{\vec k}(-)\mathring{\vec p};\mathring{\vec p}) = -g_V^2\frac{2m}{\Delta_0-m} - g_V^2\frac{(\vec\sigma_1\vec\Delta)(\vec\sigma_2\vec\Delta)-(\vec\sigma_1\vec\sigma_2)\vec\Delta^2}{\vec\Delta^2} -$$
$$- g_V^2\frac{i(\vec\sigma_1+\vec\sigma_2)}{m^2}\cdot[\mathring{\vec p}\times\vec\Delta]\cdot\left\{\frac{2\mathring p_0}{\Delta_0-m}+\frac{1}{\vec\Delta^2}\right\} - \tag{2.5}$$
$$- g_V^2\frac{2}{m^2}\,\frac{\mathring p^2(\Delta_0+m)+2\mathring p_0(\mathring{\vec p}\vec\Delta)-2m^3}{\Delta_0-m} - g_V\frac{2\mathring{\vec p}^2}{m^2} -$$
$$- g_V^2\{i(\vec\sigma_1+\vec\sigma_2)\cdot[\mathring{\vec p}\times\vec\Delta]^2/\vec\Delta^2\cdot m^2.$$

The variable $\Delta_\mu = (\Lambda^{-1}_{\mathring p}\mathring k)_\mu$ in (2.5) is defined:

$$\vec\Delta = \overrightarrow{(\Lambda^{-1}_{\mathring p}\mathring k)} \equiv \mathring{\vec k} - \frac{\mathring{\vec p}}{m}\left[\mathring k_0 - \frac{\mathring{\vec k}\mathring{\vec p}}{\mathring p_0+m}\right] \equiv \mathring{\vec k}(-)\mathring{\vec p}, \tag{2.6}$$

$$\Delta_0 = (\Lambda^{-1}_{\mathring p}\mathring k)_0 = (\mathring k_0\mathring p_0 - \mathring{\vec k}\mathring{\vec p})/m, \tag{2.7}$$

$$t = (p-k)^2 = 2m^2 - 2m\Delta_0. \tag{2.8}$$

As we can see the first two terms of the quasipotential $V_{(2)}(\mathring{\vec k}(-)\mathring{\vec p};\mathring{\vec p})$ (2.5) are the local functions in the Lobachevsky momentum space, i.e., they depend only on the difference $(-)$ of two vectors $\mathring{\vec k}(-)\mathring{\vec p} = \vec\Delta$ in this space [15]. The other terms depend not only on the momentum transfer in the Lobachevsky space $\vec\Delta$, but also on the vector $\mathring{\vec p}$.

Let us perform an analogous "remove" of the polarization indices of the wave function $\Psi_{\sigma_1\sigma_2}(\mathring{\vec p})$. It can be performed with the help of the Wigner rotations

$$\Psi_{\sigma_{1\mathring p}\sigma_{2\mathring p}}(\mathring{\vec p}) = \sum_{\sigma_1,\sigma_2=\pm 1/2} D^{1/2}_{\sigma_{1\mathring p}\sigma_1}\{V^{-1}(\Lambda_{\lambda\mathcal{P}},p_1)\}\times$$
$$\times D^{1/2}_{\sigma_{2\mathring p}\sigma_2}\{V^{-1}(\Lambda_{\lambda\mathcal{P}},p_2)\}\Psi_{\sigma_1\sigma_2}(\mathring{\vec p}). \tag{2.9}$$

After removing the spins of the particles and their spin indices on one and the same momentum $\mathring{\vec p}$ (what corresponds to passing to their quantization on one and the same axis taken along the vector $\mathring{\vec p}$), we can perform a covariant summation of the spins [14]. In this way we pass to the wave function that is characterized by the total spin S and its projection $\sigma_{\mathring p}$ [9, III]:

$$\Psi_{S\sigma_{\mathring p}}(\mathring{\vec p}) = \sum_{\sigma_{1\mathring p},\sigma_{2\mathring p}=\pm 1/2}\left\langle\frac{1}{2},\frac{1}{2};\sigma_{1\mathring p},\sigma_{2\mathring p}\,\middle|\,S\sigma_{\mathring p}\right\rangle\Psi_{\sigma_{1\mathring p};\sigma_{2\mathring p}}(\mathring{\vec p}). \tag{2.10}$$

For the state with the total spin $S=0$ and $\sigma_{\mathring p}=0$, we find that the corresponding WF has the form:

$$\Psi_{0,0}(\mathring{\vec p}) = \frac{1}{\sqrt{2}}\{\Psi_{\frac12\mathring p,-\frac12\mathring p}(\mathring{\vec p}) - \Psi_{-\frac12\mathring p,\frac12\mathring p}(\mathring{\vec p})\} \equiv \Psi^{S=0}_M(\mathring{\vec p}). \tag{2.11}$$

As has been shown in refs. [9,III] after removing in equation (1.1) all the spin indices σ_i and σ_j on one and the same momentum $\mathring{\vec p} \equiv \vec\Delta_{p,m\lambda\mathcal{P}}$, equation (1.1) for the state with the total spin $S=0$ defined by (2.10) takes the form:

$$2\Delta^\circ_{p,m\lambda\mathcal{P}}(M-2\Delta^\circ_{p,m\lambda\mathcal{P}})\Psi^{S=0}_M(\vec\Delta_{p,m\lambda\mathcal{P}}) =$$
$$= \frac{1}{(2\pi)^3}\int\frac{d^3\vec\Delta_{k,m\lambda\mathcal{P}}}{2\Delta^\circ_{k,m\lambda\mathcal{P}}}V^{S=0}(\vec\Delta_{k,m\lambda\mathcal{P}}(-)\vec\Delta_{p,m\lambda\mathcal{P}};\mathring{\vec p})\Psi^{S=0}_M(\vec\Delta_{k,m\lambda\mathcal{P}}), \tag{2.12}$$

where

$$V^{S=0}(\vec{\Delta}_{k,m\lambda\mathcal{P}}(-)\vec{\Delta}_{p,m\lambda\mathcal{P}};\overset{\circ}{\vec{p}})=\sum_{\sigma_{1\overset{\circ}{p}},\sigma_{2\overset{\circ}{p}}=\pm 1/2}\ \sum_{\nu_{1\overset{\circ}{p}},\nu_{2\overset{\circ}{p}}=\pm 1/2}\times$$

$$\times<\frac{1}{2},\frac{1}{2};\sigma_{1\overset{\circ}{p}}\sigma_{2\overset{\circ}{p}}|0,0>V^{\sigma_{1\overset{\circ}{p}}\sigma_{2\overset{\circ}{p}}}_{\nu_{1\overset{\circ}{p}}\nu_{2\overset{\circ}{p}}}(\vec{\Delta}_{k,m\lambda\mathcal{P}}(-)\vec{\Delta}_{p,m\lambda\mathcal{P}};\overset{\circ}{\vec{p}})\cdot<\frac{1}{2},\frac{1}{2};\sigma_{1\overset{\circ}{p}}\sigma_{2\overset{\circ}{p}}|0,0>. \tag{2.13}$$

Simple calculations show that the second term in (2.5) gives the contribution to the matrix element (2.13) that equals to

$$-g_V^2\frac{2\vec{\Delta}^2}{\vec{\Delta}^2}=-2g_V^2 \tag{2.14}$$

while the third and the last terms give a zero contribution. The fourth term in (2.5), which contains the terms with the orbital motion, can be transformed with the help of the identity

$$\frac{\overset{\circ}{\vec{k}}\overset{\circ}{\vec{p}}}{\Delta_0-m}=\frac{\overset{\circ}{k}_0\overset{\circ}{p}_0-m^2}{\Delta_0-m}-m \tag{2.15}$$

to the form

$$2\frac{g_V^2}{m^2}\cdot\frac{\overset{\circ}{p}_0(\Delta_0+m)+2\overset{\circ}{p}_0(\overset{\circ}{\vec{p}}\overset{\circ}{\vec{\Delta}})-2m^3}{\Delta_0-m}=$$

$$=2g_V^2\left[\frac{(\overset{\circ}{p}_0\overset{\circ}{k}_0-m^2)}{m^2}\cdot\frac{1}{\Delta_0-m}-\frac{\overset{\circ}{p}{}_0^2}{m^2}\right]. \tag{2.16}$$

Combining together the first and the fifth terms in (2.5) and adding them to (2.14) and (2.16), we find the final expression

$$V^{S=0}(\overset{\circ}{\vec{k}}(-)\overset{\circ}{\vec{p}};\overset{\circ}{\vec{p}})=g_V^2\left(2m-\frac{4\overset{\circ}{k}_0\overset{\circ}{p}_0}{m}\right)\frac{1}{\Delta_0-m}. \tag{2.17}$$

So equation (2.13) can be written in the form

$$2\overset{\circ}{p}_0(M-2\overset{\circ}{p}_0)\Psi_M^{S=0}(\overset{\circ}{\vec{p}})=$$

$$=\frac{1}{(2\pi)^3}\int\frac{d^3\overset{\circ}{\vec{k}}}{2\overset{\circ}{k}_0}(4\overset{\circ}{p}_0\overset{\circ}{k}_0-2m^2)V_0(\overset{\circ}{\vec{k}}(-)\overset{\circ}{\vec{p}})\Psi_M^{S=0}(\overset{\circ}{\vec{k}}), \tag{2.18}$$

where

$$V_0(\overset{\circ}{\vec{k}}(-)\overset{\circ}{\vec{p}})=-\frac{2g_V^2}{Q^2}=\frac{-2g_V^2}{2m^2-2m\Delta_0}. \tag{2.19}$$

Expanding $\Psi_M^{S=0}(\overset{\circ}{\vec{p}})$ and V_0 in partial waves *

$$\Psi_M^{S=0}(\overset{\circ}{\vec{p}})=\sum_{\ell=0}^{\infty}(2\ell+1)i^\ell\frac{1}{|\overset{\circ}{\vec{p}}|}\phi_{M\ell}(\overset{\circ}{p})P_\ell(\vec{n}_{\overset{\circ}{p}}), \tag{2.20}$$

$$V^{S=0}(\overset{\circ}{\vec{k}}(-)\overset{\circ}{\vec{p}},\overset{\circ}{\vec{p}})=\sum_{\ell=0}^{\infty}\frac{(2\ell+1)}{4\pi}V_\ell(\overset{\circ}{k},\overset{\circ}{p})P_\ell(\cos\widehat{\vec{n}_{\overset{\circ}{p}}\vec{n}_{\overset{\circ}{k}}}) \tag{2.21}$$

we find for $\phi_{M\ell}(\overset{\circ}{p})$ an equation

$$2\overset{\circ}{p}_0(M-2\overset{\circ}{p}_0)\phi_{M\ell}(\overset{\circ}{p})=$$

$$=\frac{g_V^2}{(2\pi)^2}\int_0^\infty\frac{d\overset{\circ}{k}}{2\overset{\circ}{k}_0}(2\overset{\circ}{p}_0\overset{\circ}{k}_0-m^2)Q_\ell\left(\frac{\overset{\circ}{k}_0\overset{\circ}{p}_0-m^2}{\overset{\circ}{k}\overset{\circ}{p}}\right)\phi_{M\ell}(\overset{\circ}{k}). \tag{2.22}$$

In the nonrelativistic limit $\overset{\circ}{k}_0\to mc^2+\overset{\circ}{\vec{k}}{}^2/2m$

$$Q_\ell\left(\frac{k_0p_0-m^2c^4}{pkc^2}\right)\to Q_\ell\left(\frac{k^2+p^2}{2kp}\right) \tag{2.23}$$

and the equation transforms into the Schrödinger equation written in the momentum representation

$$\left(\frac{\vec{p}^2}{2m}+E_{bound}\right)\phi_{E_{bound},\ell}(p)=$$

$$=-\frac{g_V^2}{(2\pi)^2}\int_0^\infty dk\,Q_\ell\left(\frac{k^2+p^2}{2kp}\right)\phi_{E_{bound},\ell}(k). \tag{2.24}$$

It is easy to show that an equation analogous to (2.18) appears if we substitute into the main equation (1.2) the wave function, chosen in the form

$$\Psi_M^{\sigma_1\sigma_2}(\overset{\circ}{\vec{p}})=\bar{u}_q^{\sigma_1}(p_1)\gamma_5\bar{u}_{\bar{q}}^{\sigma_2}(p_2)\cdot\frac{\phi_M(\overset{\circ}{\vec{p}})}{2\overset{\circ}{p}_0},$$

$$\overset{\circ}{p}_0=\frac{\mathcal{P}\cdot p_1}{M}, \tag{2.25}$$

and take the quasipotential V in the same form (2.1) and (2.2). Really, in this case we have

$$(M-2\overset{\circ}{p}_0)\bar{u}_q(p_1)\gamma_5\bar{u}_{\bar{q}}(p_2)\phi_M(\overset{\circ}{\vec{p}})=\frac{1}{(2\pi)^3}\int\frac{d^3\overset{\circ}{\vec{k}}}{(2\overset{\circ}{k}_0)^2}\times \tag{2.26}$$

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*Due to ref. /12/ this expansion has an invariant nature for details (see also ref. /9,III/).

$$\times \bar u_q(p_1)\gamma^{\mu} u_q(k_1)\cdot u_{\bar q}(k_2)\gamma_{\mu} \bar u_q(p_2)\cdot V_0(\overset{\circ}{\vec k}(-)\overset{\circ}{\vec p}) \times$$
$$\times \bar u_q(k_1)\gamma_5 \bar u_{\bar q}(k_2)\phi_M(\overset{\circ}{\vec k}). \qquad (2.26)$$

Multiplying (2.26) by $u_{\bar q}(p_2)\gamma_5 u_q(p_1)$ from the left-hand side and performing a summation of polarizations, we arrive at

$$2(2\overset{\circ}{p}_0)^2(M-2\overset{\circ}{p}_0)\phi_M(\overset{\circ}{\vec p}) =$$
$$= \frac{1}{(2\pi)^3}\int \frac{d^3\overset{\circ}{\vec k}}{(2\overset{\circ}{k}_0)^2} A(p,k)\, V_0(\overset{\circ}{\vec k}(-)\overset{\circ}{\vec p})\cdot\phi_M(\overset{\circ}{\vec k}),$$

where

$$A(p,k) = Sp\{(\hat p_2 - m)\gamma_5(\hat p_1 + m)\gamma^{\mu}(\hat k_1 + m)\gamma_5(\hat p_2 - m)\gamma_{\mu}\} = \qquad (2.27)$$
$$= -4\{4(p_1p_2)(k_1k_2) - 4m^2(k_1k_2) + 4m^2(p_1p_2) -$$
$$- 2m^2(p_1+p_2)(k_1+k_2) + 4m^4\},$$
$$A(p,k) = 4\cdot 2\overset{\circ}{p}_0\cdot 2\overset{\circ}{k}_0(2\overset{\circ}{p}_0\overset{\circ}{k}_0 - m^2). \qquad (2.28)$$

The substitution of (2.28) into (2.26) leads us to equation (2.18).

Now let us see what asymptotic behaviour has the wave function, which satisfies equation (2.18). First we shall consider the spherically symmetric wave function $\phi_M(|\overset{\circ}{\vec k}|)$ alone and pass in (2.18) to new variables, the rapidities of quarks χ, by introducing the spherical coordinates on the mass-shell hyperboloid (1.5):

$$\overset{\circ}{p}_0 \equiv \Delta^{\circ}_{p,m\lambda\wp} = m\,ch\chi_p; \qquad \overset{\circ}{k}_0 \equiv \Delta^{\circ}_{k,m\lambda\wp} = m\,ch\chi_k$$
$$\overset{\circ}{\vec p} \equiv \vec\Delta_{p,m\lambda\wp} = \vec n_{\overset{\circ}{p}}\cdot m\,sh\chi_p; \qquad \overset{\circ}{\vec k} \equiv \vec\Delta_{k,m\lambda\wp} = \vec n_{\overset{\circ}{k}}\cdot m\,sh\chi_k, \qquad (2.29)$$
$$\vec n_p = \vec p/|\vec p|; \qquad \vec n_k = \vec k/|\vec k|.$$

After the integration over the polar angle in (2.18), we arrive at:

$$ch\chi_p\left(\frac{M}{2M} - ch\chi_p\right)\phi_M(\chi_p) =$$
$$= \frac{m^2}{2(2\pi)^3}\int_0^{\infty} d\chi_k\{ch(\chi_p-\chi_k) + ch(\chi_p+\chi_k) - 1\}\times$$
$$\times \int_{|\chi_p-\chi_k|}^{\chi_p+\chi_k} dy\, sh y\cdot V_0\left(2m\,sh\frac{y}{2}\right)\phi_M(\chi_k). \qquad (2.30)$$

Here we have introduced with the help of the relation

$$\Psi_M^{S=0}(|\vec\Delta_{p,m\lambda\wp}|) \equiv \Psi_M^{S=0}(m\,sh\chi_p) = \frac{4}{m\,sh\chi_p}\phi(\chi_p). \qquad (2.31)$$

a new wave function $\phi(\chi_p)$ and made use of the next parametrization of the square of momentum transfer

$$q^2 = (p_1-k_1)^2 = 2m^2 - 2m\Delta_0 = 2m^2 -$$
$$- 2m\sqrt{m^2 + (\overset{\circ}{\vec p}(-)\overset{\circ}{\vec k})^2} = \left(2m\,sh\frac{y}{2}\right)^2, \qquad (2.32)$$

where

$$ch y = ch\chi_p ch\chi_k - (\vec n_p\vec n_k)sh\chi_p\cdot sh\chi_k. \qquad (2.33)$$

In terms of these variables for the quasipotential of the one-photon exchange in electrodynamics, we find

$$V_0^{QED}\left(2m\,sh\frac{y}{2}\right) = -\frac{4\pi\alpha}{q^2} = -\frac{\pi\alpha}{m^2sh^2 y/2}, \qquad (2.34)$$

while for the quasipotential, which corresponds to the one gluon-exchange in QCD, we find

$$V_0^{QCD}\left(2m\,sh\frac{y}{2}\right) = -\frac{(4\pi)^2}{\beta_0 Q^2\ln Q^2/\Lambda^2} =$$
$$= -\frac{(2\pi)^2}{\beta_0 m^2 sh^2\frac{y}{2}\ln\left(\frac{2m}{\Lambda}sh\frac{y}{2}\right)}, \qquad (2.35)$$

where $\beta_0 = 11 - \frac{2}{3}n_f$ (n_f is the number of flavours).

Now, passing in the r.h.s. of (2.30) to the limit $|\vec\Delta_{p,m\lambda\wp}|\to\infty$ under the integral sign, we find the asymptotics of the wave function at $\chi_p\to\infty$

$$\phi_M(\chi_p) \simeq \frac{m^2 sh\chi_p V_0(2m\,sh\chi_p/2)}{(2\pi)^2\left(\frac{M}{2m} - ch\chi_p\right)}\int_0^{\infty} d\chi_k\cdot\chi_k\cdot\phi_M(\chi_k). \qquad (2.36)$$

From (2.36) we conclude that in the case of the "QED" quasipotential" (2.34), the asymptotics has the form:

$$\phi_M^{QED}(\chi_p) \simeq \frac{\alpha \cdot \mathrm{sh}\chi_p}{\pi(\mathrm{ch}\chi_p - \frac{M}{2m})} \cdot \frac{1}{(\mathrm{ch}\chi_p - 1)} \cdot \int_0^\infty d\chi_k \cdot \chi_k \cdot \phi^{QED}(\chi_k) \simeq$$

$$\simeq \mathrm{const}/\mathrm{ch}\chi_p , \qquad (2.37)$$

and in the case of the QCD quasipotential (2.35)

$$\phi_M^{QCD}(\chi_p) \simeq \frac{\mathrm{sh}\chi_p}{2\beta_0(\mathrm{ch}\chi_p - \frac{M}{2m})} \cdot \frac{1}{(\mathrm{ch}\chi_p - 1)} \times \qquad (2.38)$$

$$\times [\ln(\frac{2m}{\Lambda}\mathrm{sh}\frac{\chi_p}{2})]^{-1} \cdot \int_0^\infty d\chi_k \cdot \chi_k \, \phi_M^{QCD}(\chi_k) \simeq$$

$$\simeq \mathrm{const}/\chi_p \cdot \mathrm{ch}\chi_p .$$

3. FORMULATION OF AN EQUATION IN THE RELATIVISTIC CONFIGURATIONAL REPRESENTATION

Let us perform in equation (2.18) a transition to the relativistic configurational representation introduced for the first time in ref.[15] with the help of an expansion of the wave function $\Psi_M(\overset{\circ}{\vec{p}})$ on the Lorentz group

$$\Psi_M(\overset{\circ}{\vec{p}}) = \int d^3\vec{r}\, \xi^*(\overset{\circ}{\vec{p}}, \vec{r})\, \Psi_M(\vec{r}) , \qquad (3.1)$$

$$\Psi_M(\vec{r}) = \frac{1}{(2\pi)^3} \int \frac{d^3\overset{\circ}{\vec{p}}}{2\overset{\circ}{p}_0} \xi(\overset{\circ}{\vec{p}}, \vec{r})\, \Psi_M(\overset{\circ}{\vec{p}}) , \qquad (3.2)$$

where the functions

$$\xi(\overset{\circ}{\vec{p}}, \vec{r}) = [\frac{\overset{\circ}{p} - \overset{\circ}{\vec{p}}\vec{n}}{m}]^{-1-irm} ; \quad \vec{r} = r\vec{n}; \quad \vec{n}^2 = 1 \qquad (3.3)$$

realize the principal series of the unitary representations of SO (3.1) group[16] and compose an orthogonal and complete system of functions on the mass-shell hyperboloid (1.5). With the help of the relation[15]

$$\hat{H}_0 \xi(\overset{\circ}{\vec{p}}, \vec{r}) = 2(\overset{\circ}{p}_0)\, \xi(\overset{\circ}{\vec{p}}, \vec{r}) , \qquad (3.4)$$

where

$$\hat{H}_0 = 2m\,\mathrm{ch}(\frac{i}{m}\frac{\partial}{\partial r}) + \frac{2i}{r}\mathrm{sh}(\frac{i}{m}\frac{\partial}{\partial r}) - \frac{\Delta_{\theta,\phi}}{mr^2} e^{\frac{i}{m}\frac{\partial}{\partial r}} \qquad (3.5)$$

is the differential-difference (with step proportional to i/m) operator of the free Hamiltonian, equation (2.18) can be written in the form

$$\hat{H}_0(M - \hat{H}_0)\Psi_M^{S=0}(\vec{r}) = \{\hat{H}_0 \frac{V_0(r)}{2m^2}\hat{H}_0 - V_0(r)\}\Psi_M^{S=0}(\vec{r}). \qquad (3.6)$$

So, in the case of spin particles an interaction term $V_0(r)$ can enter an equation in a more complicated way than it takes place in a case of scalar particles.

Now we pass to the partial-wave expansion of the wave function

$$\Psi_M^{S=0}(\vec{r}) = \sum_{\ell=0}^{\infty} (2\ell+1)\, i^\ell \frac{1}{r} \phi_{M,\ell}^{S=0}(r)\, P_\ell(\vec{n}) . \qquad (3.7)$$

If we shall restrict our consideration to the $\ell = 0$ case, then for the wave function $\phi_{M,\ell=0}^{S=0}(r)$ a radial equation will have the form

$$\hat{H}_0^{rad}(M - \hat{H}_0^{rad})\, \phi_{M,\ell=0}^{S=0}(r) = \qquad (3.8)$$

$$= \{\hat{H}_0^{rad}\frac{V_0(r)}{2m^2}\hat{H}_0^{rad} - V_0(r)\}\phi_{M,\ell=0}^{S=0}(r) ,$$

where for $\ell = 0$

$$\hat{H}_{0,\ell=0}^{rad} = 2m\,\mathrm{ch}(\frac{i}{m}\frac{\partial}{\partial r}) . \qquad (3.9)$$

4. CONCLUSION

In the present paper we have obtained the covariant equation for the wave function of the bound state system, composed of quark and antiquark with spins 1/2, when this system has a total spin S=0. This equation describes a relative motion of two quarks in π-meson. We have studied the asymptotic behaviour of the wave function in the momentum representation at large values of the relative momentum. In the subsequent paper we shall study in detail the behaviour of the wave function for the case of confining potentials and the mass spectra of a bound state.

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REFERENCES

1. Salpeter E.E., Bethe H.A. Phys.Rev., 1951, v.84, No. 6, p. 1232-1248.
2. Logunov A.A., Tavkhelidze A.N. Nuovo Cim., 1963, v.29, No.2, p. 380-400.

3. Logunov A.A. et al. Theor.Math. Phys., 1971, v.6, No.2, p. 157-165.
4. Kadyshevsky V.G. ZhETF, 1968, v. 46, No.2, p. 654-662.
5. Kadyshevsky V.G. Nucl.Phys., 1968, v. B6, No.12, p.125-148
6. Matveev V.A., Muradyan R.M., Tavkhelidze A.N. JINR, E2-3498, Dubna, 1967.
7. Faustov R.N. Annals of Physics, 1973, v. 78, No.1, p. 176-189.
8. Kadyshevsky V.G., Mateev M.D. Nuovo Cim., 1968, v. 55A, No.2, p. 275-300.
9. Skachkov N.B. JINR, E2-81-294, E2-81-308, E2-81-399, Dubna 1981.
10. Savrin V.I., Skachkov N.B. Preprint TH 2822-CERN, Geneva, CERN, 1980; Lett.Nuovo Cim., 1980, v. 29, No. 4, p. 363-368.
11. Linkevich A.D., Savrin V.I., Skachkov N.B. JINR, E2-81-611, Dubna, 1981.
12. Shirokov Yu.M. ZhETF, 1958, v. 35, No.4, p. 1005-1013.
13. Kvinikhidze A.N., Stoyanov D.Ts. JINR, E2-5746, Dubna, 1971.
14. Shirokov Yu.M. ZhETF, 1951, v.21, No.6, p. 746-760; Cheshkov A.A., Shirokov Yu.M. ZhETF, 1963, v. 44, No. 6, p. 1982-1992.
15. Kadyshevsky V.G., Mir-Kasimov R.M., Skachkov N.B. Nuovo Cim., 1968, v. 55A, No.2, p. 233-257; Sov.Journ. Particles and Nuclear, 1972, v. 2, No.3, p. 635-678.
16. Shapiro I.S. Sov. Journ.Doklady 1956, v. 106, No.4, p. 647-649.

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Ковариантное трехмерное уравнение для волновой функции π-мезона в составной модели спинорных кварков

Получено ковариантное трехмерное уравнение для волновой функции псевдоскалярной частицы, составленной из двух кварков с равными массами и спинами 1/2. Исследована асимптотика решений этого уравнения в импульсном представлении в случае, когда взаимодействие между кварками осуществляется за счет обмена одним глюоном.

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Savrin V.I., Skachkov N.B., Tyumenkov G.Yu. E2-82-122

Covariant Three-Dimensional Equation for the Wave Function of π-Meson in the Composite Model of Spinor Quarks

A covariant three dimensional equation is derived for a wave function of a pseudoscalar particle, compoused of two equal mass quarks with spins 1/2. An asymptotics of the solution of this equation is found in the momentum representation in the case of quarks interaction chosen in a form of a one gluon exchange amplitude.

The investigation has been performed at the Laboratory of the Theoretical Physics, JINR.

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